Harmonic Functions with Cauchy–Riemann Boundary Data

Problem 06-004, *by* DAVID L. RUSSELL[1] (Virginia Tech, Blacksburg, VA).

Let Ω be a bounded, simply connected domain in the plane with smooth boundary. Let u, v be harmonic functions in Ω, of class C^1 up to the boundary, and let g be a continuous function on $\partial\Omega$. Let $n = (n_1, n_2)$ be the exterior unit normal, and let $t = (-n_2, n_1)$ be the unit tangent vector. Assume that u and v satisfy the boundary conditions

$$\frac{\partial u}{\partial n} - \frac{\partial v}{\partial t} = gn_1,$$

$$\frac{\partial u}{\partial t} + \frac{\partial v}{\partial n} = gn_2.$$

Prove that $g = 0$ and $u + iv$ is analytic.

Status. A solution is known. Other solutions are welcome.

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