IP1

How Applied Algebraic Geometry is Useful in Pure Mathematics

For historical reasons, the culture of pure algebraic geometry has often been quite distant from applications, and "applied" methods. While there are some good reasons for the distinction between the pure and applied side of the subject, this division is, happily, gradually eroding. I will describe some examples of where in theoretical advances have been built on insights and experiences coming from the more applied side of the subject. For this reason, it is worth our time to learn to talk to people in different parts of the subjects, even if the cultural and linguistical differences sometimes make it challenging.

Ravi Vakil Department of Mathematics Stanford University vakil@math.stanford.edu

IP2

A Tale of Two Theorems

I will explain and draw connections between the following two theorems: (1) Hilbert's theorem on nonnegative polynomials and sums of squares, and (2) Classification of varieties of minimal degree by Del Pezzo and Bertini. This will result in the classification of all varieties on which nonnegative polynomials are equal to sums of squares. Along the way I will provide an introduction to Convex Algebraic Geometry. The talk is based on joint work with Greg Smith and Mauricio Velasco.

Greg Blekherman Georgia Institute of Technology greg@math.gatech.edu

IP3

On k-apart Configuration Spaces

k-apart - or no-k-equal configuration spaces - are formed by tuples of points in a topological space with no more than k coinciding. They appeared as a model problem in theoretical computer sciences, and are very useful in other applications, such as motion planning in robotics. I will survey some old and new results in the area.

Yuliy Baryshnikov

University of Illinois at UrbanaChampaign Department of Mathematics ymb@uiuc.edu

IP4 Numerics and Algebraic Geometry

Numerical methods have increasingly proven to be both helpful and necessary in Algebraic Geometry. Conversely geometrical and algebraic tools have led to new algorithms providing advances in more applied areas such as engineering and medical science. We will present a few examples showing this fruitful interplay. Classical theory from algebraic geometry can be used in Kinematics. Established tools form topology apply to give a numerical cell-decomposition of solution sets. Numerical methods, on the other hand, are essential tools for efficient algorithms to compute invariants of algebraic varieties, like Chern classes or the Euler characteristics. The talk is based on joint work with Besana, Eklund, Hauenstein, Peterson, Sommese and Wampler.

Sandra Di Rocco Department of Mathematics KTH dirocco@math.kth.se

IP5

Algebraic Geometry in System Biology

Systems biology aims to understand complex systems and the mechanisms that are responsible for specific behaviors, such as multi-stationarity or oscillation. Typical mathematical models of biological systems produce polynomial systems of equations. In recent years tools from algebraic geometry are increasingly being applied to understand such polynomial systems and extract information that are relevant for the design of experiments/systems and the analysis of experimental data. In this talk I will review recent results, and discuss some of the challenges we are facing.

Carsten Wiuf

 $\label{eq:constraint} \begin{array}{l} \text{University of Copenhagen}\\ wiuf@math.ku.dk \end{array}$

IP6

Cluster Algebra and Complex Volume of Knots

The cluster algebra was introduced by Fomin and Zelevinsky around 2000. The characteristic operation in the algebra called 'mutation' is related to various notions in mathematics and mathematical physics. In this talk I review a basics of the cluster algebra, and introduce its application to study the complex volume, (hyperbolic volume) + i (Chern-Simons invariant), of knot complements in S^3 . We formulate the ideal tetrahedral decomposition of hyperbolic 3-manifolds in terms of the cluster algebra, where a mutation produces an ideal tetrahedron. This talk is based on joint work with Kazuhiro Hikami (Kyushu University).

<u>Rei Inoue</u> Chiba University reiiy@math.s.chiba-u.ac.jp

IP7

Speeding up Lattice Reduction with Numerical Linear Algebra Techniques

A lattice is the set of all integer linear combinations of some linearly independent vectors. Visually, it is an infinite grid of regularly spaced points. Lattices have many applications in computer science. For example, they frequently appear in computer algebra (e.g., to factor rational polynomials), and in cryptography (both to break and design cryptographic protocols). The LLL algorithm, named after its authors Arjen Lenstra, Hendriz Lenstra and Lszl Lovsz, enables the computation of a good representation, or basis, of a given lattice: This representation provides decent intrinsic information on the lattice under scope. Numerous applications were found right after the discovery of the LLL algorithm, which motivated the search of algorithmic improvements. Today, the most efficient approach relies, internally, on low precision floating-point computations, leading to a numeric-algebraic hybrid algorithm. In this talk, I will first give an introduction to lattices, and then describe the hybrid numeric-algebraic approach underlying the modern variants of the LLL algorithm. This talk relies on joint works with Xiao-Wen Chang, Phong

Nguyen, Andrew Novocin, Ivan Morel and Gilles Villard

<u>Damien Stehlé</u> École Normale Supérieure de Lyon damien.stehle@ens-lyon.fr

IP8

Multivariate Polynomial Interpolation provides Surprising Combinatorial Insights: Zonotopal Algebra and Beyond

I will survey recent developments connecting multivariate polynomial interpolation, special polynomial ideals, geometry and combinatorics of hyperplane arrangements, vector partition functions and matroid theory. These connections originated in the theory of multivariate splines and led to the construction of so-called zonotopal algebra(s), which in turn shed light on enumerative problems related to graphs and matroids. However, many interesting open questions remain.

Olga Holtz

University of California, Berkeley Technische Universitaet Berlin holtz@math.tu-berlin.de

$\mathbf{CP1}$

Support Function Based Description of Topology and Approximation of Real Algebraic Curves

The support function representation can considerable simplify each of main phases of the problem of describing the topology of real algebraic curves (the determination of critical points and the reconstruction of their connectivity). Critical points of each type can be described as the solutions of a system of equations and their connectivity can be determined in a geometrical way. This representation allow us to find an approximation which also approximate the curvature, specially preserves cusps.

<u>Eva Cernohorská</u>

Faculty of Mathematics and Physics Charles University in Prague cernohorska.eva@gmail.com

Zbynek Sir Charles University in Prague Czech Republic zbynek.sir@gmail.com

CP1

A Dynamical System Which Produces Mutually Unbiased Bases and An Application of Persistent Homology

We present a novel algorithm, thought of as a discrete-time dynamical system on a space of unitary matrices with fixed dimension, which (often) converges to a (numerically) mutually unbiased basis. This algorithm serves as an efficient method to sample elements from the space of mutually unbiased bases in a fixed dimension. The persistent homology of these sample sets is computed to extract course topological features of the underlying spaces from which the samples are drawn.

<u>Francis C. Motta</u> Colorado State University motta@math.colostate.edu

CP1

Free Tilings on Genus-3 Surfaces and Resulting Crystalline Patterns

It is well established that crystalline 3-periodic nets can be systematically catalogued using Delaney-Dress tiling theory. One approach is to enumerate two-dimensional hyperbolic (\mathbf{H}^2) tilings then project those patterns into three-dimensional Euclidean space (\mathbf{E}^3) via triply periodic minimal surfaces (see epinet.anu.edu.au). We extend this to investigate 3d structures that emerge from the systematic enumeration of "free tilings' of \mathbf{H}^2 . These hyperbolic patterns consist of unbounded tiles with translational symmetry. The results in 3d are novel multicomponent interwoven periodic nets and filaments. Some of the simpler examples of these structures are known to chemists as highly porous metal-organic frameworks and rod packings.

Vanessa Robins

Australian National University Vanessa.Robins@anu.edu.au

Myfanwy Evans Universitaet Erlangen-Nuernberg myfanwy.e.evans@physik.uni-erlangen.de

Stuart Ramsden, Stephen Hyde RSPhysSE Australian National University stuart.ramsden@anu.edu.au, stephen.hyde@anu.edu.au

CP1

Construction of Lorentz-Conformal Coordinate Transformations

We use solutions of the wave equation to construct Lorentzorthogonal coordinate transformations of the plane as well as triply-orthogonal coordinate systems in Lorentz space. This construction may be translated to an approach in which the initial data for constructing such a system consists of a spectral curve, which may be singular and reducible. Such systems have applications to systems of differential equations with Lorentzian symmetries.

Patrick Shipman Department of Mathematics Colorado State University shipman@math.colostate.edu

CP1

Geometrically Minimal Realizations for Linear Control Systems over Boolean Semiring

A problem of description of all (minimal) realizations of given i-o map in a category of systems over semiring with free state semimodule is studied. Our approach is based on the study of morphisms in the category of realizations. We have classified all reductions realizable in the category of boolean systems with free states and studied some reductions in category of free systems, arising from the non-free case. Realizations for various classes of i-o maps are studied.

<u>Oleg O. Vasil'ev</u> <u>Institute of Control Sciences RAS</u> ovasilev@gmail.com

$\mathbf{CP2}$

Elegant Expressions and Formulae for Riemann Zeta, Dirichlet Beta, Euler Numbers and Other Mathematical Functions

New techniques are being developed to derive simple formulae for calculating the values of Riemann zeta, Dirichlet beta, Euler numbers and other special functions. These techniques may be used to provide simple proofs of the values of zeta(2n) and beta(2n+1). A new concept for deriving the values of Euler numbers is also discovered. This new concept is being used to establish some new identities. We attempt to find a closed form expression for $\zeta(2n+1)$ and discover a most intriguing result.

<u>Michael A. Idowu</u> University of Abertay Dundee, United Kingdom m.idowu@abertay.ac.uk

$\mathbf{CP2}$

List Decoding of Repeated Codes

We present a decoding algorithm for hard-decision decoding of repeated codes, based on the soft-decision algorithm of Koetter and Vardy. Lower bounds for decoding performance are given for best case, worst case and expected case. Computer simulations show that the performance is somewhat better than these bounds, particularly for higher rates. We compare performance and decoding time of a repeated Reed-Solomon code and standard Reed-Solomon code (over a larger field). The decoding performance is relatively close, but the repeated code requires much less time.

<u>Michael E. O'Sullivan</u> San Diego State University mosulliv@mail.sdsu.edu

Fernando Hernando Universitat Jaume carrillf@mat.uj.es

Diego Ruano Aalborg University diego@math.aau.dk

CP2

Doubly Adapted Bases for the Symmetric Group

Adapted bases play an important computational role in many applications of the representation theory of finite groups. In this talk, I will describe an interesting "doubly adapted" (with respect to the usual left action and right action) basis for the regular representation of the symmetric group. I will then explain why we think such bases might be the key to a new approach to creating fast Fourier transforms for finite groups.

Michael E. Orrison Harvey Mudd College orrison@hmc.edu

Michael Hansen N/A mhansen@gmail.com

Masanori Koyama University of Wisconsin-Madison koyama@math.wisc.edu

MS1

Identifiability and Parameter Estimation in Modeling Disease Dynamics

Connecting differential equation models with data to yield predictive results requires a variety of parameter estimation, identifiability, and uncertainty quantification techniques. Identifiability analysis addresses the question of whether it is possible to uniquely recover the model parameters from a given set of data. In this talk, I will discuss some recent work using and developing identifiability methods based on tools from computational differential algebra and systems theory, and present several clinical and public health applications to problems in human disease, including cholera and other infectious disease transmission processes.

Marisa Eisenberg University of Michigan marisae@umich.edu

$\mathbf{MS1}$

Identifiability of Mechanical Systems in Cardiovascular Modeling

Cardiovascular (CV) modeling of the blood pressure regulation system requires mathematical description of the viscoelastic mechanical properties of the arterial wall as well as the coupling of the nerve endings of various receptors (e.g. baroreceptors) to the wall. Those mechanical models usually involve springs (elastic elements) and dash-pots (viscous elements) in various configuration, which can be described using algebraic ODEs. An essential step in solving the inverse problem i.e. estimating model's parameters is to establish whether a model is structurally identifiable. The importance of this property is that it guarantees that for the noise-free observations and perfect model its unknown parameters can be uniquely estimated from the input/output experiments. In this talk we consider the structural identifiability problem related to the important class of mechanical models used in the CV modeling. We also show how algebraic methods can be used to design dynamically "optimal" model.

<u>Adam Mahdi</u> North Carolina State University amahdi@ncsu.edu

Nicolette Meshkat University of California, Los Angeles nicolette.meshkat@gmail.com

Seth Sullivant North Carolina State University smsulli2@ncsu.edu

MS1

Identifiable Reparameterizations of Linear Ode Systems

Identifiability concerns finding which unknown parameters of a model can be quantified from given input-output data. Many linear ODE models, used primarily in Systems Biology, are unidentifiable, which means that parameters can take on an infinite number of values and yet yield the same input-output data. We study a particular class of unidentifiable models and find conditions to obtain identifiable reparameterizations of these models. In particular, we use a graph-theoretic approach to analyze the models and show that graphs with certain properties allow a monomial scaling reparameterization over identifiable functions of the parameters.

