A Conjectured Inequality Arising in Source-Channel Image Coding

Problem 04-001, *by* RAOUF HAMZAOUI (Fachbereich Informatik und Informationswissenschaft, Universität Konstanz, Konstanz, Germany).

Let $\mathcal{V} = \{v_1, \ldots, v_m\}$ be a set of positive integers such that $v_1 < \cdots < v_m$. For each $v_i \in \mathcal{V}$ $(i = 1, \ldots, m)$, we associate a real number $p(v_i)$ with $0 < p(v_1) < \cdots < p(v_m) < 1$.

Let N be a positive integer. For $R = (r_1, \ldots, r_N) \in \mathcal{V}^N$, define

$$P_i(R) = \begin{cases} p(r_1) & \text{for} \quad i = 0, \\ p(r_{i+1}) \prod_{j=1}^{i}(1 - p(r_j)) & \text{for} \quad i = 1, \ldots, N-1, \\ \prod_{j=1}^{N}(1 - p(r_j)) & \text{for} \quad i = N. \end{cases}$$

Let f be a nonincreasing convex function. Suppose that R^* maximizes over $\mathcal{V}^N$ the cost function

$$E_N[r](R) = \sum_{i=0}^{N} P_i(R) V_i(R), \tag{1}$$

where $V_0(R) = 0$ and $V_i(R) = \sum_{j=1}^{i} r_j$ for $i = 1, \ldots, N$. Suppose that T^* minimizes over $\mathcal{V}^N$ the cost function

$$E_N[d](R) = \sum_{i=0}^{N} P_i(R) f(V_i(R)). \tag{2}$$

Prove or disprove the following conjecture.

CONJECTURE **1.** $V_N(T^*) \leq V_N(R^*)$.

The problem stems from the field of source-channel image coding [1]. Suppose that N packets of data are successively transmitted over a binary symmetric channel. All packets have the same size, and each packet consists of a variable number of information bits and protection bits. When a packet contains v_i information bits it has probability $p(v_i)$ of not being correctly recovered by the receiver. To avoid error propagation, the receiver discards the first packet that cannot be correctly recovered and all the following ones. If $f(x)$ denotes the distortion (error between the original image and the decoded image) when x information bits are decoded, then the first cost function (1) is the expected number of correctly decoded bits, while the second cost function (2) is the expected distortion.

The conjecture simply states that the strategy that minimizes (2) should send fewer information bits than the one that maximizes (1). The conjecture was erroneously given as a proposition in [2].

REFERENCES

[1] V. Stanković, R. Hamzaoui, and D. Saupe, *Fast algorithm for rate-based optimal error protection of embedded codes*, IEEE Trans. Commun., 51 (2003), pp. 1788–1795.

[2] R. Hamzaoui, V. Stanković, and Z. Xiong, *Rate-based versus distortion-based optimal joint source-channel coding*, in Proceedings of the IEEE Data Compression Conference (DCC'02), J. A. Storer and M. Cohn, eds., Snowbird, UT, 2002, pp. 63–72.

Status. This problem is open. The proposer has a proof for the case $N = 1$.