A Complete Monotonicity Conjecture

Problem 09-001, *by* Yaming Yu[1] (Department of Statistics, University of California, Irvine, CA).

Prove or disprove the following conjecture.

Conjecture. *Let* $bi(k; n, p) = \binom{n}{k} p^k (1-p)^{n-k}$ *denote the binomial mass function (pmf), and let* $po(k; \lambda) = \lambda^k e^{-\lambda}/k!$ *denote the Poisson pmf. For fixed* $\lambda > 0$*, define*

$$D_n = \sum_{k=0}^{n} bi(k; n, \lambda/n) \log \frac{bi(k; n, \lambda/n)}{po(k; \lambda)}, \qquad n > \lambda.$$

Then D_n *is a completely monotonic function of* n.

This problem arises in a study of monotonicity in classical limit theorems. Note that D_n is the relative entropy, or Kullback–Leibler divergence, between the binomial distribution $Bi(n, \lambda/n)$ and the Poisson distribution $Po(\lambda)$. It is known that D_n decreases as n increases (and $\lim_{n\to\infty} D_n = 0$). I cannot even prove that D_n is convex, let alone completely monotonic [1].

REFERENCE

[1] Y. Yu, *Convergence and monotonicity problems in an information theoretic law of small numbers*, Preprint, arXiv:0810.5203 (2009).

Status. This problem is open.

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