Nicolette Meshkat

University of California, Los Angeles nicolette.meshkat@gmail.com

Seth Sullivant North Carolina State University smsulli2@ncsu.edu

$\mathbf{MS1}$

Differential Algebra Techniques for Identifiability of Biological Systems

The aim of this talk is to stress the role of differential algebra techniques for checking identifiability of dynamic models described by nonlinear ordinary differential equations, as those arising in chemical kinetics and biological systems. Identifiability is a well-known prerequisite for the well-posedness of model parameter estimation. The proposed methodology, which applies to systems of rational ODE's, is based on computer algebra algorithm to compute a Gröbner basis of certain families of parameter-dependent polynomials which can be extracted from the model by Ritt algorithm.

Maria Pia Saccomani University of Padova pia@dei.unipd.it

$\mathbf{MS2}$

Utilising New CAD Developments for Simplification in Computer Algebra

Formulae involving inverse functions (e.g. square root) may not universally hold when restricting to single-valued functions (e.g. the positive root). This may lead computer algebra systems to be overly cautious. One solution is to construct CADs according to the branch cuts of the functions, checking the formula in each cell. We report on recent progress on the calculation of cuts and implementation of this approach utilising new theory (truth table invariant CAD).

Matthew England, Russell Bradford, James Davenport, David J. Wilson University of Bath

$\mathbf{MS2}$

Automatic Proofs of Transcendental Function Inequalities and Their Applications

We shall present an overview of some recent applications of real algebraic geometry to automatic proofs of transcendental function inequalities. In the process, we shall describe how these techniques, as implemented in the theorem proving tools MetiTarski and Z3, are being used in the formal verification of real-world cyber-physical systems, e.g., to verify the safety of geometric maneuvers at the heart of collision avoidance algorithms in next-generation autonomous (self-driving) cars.

<u>Grant O. Passmore</u> Carnegie Mellon University University of Cambridge grant.passmore@cl.cam.ac.uk

$\mathbf{MS2}$

Beyond Equational Constraints in CAD

We discuss the new concept of truth table invariant CADs (TTICADs). These have cells with invariant truth values for a list of unquantified Tarski formulae, getting closer to the minimal information required for applications. These are constructed by extending the theory of equational constraints so it applies for each clause, even if there is no overall equational constraint. We also consider how best to formulate problems for this and other CAD algorithms.

Russell Bradford, James Davenport, Matthew England, <u>David J. Wilson</u> University of Bath rjb@cs.bath.ac.uk, j.h.davenport@bath.ac.uk, M.England@bath.ac.uk, D.J.Wilson@bath.ac.uk

MS3

Random Points on Curves in \mathbb{R}^n with Application to Parameterizing QSIC

A method for finding a random point on a curve in \mathbb{R}^n will be described. Once a point is known the irreducible component containing this point can be found. Fractional linear transformations can then be applied which will often give the component in a simple form allowing parameterization. As an application a completely numerical method for finding parameterizations of Quadric surface intersection curves (QSIC) in \mathbb{R}^3 is given that does not require looking at many separate cases.

Barry H. Dayton

Northeastern Illinois University b-dayton@neiu.edu

MS3

Computing H-Bases to Precondition Polynomial Systems for Homotopy Continuation

When solving polynomial systems using homotopy continuation, it is sometimes the case that we have a large number of paths going to ∞ . We want to remove the pieces of the system that tend to ∞ in order to get a smaller number of paths with better conditioning. We present an algorithm to compute an H-basis for our polynomial system in order to remove the pieces at ∞ . Examples are given to show the removal of the infinite paths in order to simplify the homotopy continuation run.

<u>Steven L. Ihde</u> Colorado State University ihde@math.colostate.edu

Daniel J. Bates Colorado State University Department of Mathematics bates@math.colostate.edu

Jonathan Hauenstein

North Carolina State University jdhauens@ncsu.edu

MS3

A Numerical Algorithm for the Topological Euler Characteristic of Algebraic Varieties

Recent work on numerical computation of degrees of Chern and Segre classes can be used to numerically compute the topological Euler characteristic of both non-singular and singular algebraic varieties. I will explain the idea behind the algorithm and its motivation from algebraic statistics

<u>Christine Jost</u> Stockholm University jost@math.su.se

MS3

Non-convex Optimization and Numerical Homotopies

This talk will consider a couple of problems in (semidefinite) optimization and will indicate how numerical algebraic geometry can be used to help in their solution.

Chris Peterson

Colorado State University peterson@math.colostate.edu

MS4

The Number of Nonsimple Principally Polarized Abelian Surfaces over a Finite Field

There are approximately $2q^3$ principally polarized abelian surfaces over a finite field \mathbf{F}_q . We show that the number of these surfaces that are nonsimple is bounded above by $cq^{2.5}(\log q)^d$ for some constants c > 0 and d, and is bounded below by an expression of the same form. This suggests a heuristic estimate for the number of primes p < x for which a given Jacobian over \mathbf{Q} has nonsimple reduction; we present evidence in support of this heuristic.

<u>Everett W. Howe</u> Center for Communications Research however@alumni.caltech.edu

Jeff Achter Colorado State University achter@math.colostate.edu

$\mathbf{MS4}$

Computing Discrete Logarithms in the Jacobian of High-Genus Hyperelliptic Curves and Applications

We describe novel and improved versions of index-calculus algorithms for solving discrete logarithm problems in Jacobians of medium-genus to high-genus hyperelliptic curves. One important improvement is based on the effective adaptation of sieving techniques known from the number field sieve, the function field sieve, and the arithmetic of number fields to the curve setting. We will also discuss parallelization techniques and their realization in hardware, such as graphic cards. Our new algorithms are applied to concrete problem instances arising from the Weil descent attack methodology for solving the elliptic curve discrete logarithm problem, demonstrating significant improvements in practice. This is joint work with Mark D. Velichka and Michael J. Jacobson.

<u>Andreas Stein</u> Carl von Ossietzky Universität Oldenburg andreas.stein1@uni-oldenburg.de

MS4

Vertical Brauer Groups and Del Pezzo Surfaces of Degree 4

Del Pezzo surfaces X of degree 4 are smooth intersections of two quadrics in four-dimensional projective space. They are some of the simplest surfaces for which there can be cohomological obstructions to the existence of rational points, mediated by the Brauer group Br X. I will explain how to construct, for every non-constant element $\mathcal{A} \in \operatorname{Br} \mathcal{X}$, a rational genus-one fibration $X \to P^1$ such that \mathcal{A} is "vertical' for this map. This implies, for example, that if there is an obstruction to the existence of a point on X arising from A, then there is a genus-one fibration $X \to P^1$ where none of the fibers are locally soluble, giving a geometric way of "seeing' a Brauer-Manin obstruction. The construction gives a fast, practical algorithm for computing the Brauer group of X. Conjecturally, this gives a mechanical way of testing for the existence of rational points on X.

Tony Varilly-Alvarado Rice University av15@rice.edu

Bianca Viray Brown University bviray@math.brown.edu

MS4

2-torsion Brauer Classes on Surfaces with Hyperelliptic Fibrations

Using descent techniques and the purity theorem, we give a method to compute the representatives of all 2-torsion classes in the Brauer group of a surface with a hyperelliptic fibration. We will describe the method in the example of a K3 surface which is a double cover of a quadric surface in P^3 .

Brendan Creutz University of Sydney brendan.creutz@sydney.edu.au

Bianca Viray Brown University bviray@math.brown.edu

$\mathbf{MS5}$

Arithmetic Codices and Applications to Cryptography

In this talk we present the notion of arithmetic codices. These are combinatorial objects which generalize some notions studied in the areas of information-theoretically secure cryptography and algebraic complexity. Known constructions of "asymptotically good' families of codices use towers of function fields with many rational places. Furthermore, we define and study another parameter of a tower, that we call torsion limit, which also plays an important role in the asymptotics of codices.

Ignacio Cascudo Centrum Wiskunde & Informatica i.cascudo@cwi.nl

Ronald Cramer CWI Amsterdam and Leiden University, The Netherlands cramer@cwi.nl

Chaoping Xing NTU Singapore xingcp@ntu.edu.sg

MS5

On Retrieving a Representation of An Algebraic Geometry Code

Code-based cryptography is an interesting alternative to classic number-theory PKC. Some families of codes have been proposed for these cryptosystems such as algebraic geometry codes. For the so called very strong algebraic geometry codes $\mathcal{C} = \mathcal{C}_{\mathcal{L}}(\mathcal{X}, \mathcal{P}, \mathcal{E})$, where \mathcal{X} is an algebraic curve over F_q and \mathcal{P} is an *n*-tuple of mutually distinct F_q rational points of \mathcal{X} and E is a divisor of \mathcal{X} with disjoint support from \mathcal{P} we construct an equivalent representation. This equivalent representation given by $\mathcal{C} = \mathcal{C}_{\mathcal{L}}(\mathcal{Y}, \mathcal{Q}, \mathcal{F})$ can be found as follows; The *n*-tuple of points are obtained directly from a generator matrix of \mathcal{C} , where the columns are viewed as homogeneous coordinates of these points. The curve \mathcal{Y} is given by $I_2(\mathcal{Y})$, the homogeneous elements of degree 2 of the vanishing ideal $I(\mathcal{Y})$.

Edgar Martínez Moro, Irene Marquez-Corbella Universidad de Valladolid edgar@maf.uva.es, iremarquez@gmail.com

Ruud Pellikaan Technical University of Eindhoven. g.r.pellikaan@tue.nl

Diego Ruano Aalborg University diego@math.aau.dk

$\mathbf{MS5}$

Point Compression for the Trace Zero Variety

The hardness of the (hyper)elliptic curve discrete logarithm problem over extension fields lies in the trace zero variety. A compact representation of the points of this abelian variety, computed by a point compression algorithm, is needed in order to accurately assess the hardness of the discrete logarithm problem there. Such algorithms have been proposed by Lange and Silverberg. We present a new representation that is optimal in size, compatible with the structure of the variety, and allows efficient point compression and decompression.

Maike Massierer

University of Basel and University of Neuchatel maike.massierer@unibas.ch

Elisa Gorla University of Neuchatel elisa.gorla@unine.ch

$\mathbf{MS5}$

Exponentiating in Pairing Groups

Bilinear maps derived from the Weil or the Tate pairing on elliptic curves over finite fields are versatile tools in cryptography. Finding the right pairing-friendly curve is crucial for practical performance. This talk discusses the selection of curve and field parameters that enable secure and fast implementations of cryptographic pairings for the most common security levels. Based on these choices, we study exponentiations in pairing groups and show that, although the Weierstrass model is preferable for pairing computation, it can be worthwhile to map to alternative curve representations for the non-pairing group operations in protocols.

Michael Naehrig, Craig Costello, Joppe W. Bos Microsoft Research mnaehrig@microsoft.com, craigco@microsoft.com, jbos@microsoft.com

MS6

Toric Embedding and Birational Geometry

Given a toric variety, a small modification on it, the socalled flip, is described by a change of the fan structure. Considering the Cox construction of a toric variety, this type of birational modification can be computed combinatorially and have a particularly meaningful picture when the Picard number of the variety is low. I describe this briefly and show how one can get interesting results in birational classification of 3-folds using a suitable toric embedding and performing these maps.

<u>Hamid Ahmadinezhad</u> Radon Institute (RICAM) Austrian Academy of Sciences hamid.ahmadinezhad@ricam.oeaw.ac.at

MS6

(Convex) Normal Lattice Polytopes

We show that for lattice polytopes P there is no difference between being 3– and k-convex normal ($k \ge 3$) and improve the bound of how long edges of a lattice polytope Phave to be, so that P is known to be normal, by a factor of 2. The original bound was proved by Gubeladze in his 2009 paper "Convex normality of rational polytopes with long edges".

Jan Hofmann, Petra Meyer

Goethe University Frankfurt am Main

jh of mann @math.uni-frank furt.de, petrameyer 86 @gmx.net

MS6

Combinatorial Mutations and Fano Manifolds

Given a Laurent polynomial f, one can form the period π_f : a function of one complex variable that plays an important role in Mirror Symmetry for Fano manifolds. Mutations are a particular class of birational transformations acting on Laurent polynomials in two variables that preserve the period and, at least in two-dimensions, are closely connected with cluster algebras. I will give a combinatorial description of mutation acting on the Newton polytope Pof f, and and use this to establish many basic facts about mutations.

Alexander M. Kasprzyk

Imperial College London a.m.kasprzyk@imperial.ac.uk

MS6 New Developments in LattE Integrale

The software package LattE has been known as the first implementation of Barvinok's 1994 algorithm for computing generating functions of polyhedra, counting lattice points, and computing Ehrhart series. We report on the new developments in the latest version, LattE integrale, which complements these discrete generating function techniques by new implementations of continuous and intermediate ("mixed integer") generating function algorithms for computing integrals of polynomial functions over polytopes and computing the top k coefficients of generalized Ehrhart quasipolynomials.

