Two Conjectured Bernoulli Number Identities

Problem 11-001, *by* Huizeng Qin[1] (College of Science, ShanDong University of Technology, Zibo, ShanDong 255049, China).

1) Prove or disprove that for all $m > 4$,

$$\sum_{k=1}^{\left[\frac{m-1}{2}\right]} \left(\frac{1}{m-2k} + H_{2k} \binom{m}{2k} \right) B_{2k} = \frac{m}{2} - \frac{1}{m} - \frac{1}{2}\frac{1}{m-1}, \tag{1}$$

where $H_n = 1 + 1/2 + \cdots + 1/n$ is the nth harmonic number and $B_n = \lim_{x\to 0} \frac{d^n}{dx^n} \frac{x}{e^x - 1}$ is the nth Bernoulli number.

2) Prove or disprove that for all $m > 4$, and $k = 1, 2, \ldots, m-3$,

$$\begin{aligned} &\sum_{j=1}^{\left[\frac{m-k-1}{2}\right]} \left(\frac{1}{(m-2j-k)(k+2j)} \binom{k+2j}{k} + \frac{1}{m} \binom{m}{k} \binom{m-k}{2j} H_{2j+k-1} \right) B_{2j} \\ &\quad = \frac{1}{k^2} \binom{m-1}{k-1} - \frac{1}{(m-k)k} - \frac{1}{2(m-1-k)} + \frac{m-k-2}{2(m-k)} \binom{m-1}{k} H_k. \end{aligned} \tag{2}$$

Status. This problem is open.

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