Asymptotic Behavior of a Sum of Cosecants

Problem05-001, by HONGWEI CHEN (Christopher Newport University, Newport News, VA). Prove that

$$\sum_{k=1}^{n-1} \csc\left(\frac{k\pi}{n}\right) = \frac{2n}{\pi} \left(\ln n + \gamma - \ln\left(\frac{\pi}{2}\right)\right) + O(1), \qquad n \to \infty.$$

Remark. The exact sum

$$\sum_{k=1}^{n-1} \sin\left(\frac{k\pi}{n}\right) = \cot\left(\frac{\pi}{2n}\right)$$

is well known. This motivated the proposer to investigate the corresponding sum of cosecants.

Status. The proposer has a solution. Other solutions are welcome.