**A Derivative Conjecture**

*Problem* 12-003 *by* Fuchang Gao[1] (University of Idaho, Moscow, ID), Lixing Han[2] (University of Michigan-Flint, Flint, MI), and Kenneth Schilling[3] (University of Michigan-Flint, Flint, MI).

For a given positive integer $p$, let $t$ be a primitive $p$th root of unity and let $a = \exp(te^{-t})$. Define

$$f_1(z) = a^z, \qquad f_{k+1}(z) = a^{f_k(z)}, \quad k = 1, 2, \ldots, p-1.$$

It is easy to check that $f_p(e^t) = e^t$ and $f_p'(e^t) = 1$. In [1], it is proved that

$$f_p^{(k)}(e^t) = 0 \quad \text{for} \quad k = 2, 3, \ldots, p,$$

where $f_p^{(k)}$ denotes the $k$th derivative of $f_p$. Prove or disprove the following conjecture.

Conjecture. $f_p^{(p+1)}(e^t) \neq 0$ *for all* $p \geq 1$.

*Remark.* An affirmative result for the conjecture will lead to a precise characterization of the rate of convergence, provided it is sublinear. For details, see [1].

*Status.* This problem is open.

## REFERENCES

[1] F. Gao, L. Han, and K. Schilling, *On the rate of convergence of iterated exponentials*, J. Appl. Math. Comput., 39 (2012), pp. 89–96.

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