**A Cosine Integral Series**

*Problem* 08-001, *by* Ovidiu Furdui[1] (The University of Toledo).

The **cosine integral** is defined by

$$\mathrm{Ci}(x) = -\int_x^\infty \frac{\cos t}{t}\,dt.$$

Prove that if $0 < a \le 2\pi$, then

$$\sum_{n=1}^\infty \frac{\mathrm{Ci}(an)}{n^2} = -\frac{\pi^2}{6}\ln\frac{2\pi}{a} + 2\pi^2 \ln A - \frac{\pi a}{2} + \frac{a^2}{8}$$

$$= \frac{\pi^2}{6}\ln a - \zeta'(2) + \frac{\pi^2\gamma}{6} - \frac{\pi a}{2} + \frac{a^2}{8},$$

where

$$A = \exp\left[-\frac{\zeta'(2)}{2\pi^2} + \frac{\log(2\pi)}{12} + \frac{\gamma}{12}\right] = 1.28242712\ldots$$

is the **Glaisher–Kinkelin constant**.

**Remark.** The problem is motivated by the following well-known series:

$$\sum_{n=1}^\infty \frac{\cos \beta n}{n^2} = \frac{\pi^2}{6} - \frac{\pi\beta}{2} + \frac{\beta^2}{4}, \qquad 0 \le \beta < 2\pi.$$

It is natural to ask whether a similar formula holds for the cosine integral function.

*Editorial note.* Some numerical errors in the initial posting of this problem have been corrected.

*Status.* The proposer and the editors have solutions. Additional solutions are welcome.

[1]E-mail: Ovidiu.Furdui@utoledo.edu, ofurdui@yahoo.com