A Log-Concave Sequence

Problem 07-005, *by* SLAVKO SIMIC[1] (Mathematical Institute SANU, Belgrade, Serbia).

A sequence of real numbers $\{c_n\}_{n\geq 0}$ with $c_0 = 1$ generates a sequence of polynomials $\{P_n(x)\}_{n\geq 0}$ defined by

$$P_n(x) = x^n + c_1 x^{n-1} + \cdots + c_n.$$

Let A_n denote the set of zeros of $P_n(x)$.

(a) Give a direct proof that if $m \geq 1$ and the set A_m consists of real numbers only, then for $a \in A_m$ the sequence $\{P_n(a)\}_{n=0}^{m}$ is log-concave, i.e.,

$$P_n^2(a) \geq P_{n-1}(a)P_{n+1}(a), \qquad n = 1, 2, \ldots, m-1.$$

Note. The truth of this result is known from other considerations. A relatively simple, self-contained proof is desired.

(b) Under what conditions is the sequence $\{P_n(x)\}_{n=0}^{m}$ log-concave for

$$x \in [\min A_m, \max A_m]?$$

Status. This problem is open.

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