Velleda Baldoni U. Roma Tor Vergata baldoni@mat.uniroma2.it

Nicole Berline Ecole Polytechnique berline@math.polytechnique.fr

Jesús A. De Loera, Brandon E. Dutra Department of Mathematics Univ. of California, Davis deloera@math.ucdavis.edu, bedutra@ucdavis.edu

Matthias Koeppe Dept. of Mathematics Univ. of California, Davis mkoeppe@math.ucdavis.edu

Michèle Vergne Université Paris 7 Diderot Institut Mathématique de Jussieu vergne@math.jussieu.fr

MS6

Perturbation of Transportation Polytopes

We introduce a perturbation method that can be used to reduce the problem of finding the multivariate generating function (MGF) of a non-simple cone to computing the MGF of simple cones. We then give a universal perturbation that works for any transportation polytope. We apply this perturbation to the family of central transportation polytopes of order $kn \times n$, and obtain formulas for the MGFs of the feasible cones of vertices of the polytope and the MGF of the polytope. The formulas we obtain are enumerated by combinatorial objects. A special case of the formulas recovers the results on Birkhoff polytopes given by the author and De Loera and Yoshida.

<u>Fu Liu</u> UC Davis fuliu@math.ucdavis.edu

MS7

Tensor Ranks

In this talk I will present various notions of tensor ranks that are of interest in applications. I will range from classical and more modern definitions. I will also show what is known on their mutual relations by associating them some algebraic varieties/sets.

<u>Alessandra Bernardi</u> University of Torino alessandra.bernardi@unito.it

$\mathbf{MS7}$

Projective Methods for the Identifiability of Tensors II

In this talk, effective results on identifiability of tensors, based on projective methods, are shown. Further developments are proposed.

Cristiano Bocci Universita di Siena

cristiano.bocci@unisi.it

MS7

Projective Methods for the Identifiability of Tensors I

I will introduce methods of Projective Algebraic Geometry, from the theory of secant varieties, that can be applied to the study of the uniqueness of the decomposition of general tensors. Examples of tensors for which the identifiability fails, and their Geometric explanation, will be given in the talk.

<u>Luca Chiantini</u> Universita di Siena luca.chiantini@unisi.it

$\mathbf{MS7}$

Decomposition of Infinite-dimensional Tensors

We discuss a technique that allows blind recovery of signals or blind identification of mixtures in instances where they were thought to be impossible: (i) closely located or highly correlated sources in antenna array processing, (ii) highly correlated spreading codes in CDMA radio communication, (iii) nearly dependent spectra in fluorescent spectroscopy. To provide theoretical underpinnings for this technique, we discuss separable approximations of functions and an appropriate notion of rank and nuclear norm.

Lek-Heng Lim

University of Chicago lekheng@galton.uchicago.edu

Pierre Comon GIPSA-Lab CNRS Grenoble, France pierre.comon@gipsa-lab.grenoble-inp.fr

MS8

Effective Calculations of Cohomology via Spectral Sequences

We will discuss approaches for computing cohomology groups by considering examples from algebraic geometry, commutative algebra, and algebraic topology. Emphasis will be placed on interactions between spectral sequences, arising from filtered complexes, and computer calculations. Technical details will be kept to a minimum so as to make the talk accessible to a general audience.

Nathan Grieve

Queen's University nathangrieve@mast.queensu.ca

$\mathbf{MS8}$

Computation in the Intersection Ring of Flag Bundles and Isotropic Flag Bundles

We give an explicit description of the Groebner basis of the ideal of relations defining the intersection ring of a flag bundle of a vector bundle on a nonsingular variety, and also of isotropic bundles. This justifies a key algorithm in the Schubert2 package in Macaulay2 which supports computation in intersection theory and enumerative geometry.

Dan Grayson UIUC dan@math.uiuc.edu

Alexandra Seceleanu University of Nebraska - Lincoln aseceleanu@unl.edu

<u>Mike Stillman</u> Cornell University mike@math.cornell.edu

MS8

State Polytopes of Ideals and Syzygies and Geometric Invariant Theory for Moduli of Curves

A major open question in algebraic geometry is to investigate the birational geometry of the moduli space of stable genus g algebraic curves $\overline{\mathcal{M}}_g$. I will summarize how calculations of state polytopes of ideals and syzygy modules using Macaulay2, gfan, and polymake have helped us construct and explore new birational models of $\overline{\mathcal{M}}_g$.

Anand Deopurkar Columbia University anandrd@math.columbia.edu

Maksym Fedorchuk Boston College maksym.fedorchuk@bc.edu

<u>David Swinarski</u> Fordham University dswinarski@fordham.edu

MS8

Fixed Point Sets in Affine Buildings

Let G be a semisimple group over a complete, discretely valued non-Archimedean field. The Bruhat-Tits building associated to G is an infinite simplicial complex on which G acts. For example, for SL_2 , the building is an infinite tree. The building is glued from apartments, each of which is an affine space. We study fixed point sets in the building under the G-action. In each apartment, a fixed point set is an alcoved polytope associated to a root system. We will discuss various computational problems and experiments in Macaulay 2.

Annette Werner Goethe-Universität Frankfurt werner@math.uni-frankfurt.de

Josephine Yu

School of Mathematics Georgia Institute of Technology josephine.yu@math.gatech.edu

MS9

On Hyperbolicity Cones and Spectrahedra

The generalized Lax conjecture asserts that each hyperbolicity cone is a slice of the cone of positive semidefinite matrices. We will talk about recent combinatorial approaches to the conjecture.

<u>Petter Branden</u> KTH Stockholm petterbranden@gmail.com

MS9

Containment Problems for Polytopes and Spectrahedra

In this talk we study the computational question of whether a given polytope or spectrahedron, the feasible region of a semidefinite program, is contained in another one. Extending results on the polytope/polytope-case by Gritzmann and Klee, we first classify the computational complexity. To overcome the situation that the general containment problem for spectrahedra is co-NP-hard, relaxation techniques are of particular interest. The aim of this talk is to present some semidefinite conditions to certify containment.

<u>Kai Kellner</u> Goethe University Frankfurt kellner@math.uni-frankfurt.de

Thorsten Theobald J.W. Goethe-Universität Frankfurt am Main, Germany theobald@math.uni-frankfurt.de

Christian Trabandt Goethe University Frankfurt trabandt@math.uni-frankfurt.de

MS9

Positive Polynomials on Non-Compact Sets

The difference between positive polynomials and polynomials representable via sums of squares is best understood for compact sets (with many positive results) or highly noncompact sets (with corresponding negative results). Still relatively little is known for sets that lie in between. In this talk, we will give some partial results and discuss relevant examples.

<u>Daniel Plaumann</u> Universität Konstanz daniel.plaumann@uni-konstanz.de

MS9

Bounds on the Equivariant Betti Numbers of Symmetric Semi-Algebraic Sets

Abstract not available at time of publication.

<u>Cordian Riener</u> Aalto University cordian.riener@gmail.com Saugata Basu Purdue University sbasu@math.purdue.edu

MS9

When is Every Nonnegative Quadric a Sum of Squares?

We identify a numerical invariant which is an obstruction for equality and show that it vanishes iff X is a variety of minimal degree. These result generalize a well known Theorem of Hilbert characterizing the pairs (d,k) for which every nonnegative homogeneous polynomial of degree 2d in k variables is a sum of squares.

<u>Mauricio Velasco</u> Universidad de los Andes mvelasco@uniandes.edu.co

Gregoriy Blekherman Georgia Tech greg@math.gatech.edu

Gregory G. Smith Queen's University ggsmith@mast.queensu.ca

MS10 Invited. Participation Uncertain 1

Abstract not available at time of publication.

<u>Graziano Chesi</u> Department of Electrical and Electronic Engineering (CYC Bld University of Hong Kong chesi@eee.hku.hk

MS10

Bounded Symbolic-Numeric Cylindrical Algebraic Decomposition for Solving Optimization Problems

With many applications in engineering and in scientific fields, quantifier elimination (QE) has been attracting more attention these days. Cylindrical algebraic decomposition (CAD) is used as a basis for a general QE algorithm. We propose an effective symbolic-numeric cylindrical algebraic decomposition algorithm for solving polynomial optimization problems. Our approach constructs CAD only in restricted admissible regions to remove redundant projection factors and avoid lifting cells where truth values are constant over the region.

Hidenao Iwane

FUJITSU LABORATORIES LTD / Kyushu University 4-1-1 Kamikodanaka, Nakahara-ku, Kawasaki 211-8588, JAPAN iwane@jp.fujitsu.com

Hirokazu Anai FUJITSU LABORATORIES LTD Kyushu University anai@jp.fujitsu.com

MS10 Polynomial Optimization with Real Varieties

We study the optimization problem

$$\min \quad f(x) \quad s.t. \quad h(x) = 0, \ g(x) \ge 0$$

with f a polynomial and h, g two tuples of polynomials in $x \in \mathbb{R}^n$. Lasserre's hierarchy is a sequence of sum of squares relaxations for finding the global minimum f_{min} . Let K be the feasible set. We prove the following results: i) If the real variety $V_R(h)$ is finite, then Lasserre's hierarchy has finite convergence, no matter the complex variety $V_C(h)$ is finite or not. This solves an open question in Laurent's survey. ii) If K and $V_R(h)$ have the same vanishing ideal, then the finite convergence of Lasserre's hierarchy is independent of the choice of defining polynomials for the real variety $V_R(h)$. iii) When K is finite, a refined version of Lasserre's hierarchy (using the preordering of g) has finite convergence.

Jiawang Nie University of California, San Diego njw@math.ucsd.edu

MS10

Sums of Squares of Polynomials with Rational Coefficients

We consider multivariate polynomials f with coefficients in the field Q of rational numbers which are sums of squares of polynomials with real coefficients. Such f has a sum of squares representation with coefficients in some finite real extension K of Q, but not necessarily in Q itself. We shall attempt to provide more information on possible fields K, and to give sufficient conditions that imply the existence of a representation over Q.

Claus Scheiderer

Fachbereich Mathematik und Statistik Univ. Konstanz, Germany claus.scheiderer@uni-konstanz.de

MS11

Euler-Mahonian Statistics Via Polyhedral Geometry

Permutations are some of the most fundamental objects of mathematics, and a basic combinatorial statistics of a permutation $\pi \in S_n$ is the number of descents $des(\pi) :=$ $\{j: \pi(j) > \pi(j+1)\}$. Euler realized that

$$\sum_{k \ge 0} (k+1)^n t^k = \frac{\sum_{\pi \in S_n} t^{\operatorname{des}(\pi)}}{(1-t)^{n+1}}$$

and there has been various generalization of this identity, most notably when S_n gets replaced by another Coxeter group. We will illustrate how one can view Euler's identity (and its generalizations) geometrically through enumerating integer points in certain polyhedra. This gives rise to "short" proofs of known theorems, as well as new identities. This is joint work with Ben Braun (Kentucky).

<u>Matthias Beck</u> San Francisco State University mattbeck@sfsu.edu

Benjamin Braun University of Kentucky benjamin.braun@uky.edu

MS11

Towards a Classification of Restricted Lattice Walks

Completing the Classification of Walks with Small Steps in the Quarter Plane The enumeration of walks in the quarter plane with small steps is an elegant problem in combinatorics. Recently, much work has been done attempting to classify the seventy nine non-equivalent walks of this type according to analytic properties of their generating functions. We discuss this classification, its usefulness, and outline a method of classifying the final three walks not yet proven by other means. In particular, our method gives a parametrization of the generating function which illuminates the nature of its singularities, in turn proving that they are infinite in number.

Stephen Melczer

Simon Fraser University Vancouver srm9@sfu.ca

Marni Mishna Simon Fraser University mmishna@sfu.ca

MS11

Automated Asymptotics of Multivariate Generating Functions

I begin with a description of the field of Analytic Combinatorics in Several Variables, drawing on my recent book with Mark Wilson. It is well known that coefficient asymptotics of a multivariate generating function depend mainly on the geometry of the algebraic surface Q=0. One of the challenges in moving from theorems that handle most cases in practice to automated asymptotics is to combinatorialize the geometric data. This talk concerns some of the problems that arise when implementing automation.

<u>Robin Pemantle</u>

University of Pennsylvannia pemantle@math.upenn.edu

Mark Wilson University of Auckland mcw@cs.auckland.ac.nz

MS11

On the Summability of Bivariate Rational Functions

We present criteria for deciding whether a bivariate rational function in two variables can be written as a sum of two (q-)differences of bivariate rational functions. Using these criteria, we show how certain double sums can be evaluated, first, in terms of single sums and, finally, in terms of values of special functions.

Shaoshi Chen, <u>Michael F. Singer</u> Department of <u>Mathematics</u> North Carolina State University schen@amss.ac.cn, singer@math.ncsu.edu

MS12

The Dynamics of Chip-firing on Abstract Tropical Curves

Chip-firing has been studied for nearly 25 years since its independent introductions in statistical physics and graph theory. Recently, it has been employed as a combinatorial language for describing linear equivalence of divisors on graphs and tropical curves. In this talk we will address some dynamical questions which arise in this subject in the context of greedy algorithms and generalized chip-firing via open covers.

Spencer Backman

Georgia Institute of Technology spencerbackman@gmail.com

MS12

Combinatorics of the Tropical Moduli Space of Curves

A tropical curve is a metric graph marked with integer vertex weights obtained as a metrized dual graph of a semistable model of a curve over a valued field. We will review the construction tropical moduli space $M_{g,n}^{trop}$ of genus g tropical curves with n marked points and report on some combinatorial properties of these spaces.

