Questions Concerning Entire Functions of Finite Order
with Positive Taylor Coefficients

Problem 04-004, *by* SLAVKO SIMIC (Mathematical Institute SANU, Belgrade, Serbia).

Let A_ρ be the class of transcendental entire functions of order ρ, $0 \leq \rho < \infty$, with positive Taylor coefficients. For $f \in A_\rho$ let

$$T(r) := \frac{f(r)f''(r)}{(f'(r))^2},$$

and

$$c_\rho^- := \min_{f \in A_\rho}\left(\limsup_{r \to \infty} T(r)\right); \qquad c_\rho^+ := \max_{f \in A_\rho}\left(\limsup_{r \to \infty} T(r)\right).$$

1. Prove that $c_\rho^- = 1$.

2. Is it true that $c_\rho^+ < \infty$, independently of order ρ? If so, what is the exact value of c_ρ^+?

Status. The proposer has a solution of the first problem. Question 2 is open.