Solution of Problem 04-002 *by* Michael Renardy (Virginia Polytechnic Institute and State University).

1. Left neutral element: There exists an element e_L such that $e_L x = x$ for every x.

   Proof. Define e_L to be a solution of $e_L g = g$ [property (ii)]. For given x, let y be such that $gy = x$ [property (i)]. We have $e_L gy = gy$, hence $e_L x = x$.

2. Left pseudoinverse: For every x, there exists a y such that $yxz = z$ for every z.

   Proof. Pick y such that $y(xg) = g$ [property (ii)]. Now pick u such that $gu = z$ [property (i)]. It follows that $yxz = yxgu = gu = z$.

3. Right neutral element: There exists an element e_R such that $xe_R = x$ for every x.

   Proof. Define e_R such that $ge_R = g$ [property (i)]. Next, let y be such that $yx = g$ [property (ii)]. It follows that $yxe_R = ge_R = g = yx$. Now let v be the left pseudoinverse of y from step 2 above. It follows that $xe_R = vyxe_R = vyx = x$.

4. Left inverse: For every x, there exists a y such that $yx = e_R$.

   Set $z = e_R$ in step 2 above and use step 3.

5. Equality of left and right neutral element: $e_L = e_R$ (in particular, this implies uniqueness of e_L and e_R). We henceforth use e to denote $e_L = e_R$.

   Proof. $e_L e_R = e_L = e_R$.

6. Right inverse: If $yx = e$, then $xy = e$.

   Proof. Let $yx = e$. Then $yxy = ey = y$. Now let z be the left inverse of y. It follows that $xy = zyxy = zy = e$.

Also solved by Said Amghibech (Sainte-Foy, Quebec, Canada) *and the proposer.*