Melody Chan

Harvard University mtchan@math.harvard.edu

MS12

Tropical Geometry and Combinatorics in Integrable Cellular Automata

The box-ball system (BBS) is an cellular automaton given by a simple rule to move finite number of balls in boxes in a line. The integrability of the BBS is clarified by crystal base theory and tropical geometry. In this talk I will review the BBS mainly from the view point of tropical geometry. I also give an overview of the talks in this session.

<u>Rei Inoue</u> Chiba University reiiy@math.s.chiba-u.ac.jp

MS12

Rigged Configurations and Box-Ball Systems

I will explain algebraic methods for the studies on the boxball systems. The topics include Kashiwara's crystal bases for the quantum affine algebras, the rigged configurations and the tropical Riemann theta function. The purpose of the talk is to present main constructions for the simplest possible situation as we did in an introductory review: arXiv:1212.2774

<u>Reiho Sakamoto</u> Tokyo University of Science reiho@rs.tus.ac.jp

MS13

Polynomial Inequalities for Bistability in a Double Phosphorylation Network

Double phosphorylation mechanisms are important building blocks of mammalian signaling systems and bistability is considered an important property of ODE models of these systems. Bistability often arises from a saddle-node bifurcation. Establishing such a bifurcation is difficult, as parameter values are either unknown or confined to large intervals. Hence conditions guaranteeing a saddle-node bifurcation are desirable. We present such a condition in form of polynomial inequalities in the Km and Kcat values.

Carsten Conradi

Max-Planck-Institut Dynamik komplexer technischer Systeme conradi@mpi-magdeburg.mpg.de

Maya Mincheva Northern Illinois University mincheva@math.niu.edu

MS13

Identification of Multistationary Reaction Networks Modeled with Power-law Kinetics

Multistationarity in cellular systems provides a mechanism for switching between different responses and can be crucial for cellular decision making. It is in general difficult to decide whether a particular reaction network has the capacity to exhibit multiple steady states. In this talk I will present our most recent results concerning both the preclusion and identification of multistationarity in reaction networks modeled with power-law kinetics (which include mass-action kinetics).

<u>Elisenda Feliu</u>, Carsten Wiuf University of Copenhagen efeliu@math.ku.dk, wiuf@math.ku.dk

MS13

Calculating Detailed-Balanced Equilibrium by Fixed-Point Iterations and Cell Exclusion

We will report on our quest for an unconditionally convergent algorithm to compute the (detailed-balanced) equilibrium of complete networks of reversible binding reactions. Several networks used in pharmacology for simulation and parameter estimation are in this class, and methods used to compute equilibrium are often questionable or even inapplicable. We turned the algebraic equation for equilibrium into a fixed-point problem. We will present network-structural conditions that guarantee the convergence of fixed-point iterations, and a cell-exclusion algorithm that applies more generally.

Gilles Gnacadja Amgen gilles.gnacadja@gmail.com

MS13

Characterization of Steady States of General Mass Action Systems by Correspondence to Weakly Reversible Networks

In order to analyze the dynamical behavior of chemical reaction networks, it is important to first of all characterize the steady states permitted by the mechanism. In this talk, I will relate a recent steady state characterization, called toric steady states, to a classical characterization related to weakly reversible networks. I will present a graphical trick for reformulating arbitrary networks into weakly reversible networks and present conditions under which the steady states are guaranteed to be preserved under the operation.

<u>Matthew Johnston</u> University of Wisconsin Madison mjohnston3@wisc.edu

MS13

Chemical Reaction Networks As Compartmental Systems

Consider a chemical readtion network with general monotone rate functions. Conditions are given for the system to be reducable to a compartmental system. Consequently, every solution will approach an equilibrium point. Comparison is made with other approaches.

David Siegel

University of Waterloo Department of Applied Math dsiegel@math.uwaterloo.ca

MS14

Computational and Statistical Tradeoffs Via Convex Relaxation

Processing massive datasets is usually viewed as a substantial computational challenge. However, if data are a statisticians main resource then access to more data should be viewed as an asset rather than as a burden. We describe a framework based on convex relaxation to reduce the computational complexity of an inference procedure when one has access to increasingly larger datasets, and we demonstrate the efficacy of this methodology in a class of denoising problems.

<u>Venkat Chandrasekaran</u> California Institute of Technology venkatc@caltech.edu

Michael Jordan University of California at Berkeley jordan@eecs.berkeley.edu

MS14

Differentiable, Continuous, and Combinatorial Hodge Theories

The usual Hodge theory on Riemannian manifolds, which may be described as "differentiable Hodge theory," has been very useful in physical problems like fluid dynamics and electromagnetics. In data analytic problems, we often have only some notion of proximity of the data points and some knowledge of the distribution of the data set. We discuss recent developments in "continuous Hodge theory" on metric spaces and "discrete Hodge theory" on simplicial complexes for such problems.

Lek-Heng Lim

University of Chicago lekheng@galton.uchicago.edu

MS14

It is Hard to be Strongly Faithful

Many algorithms for inferring causality rely heavily on the faithfulness assumption. The main justification for imposing this assumption is that the set of unfaithful distributions has Lebesgue measure zero, since it can be seen as a collection of hypersurfaces in a hypercube. However, due to sampling error the faithfulness condition alone is not sufficient for statistical estimation, and strong-faithfulness has been proposed and assumed to achieve uniform or highdimensional consistency. In contrast to the plain faithfulness assumption, the set of distributions that is not strongfaithful has non-zero Lebesgue measure and in fact, can be surprisingly large as we show in this talk. Taking a geometric point of view we give bounds on the Lebesgue measure of strong-faithful distributions for various classes of directed acyclic graphs. Our results imply fundamental limitations for algorithms inferring causality based on partial correlations, with the PC-algorithm as its most prominent example.

<u>Caroline Uhler</u> Institute of Science and Technology Austria caroline.uhler@ist.ac.at

Garvesh Raskutti Department of Statistics and Operations Research University of North Carolina raskutti@samsi.info

Peter Buehlmann ETH Zurich peter.buehlmann@stat.math.ethz.ch

Bin Yu University of California, Berkeley binyu@stat.berkeley.edu

MS15

On a Family of Determinantal Varieties Arising as Critical Loci in a Classical Computer Vision Problem

Scene reconstruction is a classical problem in computer vision. It is well known that there exist critical configurations of camera positions and sample points that make reconstruction not possible. Such configurations live on a family of determinantal varieties that will be introduced.

<u>Gianmario Besana</u> DePaul University gbesana@depaul.edu

MS15

Certifiable Numerical Computations in Schubert Calculus

Numerical methods for solving polynomial systems may be advantageous in avoiding the complexity of calculating a *Gröbner* basis in many applications. However, numerical solutions are approximate, and algorithms which certify them require that the system be square. Many geometric problems are presented algebraically as overdetermined systems. We use Schubert problems as prototypical overdetermined systems. Using local coordinates and duality, we give a method for formulating these problems as square systems.

<u>Nickolas Hein</u> University of Nebraska at Kearney nickhein@gmail.com

Jonathan Hauenstein North Carolina State University hauenstein@ncsu.edu

Frank Sottile Texas A&M University sottile@math.tamu.edu

MS15

Macaulay Dual Space and Numerical Primary Decomposition

Numerical algorithms for an irreducible decomposition of a polynomial ideal fall short of a full decomposition into primary components. In particular, an irreducible decomposition does not detect embedded components. The Macaulay dual space provides a numerical alternative to standard basis algorithms for computing local information at a point. We employ the Macaulay dual to examine points of interest and distinguish embedded components from other singular points on the variety.

<u>Robert Krone</u> School of Mathematics Georgia Institute of Technology rkrone3@math.gatech.edu

Jonathan Hauenstein North Carolina State University hauenstein@ncsu.edu

Anton Leykin School of Mathematics Georgia Institute of Technology leykin@math.gatech.edu

MS15

Determinantal Representations of Hyperbolic Curves via Polynomial Homotopy Continuation

A smooth curve in the real projective plane is hyperbolic if its ovals are maximally nested. By the Helton-Vinnikov Theorem, any such curve admits a definite symmetric determinantal representation. We compute such representations numerically by tracking a specially designed homotopy in a space of regular real polynomial systems. (Joint work with Daniel Plaumann)

Anton Leykin School of Mathematics Georgia Institute of Technology leykin@math.gatech.edu

MS16

Arithmetic Occult Periods

For certain types of complete intersections, the associated complex moduli space is actually a quotient of a complex ball. I will survey recent work, particularly involving the case of cubic surfaces, which suggests that this unexpected structure is the complex realization of a morphism of integral moduli spaces.

Jeff Achter

Colorado State University j.achter@colostate.edu

MS16 Crystalline Cohomology of the Igusa Tower

We relate the crystalline cohomology of the Igusa tower to classical cusp forms of finite slope. In the ordinary setting, we obtain a canonical isomorphism between the crystalline cohomology of the Igusa tower and the Iwasawa module of ordinary Λ -adic cuspforms.

Bryden Cais University of Arizona cais@math.arizona.edu

MS16

Sato-Tate Groups of Abelian Surfaces and Three-folds

We classify the Sato-Tate groups of abelian surfaces over number fields, and give partial results towards the case of threefolds. We then compare with numerical evidence supporting the generalized Sato-Tate conjecture.

Francesc Fité University of Bielefeld francesc.fite@gmail.com

Kiran S. Kedlaya University of California at San Diego kedlaya@ucsd.edu

Víctor Rotger Universitat Politècnica de Catalunya victor.rotger@upc.edu

Andrew V. Sutherland MIT drew@math.mit.edu

MS16

Colmez's Product Formula for CM Abelian Varieties

We complete a proof of Colmez, showing that the standard product formula for algebraic numbers has an analog for periods of CM abelian varieties with CM by an abelian extension of the rationals. The proof depends on explicit computations with the De Rham cohomology of Fermat curves, which in turn depends on explicit computation of their stable reductions.

<u>Andrew Obus</u> Columbia University obus@cpw.math.columbia.edu

MS16

Rational Points on Twists of Modular Curves

In this talk, I will present results about points on certain twists of the classical modular curve. Some of these twisted curves violate the Hasse principle and this violation is explained by the Brauer-Manin obstruction. In some other cases, it is possible to study Hasse principle violations via local-global trace obstructions.

Ekin Ozman University of Texas at Austin ozman@math.utexas.edu

MS17

Short Algebraic-Geometry Codes and Their Weight Distribution for Diffusion in Block Ciphers and Hash Functions

We study the notion of diffusion in block ciphers, in the case of substitution permutation networks. This is formalized with the *branch number* of a linear matrix, which should be high for good resistance against linear and differential cryptanalyses. A maximal branch number is obtained with MDS codes. Non maximality introduces codes of low genus, and their weight distribution is needed to study the resistance of the cipher.

Daniel Augot INRIA daniel.augot@inria.fr

MS17

On the Design of Wiretap Codes

Wiretap codes are families of codes that promise both reliability between the legitimate users Alice and Bob, and confidentiality in the presence of an eavesdropper Eve. We survey an error probability point of view to the design of wiretap codes for several continuous channels, and discuss the resulting code design criteria. They give rise to a serie of interesting problems, such as the understanding of a new lattice invariant, or the search for number fields with particular discriminant and regulator.

Frederique Oggier, <u>Jerome Ducoat</u> Nanyang Technological University frederique@ntu.edu.sg, jducoat@ntu.edu.sg

MS17

Orders of Central Simple Algebras as a Tool for Wireless Communications

We demonstrate how central simple algebras and their orders can be used to construct efficient space-time codes that improve the quality of wireless communications. We will provide the essential algebraic background and familiarize the audience with the transmission model for wireless fading channels. Examples of space-time codes designed especially for digital video broadcasting, an application of major current interest, will be given.

<u>Camilla Hollanti</u> Aalto University camilla.hollanti@aalto.fi

MS17

New Matrix-Based Lattice Construction Techniques

Let m and n be integers greater than 1. Given pointlattices A and B of dimensions m and n, respectively, a technique for constructing a new lattice from them of dimension either m+n-1 or m+n-2 is proposed. The technique is based on the existence of a common sub-lattice of dimension 1 or 2. Denser sphere packings than previously known ones in dimensions 52, 68, 84, 248, 520, and 4098 are obtained.

<u>Carmelo Interlando</u> San Diego State University carmelo.interlando@sdsu.edu

Probability Bounds for Algebraic Lattice Codes

Lattices which arise from totally real algebraic number fields have many applications to the coding theory of fading channels, which arise naturally in the context of wireless communications. One can naturally attach to the ring of integers of a number field K of degree n over \mathbf{Q} a lattice in \mathbf{R}^n , from which one can carve a finite codebook in a natural way. We will show how familiar number theoretic invariants of K, such as its regulator and values of its Dedekind zeta function, control the probability that the corresponding codebook provides reliable transmission over a fading channel. We will also discuss generalizations to codes built from central simple algebras.

