

## Sum of the Reciprocals of Polygonal Numbers

*Problem 07-003, by HONGWEI CHEN<sup>1</sup> (Christopher Newport University).*

Let  $p_r(n)$  be the  $n$ th  $r$ -sided polygonal number. It is well known that

$$p_r(n) = \frac{1}{2}n[(r-2)n - (r-4)].$$

For  $r \geq 5$ , prove that

$$\sum_{n=1}^{\infty} \frac{1}{p_r(n)} = -\frac{2}{r-4} \left( \gamma + \Psi \left( \frac{2}{r-2} \right) \right),$$

where  $\gamma$  is the Euler's constant and  $\Psi$  is the digamma function which is defined by

$$\Psi(x) = \frac{\Gamma'(x)}{\Gamma(x)}.$$

In a recent issue of *The PiME Journal*, the editor proposed the problem of finding the sum of the reciprocals of the pentagonal numbers. This directed my attention to this subject.

*Status.* The proposer has a solution. Other solutions are welcome.

---

<sup>1</sup>E-mail: hchen@cnu.edu