On a Circular Membrane Elastically Supported in a Part of Its Boundary

Problem 99-001, *by* LUCIO R. BERRONE (CONICET, Universidad Nacional de Rosario, Argentina (berrone@unrctu.edu.ar)).

(i) For the unit circle $\Omega = B_1(0)$ with boundary $\partial\Omega$ split in two finite families of arcs Γ_0 and Γ_1, the solution $u[\Gamma_1]$ to the boundary value problem

$$(1) \qquad \begin{cases} \Delta u(x) = 0, & x \text{ in } \Omega, \\ u(x) = 0, & x \text{ on } \Gamma_0, \\ (\partial u/\partial r)(x) = \alpha(u_0 - u(x)), & x \text{ on } \Gamma_1, \end{cases}$$

with $\alpha, u_0 > 0$, models the position of an elastic membrane which is fixed at 0 on Γ_0 and elastically supported on the remaining arcs Γ_1 (of course, $\Gamma_0 \cup \Gamma_1 = \partial\Omega$, $\Gamma_0 \cap \Gamma_1 = \varnothing$). If the family Γ_1 varies on $\partial\Omega$ so as to maintain its total measure equal to a constant C $(0 < C < 2\pi)$, prove that the height at $x = 0$ of the membrane is a maximum when Γ_1 is a single arc of length C. In other words, show that

$$(2) \qquad \max\{u[\Gamma_1](0) : |\Gamma_1| = C\} = u[\Gamma_1^*](0),$$

where $\Gamma_1^* \subset \partial\Omega$ is an arc of length C.

(ii) Is a property similar to (2) valid when the height at $x = 0$ is replaced by the maximum height reached by the membrane, $\max_{x \in \overline{\Omega}} u[\Gamma_1](x)$?

Status. The proposer has a solution for part (i). For part (ii), he conjectures that

$$\max\{\|u[\Gamma_1]\|_\infty : |\Gamma_1| = C\} = \|u[\Gamma_1^*]\|_\infty$$

holds when $\Gamma_1^* \subset \partial\Omega$ is an arc of length C.