David Karpuk Aalto University david.karpuk@aalto.fi

MS18

Light-weight Methods for Automatic Recognition in Mobile Applications

I will describe some automatic recognition problems which I came across as part of my research on portable device applications. These problems, I will argue, justify the need for a mathematical theory of shape that is amenable to discrete, noisy data. As a tentative first step towards such a theory, I will define the Pascal Triangle of a discrete grayscale image as a pyramidal arrangement of complex-valued moments and explore its geometric significance. In particular, we will observe that the entries of row k of this pyramid correspond to the Fourier series coefficients of the order k moment of the Radon transform of the image. Group actions on the plane can be naturally prolonged onto the entries of the Pascal triangle; we will propose simple tests for equivalence and self-equivalence under some common group actions.

<u>Mireille Boutin</u> School of ECE Purdue University mboutin@purdue.edu

MS18 Estimating Radar Target Invariants

Radar imaging methods require precise knowledge of the relative locations between the measurement antenna(s) and the target to be imaged. Over the past few decades, methods for determining the relative motion, based on exploiting the targets underlying geometric invariance, have been developed in the setting of a single antenna. Extension of these methods to multiple antennas is not straightforward. We discuss our attempts to address the problem by using a specific matrix of Euclidean invariants.

<u>Matthew Ferrara</u>, Gregory Arnold Matrix Research, Inc. matthew.ferrara@matrixresearch.com, greg.arnold@matrixresearch.com

Jason T. Parker AFRL/RYRT jason.parker@wpafb.af.mil

MS18

The Ideal of the Trifocal Variety

Trifocal tensors are constructed from a bilinear map built from the three view camera setup. It is natural to ask what are the minimal generators of the ideal of the algebraic variety of trifocal tensors. We answer this question using a variety of numerical and symbolic techniques from Algebra, Geometry, and Representation Theory. This is joint work with Chris Aholt (Washington)

Chris Aholt University of Washington aholtc@uw.edu

Luke Oeding

University of California Berkeley Università degli Studi di Firenze lukeoeding@gmail.com

MS18

Invariant Histograms and Signatures for Object Recognition and Symmetry Detection

I will survey recent developments in the use of groupinvariant histograms, using distances, areas, etc., and signatures, using differential invariants, joint invariants, invariant numerical approximations, etc., for object recognition and symmetry detection in images, including recent applications to automated jigsaw puzzle assembly.

Peter Olver

Mathematics Department University of Minnesota olver@math.umn.edu

MS19

On Best (r_1, \ldots, r_d) Approximation of *d*-Mode Tensors

In this talk we show that for a generic real *d*-mode tensor the best (r_1, \ldots, r_d) approximation is unique. Furthermore, for a generic symmetric *d*-mode tensor the best rank one approximation is unique, hence symmetric. Similar results hold for partially symmetric tensors. The talk is based on a recent joint paper with Giorgio Ottaviani and a recent paper of the speaker.

Shmuel Friedland

Department of Mathematics University of Illinois, Chicago friedlan@uic.edu

MS19

Computational Complexity of Tensor Problems

Frequently, a problem in science or engineering can be reduced to solving a linear (matrix) system of equations and inequalities. Other times, solutions involve the extraction of certain quantities from matrices such as eigenvectors or singular values. Recently, there has been a flurry of work on multilinear analogues to these basic problems of linear algebra. These tensor methods have found applications in many fields, including computational biology, neuroimaging, phylogenetics, signal processing, spectroscopy, wireless communications, and other areas. Thus, tensor generalizations to the standard algorithms of linear algebra have the potential to substantially enlarge the arsenal of core tools in numerical computation. We explain what is known about the computational complexity of tensor problems. In particular, we show that many naturally occurring tensor problems are NP-hard, both to decide whether solutions exist and to approximate them when they do.

Christopher Hillar University of California, Berkeley chillar@msri.org

Lek-Heng Lim University of Chicago lekheng@galton.uchicago.edu

MS19

Tensor Decomposition, Low Rank Structured Matrix Approximation and Applications

The tensor decomposition problem can be interpreted as a rank completion problem on structured matrices. We will detail this transformation and show how it can be exploited in a decomposition algorithm, using a flat extension property. In the case of small rank, this yields a direct method to compute the unique decomposition. The problem of approximation by tensors of small rank will be discussed and applications in numerical computation will be given.

<u>Bernard Mourrain</u> GALAAD INRIA Sophia Antipolis Bernard.Mourrain@sophia.inria.fr

MS19

Counting Singular Vectors of a Multidimensional Tensor

A rectangular matrix of size $m \times n$ has in general $\min(m, n)$ singular vector pairs. The singular vector pairs correspond to the critical points of the function which contracts the matrix over the product of spheres. For a tensor of size $2 \times 2 \times n$ there are 6 singular vector triples for n = 2and 8 singular vector triples for n > 2. We give a formula counting the number of singular vector d-ples for any general multidimensional tensor, considering also the (partially) symmetric case. The technique is to reduce to a Chern class computation of a suitable vector bundle. This is joint work with Shmuel Friedland.

Giorgio Ottaviani University of Firenze ottavian@math.unifi.it

Shmuel Friedland friedlan@uic.edu university of illinois at chicago

MS19

Direct Sum Decomposability of Polynomials

A form $F = F(x_1, \ldots, x_n)$ is decomposable as a direct sum if, possibly after a linear change of coordinates, $F = F_1(x_1, \ldots, x_k) + F_2(x_{k+1}, \ldots, x_n)$. For example, $xy = \frac{1}{4}(x+y)^2 - \frac{1}{4}(x-y)^2$ and the 2 × 2 determinant ad-bc are direct sums. General forms are indecomposable as direct sums, but this can be hard to show for particular forms. We give an interesting necessary criterion for a form to be a direct sum and use it to answer a question of Shafiei. We investigate the indecomposable forms satisfying our criterion.

Weronika Buczyńska, Jarosław Buczyński IMPAN wkrych@mimuw.edu.pl, jabu@mimuw.edu.pl

Zach Teitler Boise State University

zteitler@boisestate.edu

MS20

Effective Computing in Rings with Infinite Numbers of Variables

Computing with ideals in polynomial rings having infinite numbers of variables is usually assumed to be an impossible task. For instance, algorithmic termination guarantees typically require Noetherianity (finite generation of ideals) hypotheses, which are violated when there are more than a finite number of indeterminates. However, if the ideals in question have extra structure, such as being invariant under a group action, it is sometimes possible for symbolic algorithms to terminate. We discuss this phenomenon in the context of ideals invariant under the symmetric group and demonstrate software that can effectively solve nontrivial problems of symbolic algebra involving infinite numbers of variables.

Christopher Hillar University of California, Berkeley chillar@msri.org

Robert Krone, Anton Leykin School of Mathematics Georgia Institute of Technology rkrone3@math.gatech.edu, leykin@math.gatech.edu

Seth Sullivant North Carolina State University smsulli2@ncsu.edu

MS20

Geometry of Wachspress Surfaces

Let P_d be a convex polygon with d vertices. The associated Wachspress surface W_d is a fundamental object in geometric modeling, defined as the image of the rational map from the projective plane to projective d-1 space, determined by the Wachspress barycentric coordinates. We show that the Wachspress surface is always smooth, and for d > 4 is isomorphic to the blowup of the projective plane at points corresponding to intersections of lines determining the edges of the polygon, which are not vertices of the polygon. We determine the ideal of polynomials vanishing on W_d , and other algebraic invariants (betti numbers and regularity).

Corey Irving Mathematics Department Santa Clara University cfirving@scu.edu

<u>Hal Schenck</u> Mathematics Department University of Illinois schenck@math.uiuc.edu

Bounds on Projective Dimension

In commutative algebra, computing resolutions (e.g. by using computer algebra systems) has become essential. In order to understand the complexity of these computations, it is important to be able to bound the homological invariants. There has been a lot of interest, motivated both from a theoretical and a computational perspective, in finding bounds on the projective dimension of homogeneous ideals of polynomial rings in terms of data readily apparent before one computes a resolution. Hilbert's syzygy theorem is a classical example. More recently, Stillman asked whether the projective dimension of an ideal could be bounded in terms of the degrees of the generators, without involving the number of variables of the ambient ring. This and the similar question for regularity are still open in complete generality. I will discuss some approaches and recent progress on these questions.

<u>Alexandra Seceleanu</u> University of Nebraska aseceleanu2@math.unl.edu

MS20

Ghosts of the Jacobian Ideal and Graphic Arrangements

The Jacobian ideal $J(f) = (\frac{\partial}{\partial x_i}f)_{i=1}^n$ in $S = k[x_1, \ldots, x_n]$ encodes much more than just the singularities of a hypersurface f. The ring S/J(f) is Cohen-Macaulay exactly when the module of logarithmic 1-forms Ω^1 is a free Smodule. We study families of arrangements where the codimension of some $Ext^i(S/J(f), S)$ is greater than i. We call the associated primes of these Ext-modules ghosts. The goal is to find necessary and/or sufficient combinatorial conditions for J to have a ghost.

<u>Max Wakefield</u> US Naval Academy wakefiel@usna.edu

MS21

Dimensional Differences Between Faces of Nonnegative Polynomials and Sums of Squares

We study dimensions of the faces of the cone of nonnegative polynomials and sums of squares. In particular, we establish dimensional differences between these faces yielding nonnegative polynomials that are not sums of squares. In several important special cases we will give an explicit and constructive answer to the question of when these gaps occur for the first time and how to use them to build nonnegative polynomials that are not sums of squares. (Based on joint work with G. Blekherman and M. Kubitzke)

<u>Sadik Iliman</u> Goethe University Frankfurt iliman@math.uni-frankfurt.de

Grigoriy Blekherman Georgia Tech greg@math.gatech.edu

Martina Kubitzke Goethe University Frankfurt kubitzke@math.uni-frankfurt.de

MS21

The A-Truncated K-Moment Problem

Let \mathcal{A} be a finite subset of N^n , and K be a compact semialgebraic set. An \mathcal{A} -tms is a vector y indexed by elements in \mathcal{A} . The \mathcal{A} -TKMP problem studies whether a given yadmits a K-measure or not. This paper proposes a numerical algorithm for solving \mathcal{A} -TKMPs. It is based on finding a flat extension of y by solving a hierarchy of semidefinite relaxations, whose objective R is generated in a certain randomized way. If y admits no K-measures and $R[x]_A$ is K-full, we can get a certificate for the nonexistence of representing measures. If y admits a K-measure, then for almost all generated R, we prove that: i) we can asymptotically get a flat extension of y; ii) under a general condition that is almost sufficient and necessary, we can get a flat extension of y. The SOEP and CP decomposition problems can be solved as an \mathcal{A} -TKMP.

Jiawang Nie

University of California, San Diego njw@math.ucsd.edu

MS21

Computing Upper Bounds for Densest Polytope Packings

I will discuss the problem of finding upper bounds for the maximum density of a packing of translative or congruent copies of polytopes in Euclidean space. In the last years, the case of regular tetrahedra got quite some attention (but see also Hilbert's 18th problem) and only very weak upper bounds are known here. In the talk I will present computational strategies on how to compute upper bounds. For this techniques from combinatorial optimization (Lovasz theta number), harmonic analysis (positive type functions for locally compact groups), and polynomial optimization (sums of squares invariant under Coxeter groups) will be used. (based on joint work with Cristobal Guzman and Fernando Mario de Oliveira Filho)

<u>Frank Vallentin</u> Delft University of Technology f.vallentin@tudelft.nl

MS21

A Concrete Approach to Hermitian Determinantal Representations

If a Hermitian matrix of linear forms is positive definite at some point, then its determinant is a hyperbolic hypersurface. In 2007, Helton and Vinnikov proved a converse in three variables, namely that every hyperbolic plane curve has a definite Hermitian determinantal representation. In this talk I will discuss a concrete proof of this statement and a method for computing these determinantal representations in practice. This involves relating the definiteness of a matrix to the real topology of its minors and extending a classical construction of Dixon from 1902. This is joint work with Daniel Plaumann, Rainer Sinn, and David Speyer.

Cynthia Vinzant University of Michigan Dept. of Mathematics vinzant@umich.edu

Polar Varieties and Algebraic Certificates

Polar varieties are traditionally used to compute real solutions of polynomial systems of equations. We show how to modify their definitions so that modified polar varieties allow to compute algebraic certificates of positivity (by means of sums of squares decompositions) of non-negative polynomials over real algebraic sets.

Aurelien Greuet Univ. Versailles-Saint-Quentin aurelien.greuet@uvsq.fr

Feng Guo UCSD f1guo@ucsd.edu

Mohab Safey El Din University Pierre and Marie Curie Mohab.Safey@lip6.fr

Lihong Zhi Academia Sinica lzhi@mmrc.iss.ac.cn

MS22

Inequality Proving and Global Optimization Via a Simplified CAD Projection

Let $X_n = (x_1, \ldots, x_n)$ and $f \in \mathbf{R}[\mathbf{X}_n, \mathbf{k}]$. The problem of finding all k such that $f(X_n, k) \ge 0$ for all $X_n \in \mathbf{R}^n$ is considered in this paper, which obviously takes as a special case the problem of computing the global infimum or proving the semi-definiteness of a polynomial. For solving the problems we propose a simplified Brown's CAD projection operator, called Nproj, of which the projection scale is always no larger than that of Brown's. For many problems, especially when $n \geq 3$, the scale of Nproj is much smaller than that of Brown's. As a result, the lifting phase is also simplified. Some new algorithms based on Nproj for solving those problems are designed and proved to be correct. Comparison to some existing tools on some examples is reported to illustrate the effectiveness of our new algorithms. This is joint work with Jingjun Han and Zhi Jin at Peking University.

