

## A Convexity Question in Matrix Analysis

*Problem 09-002, by DAVID L. RUSSELL<sup>1</sup> (Virginia Tech, Blacksburg, VA).*

Let  $\mathbf{B}$  be an  $n \times m$  real matrix of maximal rank  $m \leq n$ . Let  $\mathbf{\Lambda} = \text{diag}(\lambda_1, \lambda_2, \dots, \lambda_n)$ . Prove that the scalar-valued function

$$\mathcal{C}(\mathbf{\Lambda}) = \text{Tr}((\mathbf{B}^* \mathbf{\Lambda} \mathbf{B})^{-2})$$

is convex for  $\mathbf{\Lambda} = (\lambda_1, \lambda_2, \dots, \lambda_n) \in (\mathbb{R}^+)^n$ , the open positive orthant in  $\mathbb{R}^n$ .

*Status.* A solution is known. Other solutions are welcome.

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