An Integral Identity from Physics

Problem 12-002 *by* Z. K. Silagadze[1] (Budker Institute of Nuclear Physics and Novosibirsk State University, Novosibirsk, Russia).

Prove

$$\frac{\pi^2}{6} = \zeta(2) = \int_0^\infty \frac{dx}{x} \int_0^x \frac{dy}{y} \{\cos(x-y) - \cos x\}.$$

The proposer came across this result through the remarkable Bloch–Rojo correspondence that exists between the quantum mechanical Landau–Zener problem and the classical problem of a 2-sphere rolling on a Cornu spiral.

Status. The proposer has a solution. Other solutions are welcome.

REFERENCES

[1] A. M. Bloch and A. G. Rojo, *Kinematics of the rolling sphere and quantum spin*, Commun. Inf. Syst., 10 (2010), pp. 221–238.

[2] A. M. Bloch and A. G. Rojo, *The rolling sphere, the quantum spin, and a simple view of the Landau-Zener problem*, Am. J. Phys., 20 (2011), pp. 1011–1022.

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