<u>Bican Xia</u> Peking University xbc@math.pku.edu.cn

MS22

Exact Safety Verification of Interval Hybrid Systems Based on Symbolic-Numeric Computation

In this talk, we address the problem of safety verification of interval hybrid systems in which the coefficients are intervals instead of explicit numbers. A hybrid symbolicnumeric method, based on SOS relaxation and interval arithmetic certification, is proposed to generate exact inequality invariants for safety verification of interval hybrid systems. As an application, an approach is provided to verify safety properties of non-polynomial hybrid systems. Experiments on the benchmark hybrid systems are given to illustrate the efficiency of our method. This is joint work with Min Wu and Wang Lin.

Zhengfeng Yang

Shanghai Key Laboratory of Trustworthy Computing

zfyang@sei.ecnu.edu.cn

Min Wu, Wang Lin Shanghai Key Laboratory of Trustworthy Computing East China Normal University mwu@sei.ecnu.edu.cn, linwang@wzu.edu.cn

MS22

Computing Rational Solutions of Linear Matrix Inequalities

Consider a $(D \times D)$ symmetric matrix A whose entries are linear forms in $\mathbf{Q}[\mathbf{X}_1, \ldots, \mathbf{X}_k]$ with coefficients of bit size $\leq \tau$. We provide an algorithm which decides the existence of rational solutions to the linear matrix inequality $A \succeq 0$ and outputs such a rational solution if it exists. This problem is of first importance: it can be used to compute algebraic certificates of positivity for multivariate polynomials. Our algorithm runs within $(k\tau)^{O(1)}2^{O(\min(k,D)D^2)}D^{O(D^2)}$ bit operations; the bit size of the output solution is dominated by $\tau^{O(1)} 2^{O(\min(k,D)D^2)}$. These results are obtained by designing algorithmic variants of constructions introduced by Klep and Schweighofer. This leads to the best complexity bounds for deciding the existence of sums of squares with rational coefficients of a given polynomial. We have implemented the algorithm; it has been able to tackle Scheiderer's example of a multivariate polynomial that is a sum of squares over the reals but not over the rationals; providing the first computer validation of this counter-example to Sturmfels' conjecture.

Lihong Zhi, Qingdong Guo Academia Sinica Izhi@mmrc.iss.ac.cn, qdguo@mmrc.iss.ac.cn

Mohab Safey El Din UPMC, Univ Paris 6 Mohab.Safey@lip6.fr

MS23

Computing Persistent Homology in Chunks

We present a parallel algorithm for computing the persistent homology of a filtered chain complex. Our approach differs from the commonly used reduction algorithm by first computing persistence pairs within local chunks, then simplifying the unpaired columns, and finally applying standard reduction on the simplified matrix. The approach generalizes a technique by Günther et al., which uses discrete Morse Theory to compute persistence; we derive the same worst-case complexity bound in a more general context. The algorithm employs several practical optimization techniques which are of independent interest. Our sequential implementation of the algorithm is competitive with state-of-the-art methods, and we improve the performance through parallelized computation.

Ulrich Bauer

Institute of Science and Technology (Austria) ulrich.bauer@ist.ac.at

Michael Kerber Stanford University mkerber@stanford.edu

Jan Reininghaus IST Austria jan.reininghaus@ist.ac.at

Computational (co)Homology in Electromagnetic Modelling and Material Analysis

In this talk we show engineering and material science applications of topology. With so called Discrete Geometric Approach discrete counterparts of Maxwell's laws will be provided. We show how cohomology generators make them consistent. Some cohomology algorithms will be stressed. Later we will show efficient ways of computing homology of nodal domains. We will motivate their importance in analyzing so called spinodal decomposition in alloys. This is joint work with Ruben Specogna and Thomas Wanner.

<u>Pawel Dlotko</u> Institute of Computer Science Jagiellonian University

pawel.dlotko@ii.uj.edu.pl

MS23

One the Persistent Homology of Time-Delay Embeddings

We present in this talk a theoretical framework for the study of the persistent homology of time-delay embeddings. In particular, we propose maximum persistence as a measure of periodicity at the signal level, present structural theorems for the resulting diagrams, and derive estimates for their dependency on window size and embedding dimension. We apply this methodology to quantifying periodicity in synthetic signals, and present comparisons with state-of-the-art methods in gene expression analysis.

<u>Jose Perea</u>, John Harer Duke University joperea@math.duke.edu, harer@math.duke.edu

MS23

New Topological Methods for Robotic Grasping and Machine Learning

This talk focusses on our recent efforts in adapting ideas from topology for applications in robotic grasping and machine learning. We will present some of the research questions and challenges that one faces in these fields and we will discuss how techniques from topology might be used there. In particular, we will discuss an application of approximately shortest homology generators for grasping and we will describe how topological constraints can be used in density estimation.

Florian T. Pokorny KTH Royal Institute of Technology fpokorny@kth.se

MS23

A Categorical Approach to Multipersistent Homology

Multipersistent homology represents an interesting challenge both from an applied and a theoretical point of view. Introduced by G.Carlsson and A.Zomorodian, this method analyzes a point cloud (noisy data set in a metric space) through two or more parameters. Classically, it is studied in the category of multigraded modules over the polynomial ring. We will rephrase multipersistence in the category of functors from N^r to vector spaces, give a constructive method to build free minimal resolutions and characterize Betti numbers.

Martina Scolamiero Politecnico di Torino martina.scolamiero@gmail.com

Wojciech Chacholski KTH Royal Institue of Technology wojtek@kth.se

<u>Francesco Vaccarino</u> Politecnico di Torino francesco.vaccarino@polito.it

MS24

Exotic Cluster Structure in Gl_n

Abstract not available at time of publication.

<u>Michael Gekhtman</u> University of Notre Dame mgekhtma@nd.edu

MS24

Tropical Curves in the Planar Dimer Model

Associated to a bipartite, periodic, edge-weighted planar graph is a bivariate polynomial P(z, w), which is the determinant of a certain adjacency matrix arising in the dimer model. In an appropriate "low temperature" limit the curve P = 0 becomes a tropical curve. Various features of the tropical curve have corresponding probabilistic interpretations for the resulting dimer model.

Richard Kenyon Brown University rkenyon@math.brown.edu

MS24

Generalized Discrete Toda Lattices

I will discuss a discrete-time dynamical system which generalizes the discrete Toda lattice. In a continuous limit this dynamical system becomes a variation of the Toda lattice. In the tropical limit this dynamical system gives a variation of the box-ball system. There are also connections to a combinatorial recurrence known as the octahedron recurrence, and to the study of graphs and networks embedded in a torus. This is joint work with Rei Inoue and Pavlo Pylyavskyy.

<u>Thomas Lam</u> University of Michigan tfylam@umich.edu

MS24

Higher-Dimensional Analogues of Tropical Cluster Combinatorics

Various tropical features of cluster structures (exchange relations, g-vectors) generalize beyond the setting of cluster algebras. I will recall the tropical structures of interest, specialized to the case of a polygon, and discuss what happens when the polygon is replaced by a higher-dimensional cyclic polytope. This is based on arXiv:1001.5437 and subsequent work.

Steffen Oppermann

Norwegian University of Science and Technology steffen.oppermann@math.ntnu.no

Hugh Thomas University of New Brunswick hthomas@unb.ca

MS25

Identifiability of Linear Structural Equation Models

This talk presents combinatorial conditions for generic identifiability of linear structural equation models. These models relate random variables of interest via a linear equation system and can be represented by a graph with two types of edges that correspond to non-zero coefficients in the linear equations and correlations among noise terms. Identifiability holds if the coefficients and correlations associated with the edges of the graph can be uniquely recovered from the covariance matrix they define.

<u>Mathias Drton</u> Department of Statistics

University of Washington md5@uw.edu

Rina Foygel Department of Statistics University of Chicago rina@uchicago.edu

Jan Draisma

Eindhoven University of Technology, The Netherlands Department of Mathematics and Computer Science j.draisma@tue.nl

MS25

Identifiability of Structural Equation Models on 6 Random Variables

Structural equation models (SEMs) are used to formalize a variety of causal queries as certain types of probability distributions. A central problem in SEMs is the analysis of identification. A model is identified if it only admits a unique parametrization to be compatible with a given data set. Here, we present the results of a computational experiment to find all the algebraically *d*-identified SEM models on at most six random variables.

Luis D. Garcia-Puente Sam Houston State University ldg005@shsu.edu

MS25

Algebraic Theory for Discrete Models in Systems Biology

Systems biology aims to explain how a biological system functions by investigating the interactions of its individual components from a systems perspective. Modeling is a vital tool as it helps to elucidate the underlying mechanisms of the system. Many discrete model types can be translated into the framework of polynomial dynamical systems (PDS), that is, time- and state-discrete dynamical systems over a finite field where the transition function for each variable is given as a polynomial. This allows for using a range of theoretical and computational tools from computer algebra, which results in a powerful computational engine for model construction, parameter estimation, and analysis methods.

Franziska B. Hinkelmann ICAM Virginia Tech fhinkel@vt.edu

MS25

Scaling Invariants and Symmetry Reduction of Dynamical Systems

Scalings form a class of group actions that have theoretical and practical importance. A scaling is accurately described by a matrix of integers. Tools from linear algebra over the integers are exploited to compute their invariants, rational sections (a.k.a. global cross-sections), and offer an algorithmic scheme for the symmetry reduction of dynamical systems. A special case of the symmetry reduction algorithm applies to reduce the number of parameters in physical, chemical or biological models: we provide a monomial reparametrisation of the variables that leads to a model that depends on less parameters. This can be understood as an (algorithmic) non-dimensionalisation. We illustrate the technique on models from the mathematical biology literature.

Evelyne Hubert INRIA Méditerranée evelyne.hubert@inria.fr

George Labahn University of Waterloo glabahn@uwaterloo.ca

MS26

An Incremental Algorithm for Computing Cylindrical Algebraic Decomposition and Its Application to Quantifier Elimination

In this talk, we present a new approach for computing cylindrical algebraic decomposition of real space via triangular decomposition of polynomial systems. Based on it, a general quantifier elimination method is proposed. The usage of our method is illustrated by several application examples.

Changbo Chen

University of Western Ontario changbo.chen@gmail.com

Marc Moreno Maza Computer Science Department University of Western Ontario moreno@csd.uwo.ca

MS26

Turning CAD Upside Down

We present a new decision procedure for solving the existential fragment of non-linear arithmetic. The classic approach to the problem is to project the problem polynomials, followed by model construction (lifting). The new procedure performs the lifting optimistically, and performs focused model-based projection only on the polynomials that are relevant in inconsistencies. The new approach, and the leaner projection operator, are effective in practice, which we support with an extensive experimental evaluation.

Dejan Jovanovic New York University dejan@cs.nyu.edu

Leonardo de Moura Microsoft Research leonardo@microsoft.com

MS26

An Application of Quantifier Elimination to Automatic Parallelization of Computer Programs

In automatic parallelization of computer programs, the socalled polyhedron model is a powerful geometrical tool for analyzing the relation between iterations of nested loops. To be practically efficient, one should avoid a too finegrained parallelization. It is also desirable for the generated code to depend on parameters such as number of processors, cache sizes, etc. These extensions of the polyhedron model lead to the manipulation of system of nonlinear polynomial equations and the use of techniques like quantifier elimination (QE), Unfortunately, classical algorithms for QE are not suitable, since they do not always produce conjunctions of atomic formulas, while this format is required in order to generate code automatically. As we shall demonstrate in this talk, this issue is addressed by our recent algorithm for computing cylindrical algebraic decomposition (C. Chen and M. Moreno Maza, 2012). Indeed, this algorithm supports QE in a way that the output of a QE problem has the form of a case discussion: this is appropriate for code generation.

Marc Moreno Maza

Computer Science Department University of Western Ontario moreno@csd.uwo.ca

Changbo Chen University of Western Ontario changbo.chen@gmail.com

MS26

Relative Equilibria in the Four-Vortex Problem with Two Pairs of Equal Vorticities

The N-vortex problem concerns the dynamics of N point vortices moving in the plane. Of particular interest in this problem are solutions that appear fixed when viewed in an uniformly rotating frame. Such solutions are called relative equilibria. In our work we gave a complete classification of the relative equilibria in the four vortex problem with two pairs of equal vorticities. In this talk I will outline the problem and describe some of the methods used to obtain our results, with particular emphasis to the triangular decomposition of semi-algebraic systems.

