For $p > 2$ Hausdorff–Young Does Not Hold for the Laplace Transform on $L^p(0,\infty)$

Problem 11-007, *by* Anatoli Merzon[1] (Universidad Michoacána de San Nicolás de Hidalgo, Morelia, Michoacán, Mexico) and Sergey Sadov[2] (Department of Mathematics and Statistics, Memorial University of Newfoundland, St. John's, Canada)

Given $p > 2$, provide an example of $f \in L^p(0,\infty)$ such that the Laplace transform

$$F(s) = \int_0^\infty f(x)\, e^{-sx}\, dx$$

does not belong to $L^q(0,\infty)$, where $1/p + 1/q = 1$.

Comment. A counterexample to the Hausdorff–Young-type statement with $p > 2$ for the Fourier transform is found, for instance, in Titchmarsch's *Introduction to the Theory of Fourier Integrals*, § 4.11. The unboundedness of the Laplace transform acting from L^p to L^q follows from Theorem 3.2 in S. Bloom, *Hardy integral estimates for the Laplace transform*, Proc. Amer. Math. Soc., 116 (1992), no. 2, 417–426. (That paper gives a fairly complete answer about boundedness of the Laplace transform between pairs of L^p spaces with general weights on $(0,\infty)$.) However, we have not seen in the literature a concrete example as is sought in this problem.

Status. The proposers have a solution. Others are welcome.

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