Another Figure Eight Volume Formula

Problem 06-001, *by* Jonathan Borwein[1] (Dalhousie University, Halifax, NS, Canada) and Marc Chamberland[2] (Grinnell College, Grinnell, IA).

There are various delightful and reasonably well-known formulas for V_8, the *figure-eight knot complement volume* in hyperbolic space—see [1]. A natural starting point is to take the volume as defined by the *Clausen function*:

$$V_8 := \sum_{n=1}^{\infty} \frac{2\,\sin(n\pi/3)}{n^2} = 2.0298832128193072500\ldots\,.$$

We obtain the following Euler sum representation for which we request a self-contained proof.

$$V_8 = \frac{4}{\sqrt{3}} \sum_{k=0}^{\infty} \left(-\frac{1}{3}\right)^k \frac{1+\frac{1}{3}+\cdots+\frac{1}{2k+1}}{2k+1}.$$

Status. The authors have a solution. Additional solutions are welcome.

REFERENCE

[1] J. M. Borwein and D. Bailey, *Mathematics by Experiment*, A K Peters, Ltd., Natick, MA, 2004.

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