Manuele Santoprete Wilfrid Laurier University msantoprete@wlu.ca

Marshall Hampton University of Minnesota, Duluth mhampton@d.umn.edu

Gareth Roberts College of the Holy Cross groberts@radius.holycross.edu

MS27

Projective Path Tracking for Homotopy Continuation Methods

The homotopy continuation methods is a large class of reliable and efficient numerical methods for solving systems of polynomial equations. An essential component in such methods is the path tracking algorithm for tracking smooth paths of one real dimension. "Divergent paths' pose a tough challenge to these algorithms. A well known remedy is to operate inside the complex projective space instead. This talk focuses on the path tracking algorithms in the complex projective space.

Tianran Chen

Michigan State University chentia1@msu.edu

Tien-Yien Li Department of Mathematics Michigan State University li@math.msu.edu

MS27

Numerically Computing Polynomial Images of Algebraic Sets with Applications

Given a polynomial map π defined on an algebraic set V, a common goal is to compute the irreducible decomposition of $\overline{\pi(V)}$. In this talk, we will discuss numerical methods for determining the dimension, degree, and other invariants for each component of this decomposition. The talk will conclude by applying these methods to systems arising in physics.

<u>Noah Daleo</u>, Jonathan Hauenstein North Carolina State University nsdaleo@ncsu.edu, hauenstein@ncsu.edu

MS27

Applications of Numerical Elimination Theory

An extensive collection of problems in algebraic geometry, algebraic statistics, biology, geophysics, linear algebra, and physics can be formulated in terms of computing the irreducible decomposition of the closure of the polynomial image of an algebraic set. Often in these applications, the polynomial map is a nonlinear mapping from hidden states to observable states. This talk will explore using various numerical algebraic geometric algorithms to solve problems arising in several of these applications.

<u>Jonathan Hauenstein</u>

North Carolina State University jdhauens@ncsu.edu

MS27

On Massively Parallel Algorithms to Track One Path of a Polynomial Homotopy

The latest generation of graphics processors delivers one Tflop of peak performance but requires massively parallel algorithms occupying thousands of threads. In previous work we obtained good speedups for two building blocks to run Newton's method: the evaluation and differentiation of polynomial systems, and the solving of linear systems in the least squares sense, using double double and quad double arithmetic. This talk will present a massively parallel algorithm to track one solution path of a homotopy defined by a polynomial system.

Jan Verschelde

Department of Mathematics, Statistics and Computer Science University of Illinois at Chicago jan@math.uic.edu

Genady Yoffe Dept. of Mathematics, Statistics, and CS University of Illinois, Chicago gyoffe2@uic.edu

Xiangcheng Yu Department of Mathematics, Statistics, and Computer Science University of Illinois at Chicago xyu30@uic.edu

MS27

Numerical Algebraic Intersection Using Regeneration

In numerical algebraic geometry, algebraic sets are represented by witness sets. This paper presents an algorithm, based on the regeneration technique, that solves the following problem: given a witness set for pure-dimensional algebraic set Z, along with a system of polynomial equations $f: Z \to C^n$, compute a numerical irreducible decomposition of $Z \cap V(f)$. Also treated is the case where Z is the cross product of two or more pure-dimensional sets, each given in terms of a witness set. Two existing algorithms, diagonal intersection and the homotopy membership test, can be seen as special cases of the new algorithm. In addition to this unification, the method also extends the range of problems that can be solved using numerical algebraic geometry.

Charles Wampler

General Motors Research Laboratories, Enterprise Systems Lab 30500 Mound Road, Warren, MI 48090-9055, USA charles.w.wampler@gm.com

Jonathan Hauenstein North Carolina State University hauenstein@ncsu.edu

MS28

K-Theory of Toric Varieties Revisited

Algebraic K-theory is a synthetic subject, drawing ideas from various mathematical disciplines. In this talk we will review algebraic K-theory of monoid rings and toric varieties, starting from the 1980s and including very recent developments. The classical part of this theory is algebraic in the true meaning of the word, allowing very meaningful application: algorithms for solving systems of linear equations with multivariate polynomial coefficients, factoring invertible matrices into elementary ones (and such algorithms have been implemented in the past). However, as we move to higher K-groups, the nature of the subject changes from algebraic to topological and even categorial. Correspondingly, the applied element quickly becomes ephemeral - but not in the sense of applications to other areas of mathematics.

Joseph Gubeladze

San Francisco State University soso@sfsu.edu

MS28

Frobenius Splitting and Toric Varieties

Frobenius splittings provide some very useful techniques for the study of algebraic varieties. After a quick review of Frobenius splittings I will present some results on the F-splitting ratio of a toric ring, an invariant closely related to the F-signature that measures the singularities of the variety.

Milena Hering

University of Connecticut milena.hering@gmail.com

Kevin Tucker Princeton University kftucker@math.princeton.edu

MS28

Equivariant Vector Bundles on T-Varieties

Klyachko has shown that there is an equivalence of categories between equivariant vector bundles on a toric variety X and collections of filtered vector spaces satisfying some compatibility conditions. I will discuss joint work with H. Süß which generalizes this equivalence to the setting of Tequivariant vector bundles on a normal variety X endowed with an effective action of an algebraic torus T. Indeed, Tequivariant vector bundles on a suitable quotient of X. This correspondence can be applied to show that T-equivariant bundles of low rank on projective space split, as well as to easily compute the space of global vector fields on rational complexity-one T-varieties.

<u>Nathan Ilten</u>

University of California, Berkeley nilten@math.berkeley.edu

MS28

The Hodge Theory of Hypersurfaces

Geometric properties of generic hypersurfaces in projective toric varieties are often determined by their corresponding lattice polytopes. Pioneering work of Danilov-Khovanskii gave combinatorial descriptions for their Euler characteristic and χ_y -characteristic in terms of combinatorics. In joint work with Alan Stapledon, we outline an alternative approach to hypersurfaces. Here, we degenerate the hypersurface into a union of linear subspaces and use the limit mixed Hodge structure to understand the cohomology. We may also explain some applications to Ehrhart theory.

<u>Eric Katz</u>

Department of Combinatorics and Optimization University of Waterloo eekatz@math.uwaterloo.ca

MS28

Syzygies and Singularities of Tensor Product Surfaces of Bidegree (2,1)

Let $\{f_1, \ldots, f_4\}$ be four bihomogeneous polynomials of bidegree (2,1), having no common basepoints on $X = P^1 \times P^1$. The f_i define a regular map from X to P^3 .

We study the associated bigraded ideal I of the f_i from the standpoint of commutative algebra, proving that there are exactly six numerical types of possible bigraded minimal free resolution. These resolutions play a key role in determining the implicit equation of the image, and we show that there is a pleasing dictionary which relates the syzygies on I to the singularities of the image.

Hal Schenck

Mathematics Department University of Illinois schenck@math.uiuc.edu

A. Seceleanu University of Nebraska aseceleanu@math.unl.edu

J. Validashti Univeristy of Illinois jvalidas@illinois.edu

MS29

Degree of Regularity of Hfe Family of Cryptosystems

In this talk, we will present the recent results on the degree of regularity of Hidden Field Equation (HFE) family of multivariate public key cryprosystems, including, HFE, HFE-, HFEv and HFEv-, in particular, the bound formulas for the degree of regularity of these systems . Furthermore we will discuss the implication of these results on the security of those multivariate public key cryptosystems under direct algebraic attack using multivariate polynomial solvers.

Jintai Ding University of Cincinnati jintai.ding@gmail.com

MS29

Degree of Regularity of Generalized HFE Cryptosystems

Ding and Hodges recently showed that the algebraic attack on Patarin's HFE system is quasi-polynomial over any base field. We extend this result to more generalized systems where the central polynomial is not necessarily quadratic (i.e., is of higher degree) over the base field. The core of the proof lies in some interesting combinatorial properties of generalized lacunary binomial coefficients.

Timothy Hodges University of Cincinnati, USA hodgest@ucmail.uc.edu

MS29

Overview of Post-Quantum Cryptography

This talk gives an overview of post-quantum cryptography to set the stage for the mini symposium. It motivates the need for cryptography resisting attacks using quantum computers and briefly presents the most relevant 4 groups of systems: code-based cryptography, lattice-based cryptography, multivariate signatures, and hash-based signatures.

Tanja Lange Technische Universiteit Eindhoven Eindhoven Institute for the Protection of Systems and Inform tanja@hyperelliptic.org

MS29

On the Practical and Asymptotic Complexity of Solving Generic Systems of Equations

Existing literature overwhelmingly list F4 /F5 algorithms as optimal general solution to MQ, or solving a system of m quadratic equations in n variables in Fq. We show that the complexity of system-solving should be estimated differently in the following cases: 1. For random or generic \mathbf{F}_2 systems, brute-force is often better. 2. For many random systems (small-field, high-degree, or somewhat overdetermined with $m \propto n$) XL and F4/F5 operates at degrees max 1 apart, and here Sparse XL variants are likely pragmatically or asymptotically better.

Bo-Yin Yang

Academia Sinica by@crypto.tw

MS30

Persistence Barcode Signatures for Image Classification

Persistent topology provides a method for understanding data sets in general, and also specifically for data sets in which the points are "shapes" of some kind. We will discuss this methodology, with some examples.

<u>Gunnar E. Carlsson</u> Stanford University gunnar@math.stanford.edu

MS30

Image Segmentation with Topological Information - Yet Another Application of Persistent Homology

Topological information could be very useful for the task of image segmentation. However, most existing works have trouble incorporating such combinatorial information into the optimization framework. Persistent homology, on the other hand, is a natural tool to quantitatively measure topological features, in the view of a given scalar function. In this talk, I will introduce how to use persistent homology in image segmentation, as well as how topological information could improve segmentation results.

<u>Chao Chen</u>

Rutgers University chao.chen@rutgers.edu

MS30

Persistence Simplification with Iterated Morse Complex Decomposition

In this talk we will combine ideas of discrete Morse theory and (persistent) homology. A finite sequence of Morse complexes each of which preserves (persistent) homology of the initial object will be constructed. The final complex in the sequence is a minimal complex with the same (persistent) homology as the initial object. Later we will show how to continue the construction to obtain barcodes of a given object. This is joint work with Hubert Wagner.

Pawel Dlotko

Institute of Computer Science Jagiellonian University pawel.dlotko@ii.uj.edu.pl

MS30

Stability of Persistence Spaces for Vector-Valued Functions

In shape analysis it is common to deal with data carrying multi-parameter information. Multidimensional persistence allows for a topological analysis of these data. Unfortunately, multidimensional persistent modules do not admit concise representations analogous to persistence diagrams in the one-parameter setting. However, there is no obstruction for multidimensional persistent Betti numbers to admit one. In this talk, the persistence space of a multifiltration is shown to generalize the concept of persistence diagram in this sense.

<u>Claudia Landi</u>

University of Modena and Reggio Emilia claudia.landi@unimore.it

Andrea Cerri IMATI-CNR andrea.cerri@ge.imati.cnr.it

MS30 Metric Geometry and Persistent Homology

We show how metric measure spaces – a mathematical model for datasets – induce certain stable persistence homology invariants whose study provides some insights into the problem of computing statistical summaries of the persistent homology of large datasets. These invariants are a family of probability distributions over persistence diagrams which turn out to be stable in the Gromov-Wasserstein sense.

Facundo Memoli

Stanford University, Mathematics Department The University of Adelaide memoli@math.stanford.edu

MS31

Extremal Betti Tables

We will discuss extremal Betti tables of resolutions in three different contexts. We begin over the graded polynomial ring, where extremal Betti tables correspond to pure resolutions and can be realized via so-called tensor complexes. We then contrast this behavior with that of extremal Betti tables over regular local rings and over a bigraded ring.

<u>Christine Berkesch</u> Duke University cberkesc@gmail.com

Daniel Erman University of Michigan danerman@umich.edu

Manoj Kummini Chennai Mathematical Institute mkummini@cmi.ac.in

MS31

A Set-theoretic Proof of the Salmon Conjecture

In 2007, motivated by applications to molecular phylogenetics, Elizabeth Allman offered the challenge of determining the polynomials that vanish on the irreducible variety of 4 x 4 x 4 complex valued tensors of border rank at most 4. In this talk, we will show that this variety is the zero set of polynomials of degrees 5, 6, and 9. We will focus mainly on the degree 6 polynomials.

<u>Elizabeth Gross</u> University of Illinois at Chicago egross7@uic.edu

Shmuel Friedland Department of Mathematics University of Illinois, Chicago friedlan@uic.edu

MS31

Tangential Varieties of Segre-Veronese Varieties

I will report on joint work with Luke Oeding, involving the determination of the homogeneous coordinate ring, and of the minimal generators of the defining ideal of the tangential variety of a Segre-Veronese variety. In the special case of a Segre variety, our results confirm a conjecture of Landsberg and Weyman.

<u>Claudiu Raicu</u> Princeton University craicu@princeton.edu

Luke Oeding UC Berkeley oeding@math.berkeley.edu

MS32

Lozenge Tilings and the Weak Lefschetz Property

Let I be a monomial ideal in K[x, y, z]. We describe a connection between I and a sequence of planar regions of the triangular grid. We use this connection to determine the presence (or absence) of the weak Lefschetz property for I. This is joint work with Uwe Nagel at the University of Kentucky.

