A Characterization of Groups

Problem 04-002, *by* Torben Maack Bisgaard (Nandrupsvej 7 st. th., DK-2000 Frederiksberg, Denmark).

Let S be a semigroup which has an element g such that

(i) for each x in S there is some y in S such that $gy = x$;

(ii) for each x in S there is some y in S such that $yx = g$.

Show that S is a group.

Note. According to Clifford and Preston [1, p. 4], Dickson [2] showed the desired conclusion in the case that in (i) one can always take $y = x$ (i.e., g is a left identity); it is permitted to use this fact in the proof.

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REFERENCES

[1] A. H. Clifford and G. B. Preston, *The Algebraic Theory of Semigroups*, Vol. I, AMS, Providence, RI, 1961.

[2] L. E. Dickson, *Definitions of a group and a field by independent postulates*, Trans. Amer. Math. Soc., 6 (1905), pp. 198–204.

Status. The proposer has a solution.