David Cook II University of Notre Dame dcook8@nd.edu

MS32

Computer-aided Unirationality Proofs

An algebraic variety is called unirational if there is dominant map from some projective space to it. Of particular interest is the case of unirational moduli spaces of certain objects since this translates to a parametrization of the objects in free parameters in terms of the dominant map. In this talk we present a recent unirationality results for Hurwitz spaces (i.e. parameter spaces of simply branched coverings of the projective line by smooth curves) and show how computer-algebra can be used to substantially ease the step of showing that the parametrizing map is dominant by using computer algebra. The implementation of such a parametrization can be useful for various applications. We present how this method enters in existence results for certain vector bundles on cubic hypersurface threefolds.

<u>Florian Gei</u>, Frank-Olaf Schreyer Universität des Saarlandes fg@math.uni-sb.de, schreyer@math.uni-sb.de

MS32

Inferring Biologically Relevant Models: Nested Canalyzing Functions

Inferring dynamic biochemical networks is one of the main challenges in systems biology. Given experimental data, the objective is to identify the rules of interaction among the different entities of the network. However, the number of possible models fitting the available data is huge, and identifying a biologically relevant model is of great interest. Nested canalyzing functions, where variables in a given order dominate the function, have recently been proposed as a framework for modeling gene regulatory networks. Previously, we described this class of functions as an algebraic toric variety. In this talk , we present an algorithm that identifies all nested canalyzing models that fit the given data.

<u>Franziska Hinkelmann</u> Ohio State University franziska.hinkelmann@gmail.com

MS32

Groebner-free Computations with Binomial Ideals

The membership problem for a pure difference binomial ideal I is graph connectivity: A binomial is contained in I iff there exists a path connecting its two monomials using binomials in I. This idea is applicable even if Gröbner basis methods are infeasible. We present a C++ implementation which computed the first counterexample to radicality of a global Markov ideal. Based on joint work with Johannes Rauh and Seth Sullivant.

Thomas Kahle

Isaac-Newton-Institute thomas.kahle@tum.de

MS32

Algebraic Statistics and Macaulay2: Running Markov Chains on Network Fibers

Sampling from a fiber of a (social) network is a crucial step in goodness-of-fit testing for network models in statistics. In order to apply recent theoretical results, one needs to be able to explore a fiber of an observed network. This talk will focus on an algorithm for p_1 random graph models. Macaulay2 plays a crucial role in the development and experimentation.

Elizabeth Gross University of Illinois at Chicago egross7@uic.edu

Sonja Petrovic Department of Statistics Pennsylvania State University petrovic@psu.edu

Despina Stasi Pennsylvania State University despina@psu.edu

MS33

Polytopes with Minimal Semidefinite Representations

Finding good semidefinite representations of polytopes is a very relevant problem for combinatorial optimization and many methodologies for doing so have been proposed. The problem of finding the smallest possible representation of a polytope has been recently recast as a problem of matrix factorization. In this talk, we will give an overview of recent developments in this area, with special focus on a characterization of polytopes which have the smallest possible representations.

Joao Gouveia Universidade de Coimbra jgouveia@mat.uc.pt

Richard Robinson, Rekha Thomas University of Washington rzr@math.washington.edu, rrthomas@uw.edu

MS33

Realizing Hyperbolicity Cones As Spectrahedra and Their Projections

Spectrahedral cones are the feasible sets of semidefinite programming. Hyperbolicity cones form a potentially larger class of convex cones. Trying to realize hyperbolicity cones as spectrahedral cones or projections thereof is an interesting and important task. Important, since it helps classifying the sets on which semidefinite programming can be performed. Interesting, since methods and results from different areas, such as algebraic geometry, positivity and sums of squares, non-commutative geometry, and more, are involved when solving such problems. I will report on recent methods and results concerning these questions.

<u>Tim Netzer</u>

University of Leipzig tim.netzer@math.uni-leipzig.de

MS33

On Elliptesque and Hyperbolesque Curves

Any five points in the plane (no four on a line) determine a unique conic section. What can be said about a curve C with the property that any five points chosen from Ceither always determine an ellipse or always determine a hyperbola?

Bruce Reznick

University of Illinois, Urbana-Champaign reznick@math.uiuc.edu

MS33

Polynomial-Sized Semidefinite Representations of Derivative Relaxations of Spectrahedral Cones

The hyperbolicity cones associated with the elementary symmetric polynomials provide an intriguing family of nonpolyhedral relaxations of the non-negative orthant which preserve its low-dimensional faces and successively discard higher dimensional facial structure. We show by an explicit construction that this family of convex cones (as well as their analogues for symmetric matrices) have polynomialsized representations as projections of slices of the PSD cone. This, for example, allows us to solve the associated linear cone program using semidefinite programming.

James Saunderson, Pablo A. Parrilo Massachusetts Institute of Technology jamess@mit.edu, parrilo@MIT.EDU

MS34

Recursive Interpolation of a Sparse Polynomial Given by a Straight-Line Program

We consider the problem of interpolating a sparse univariate polynomial given by a straight-line program. Suppose we are given a univariate polynomial f(z) represented by a straight-line program, and that we have bounds T and Don the sparsity and degree of f(z) respectively. We present a recursive, randomized algorithm for interpolating f, with complexity quasi-linear with respect to $T \log^3(D)$. This algorithm builds previous work by Giesbrecht and Roche, and Garg and Schost.

Andrew Arnold

Cheriton School of Computer Science University of Waterloo a4arnold@uwaterloo.ca

Mark Giesbrecht School of Computer Science University of Waterloo mwg@uwaterloo.ca

Dan Roche United States Naval Academy roche@usna.edu

MS34

New Approaches to Sparse Interpolation and Signal Reconstruction

Abstract not available at time of publication.

Mark Giesbrecht

School of Computer Science University of Waterloo mwg@uwaterloo.ca

MS34

Numerical Issues with Sparse Interpolation

Abstract not available at time of publication.

George Labahn University of Waterloo glabahn@cs.uwaterloo.ca

MS34

Sparse Interpolation with Noise and Outliers

We address the problem of interpolation of sparse polynomials in the presence of noise and errors in the evaluations. Our approach is based on Ben-Or & Tiwari's and Berlekamp/Massey algorithms with early termination. We show a lower bound on the number of terms required in a sequence in order to recover its minimal linear generator. We then provide an algorithm that finds this generator. A duality with Reed-Solomon decoding will be shown. Lastly, we will address the more general problem of rational function and number reconstruction with errors.

<u>Clément Pernet</u> Uniiversité de Grenoble clement.pernet@imag.fr

$\mathbf{MS34}$

Combining Tricks for Exact Sparse Interpolation

Abstract not available at time of publication.

Daniel S. Roche Dept. of Computer Science US Naval Academy roche@usna.edu

MS35

A Numerical Algorithm for Zero Counting

In memoriam of our dear friend Mario In a series of three articles we produce and analyze a numerical (iterative) algorithm for computing the exact number of real projective zeros of a system of n real homogeneous polynomial equations in n+1 variables. In the first one, we show that the number of iterations, and the cost of each iteration, depends on a condition number of the system, in addition to other usual parameters. The algorithm can be implemented with finite precision as long as the round-off unit satisfies some bound depending on the same parameters. In the second one, we show that the condition number that we introduced satisfies an Eckard-Young theorem, as it represents the inverse of the distance of the input system to the ill-posed systems. We derive from this smoothed analysis bounds for it, applying other general results. Finally, in the third one, we use specific probability techniques as a Rice theorem to obtain more precise bounds for the tail and the expected value of the condition number, considered as a random variable when imposing a probability measure on the space of polynomial systems.

Felipe Cucker City University, Hong Kong macucker@cityu.edu.hk

<u>Teresa Krick</u> Universidad de Buenos Aires krick@dm.uba.ar

Gregorio Malajovich UFRJ, Brazil gregorio@ufrj.br

Mario Wschebor Universidad de la Republica, Uruguay wschebor@cmat.edu.uy

MS35

How Far Are Archimedean Tropical Varieties from Amoebae?

There is a simple, and arguably natural way to associate a polyhedral approximation to any complex hypersurface Z(f): we call this approximation the Archimedean tropical variety ArchTrop(f). The univariate case dates back to work of Ostrowski around 1940, and the higher-dimensional case has appeared in work of Passare, Mikalkin, and other authors starting around 2000. However, the distance between Z(f) and ArchTrop(f) has not been made studied in general. We give a simple and explicit upper bound for the Hausdorff distance between Z(f) and ArchTrop(f). As an application, we show how to efficiently decide whether a given ball contains any roots of a given polynomial system.

J. Maurice Rojas, Martin Avendano Texas A&M University rojas@math.tamu.edu, mavendar@yahoo.com.ar

MS35

On the Complexity of Computing the Dimension of Semi-algebaic Sets

Let S be a semi-algebraic set defined by a system of s polynomial inequalities in n variables of degree $\leq D$. We give an algorithm that computes the real dimension of the semi-algebraic set S in $(sD)^{O(n)}$ arithmetic operations.

Mohab Safey El Din UPMC, Univ Paris 6 Mohab.Safey@lip6.fr

Elias Tsigaridas INRIA/Univ. Pierre et Marie Curie, France elias.tsigaridas@gmail.com

$\mathbf{MS35}$

On the Complexity of Solving Bivariate Polynomial Systems

We give an algorithm for solving bivariate polynomial systems over either k(T)[X, Y] or Q[X, Y] using a combination of lifting and modular composition techniques.

Esmaeil Mehrabi ORCCA Western University emehrab@uwo.ca

<u>Eric Schost</u> Computer Science Department Univ. of Western Ontario eschost@uwo.ca

Romain Lebreton LIRMM Université Montpellier 2 romain.lebreton@lirmm.fr

MS36

Computing Hypergeometric Solutions of Second Order Differential Equations with Five Singularities

If a generating function of an integer or a near-integer sequence is convergent and holonomic (satisfies a linear differential equation), then we have observed that such differential equation of order 2 or 3 has either algebraic solutions or logarithmic singularities. In the later case we can express its solution in terms of $_2F_1$ -hypergeometric functions. That gives us a way to prove whether a sequence is an integer or a near-integer sequence. In this talk we will discuss about computing $_2F_1$ -type solutions of a second order linear differential equation with five singularities.

Vijay Kunwar Florida State University Tallahassee vkunwar@math.fsu.edu

Mark van Hoeij Florida State University Tallahassee, FL hoeij@math.fsu.edu

MS36

Computing Decompositions of Hypergeometric Terms

Two algorithms will be described in this talk. The first decomposes a univariate hypergeometric term into a sum of a hypergeometric-summable term and a non-summable one. It is an improvement of Abramov-Petkovšek's algorithm for additive decompositions of hypergeometric terms. The second computes a multiplicative decomposition of a multivariate hypergeometric term. It is based on the Ore-Sato theorem, which states that such a term is the product of a rational function and several factorials.

Ziming Li

Key Laboratory of Mathematics Mechanization AMSS, Chinese Academy of Sciences zmli@mmrc.iss.ac.cn

MS36

A Combinatorial Approach to Lattice Path Asymptotics

Several approaches for finding asymptotic formulas for lattice path models restricted to various regions have recently been proposed. This talk will describe some recent progress, including algorithmic and probabilistic strategies, and then focus on our new, combinatorial point of view. Our approach uses only elementary combinatorial arguments; eliminates the case analysis that was previously necessary; and can be extended to a much larger framework, for example, models in higher dimensions, and cones other than the quarter plane. Work in progress with Samuel Johnson and Karen Yeats (Simon Fraser).

<u>Marni Mishna</u> Simon Fraser University mmishna@sfu.ca

MS36

Holonomic Integer Sequences and Transcendental Numbers

Let L be a recurrence relation, with polynomial coefficients over the rational numbers. If the quotient u(n)/v(n) converges, for every pair of non-zero sequences u, v that (a) satisfy L, (b) have rational values, and (c) have non-zero tails, then we say that L is a single-growth recurrence. Such recurrences are quite common (e.g. Apery's recurrence). If the tails of u, v, w are linearly independent then the limits of u(n)/w(n), v(n)/w(n) and 1 must be linearly independent over the rational numbers. This can be used to prove various irrationality results, including Baker's formulation of the Lindemann-Weierstrass theorem.

Mark van Hoeij Florida State University hoeij@mail.math.fsu.edu

Automatic Natural Language Mathematical Problem Solving Using Real Quantifier Elimination

We present a logic-based architecture for solving mathematical problems written in natural language. At the core of the system is a computer algebra solver that handles any formula in the language of real closed field. The natural language processing (NLP) module translates the problem text into a logical form on the basis of linguistic analysis of the text. We aim at an end-to-end problem solving system through the synthesis of the advances in linguistics, NLP, and computer mathematics. We show some empirical results on Japanese university entrance examination problems.

<u>Hirokazu Anai</u> Kyushu University h.anai@kyudai.jp

Hidenao Iwane

FUJITSU LABORATORIES LTD / Kyushu University 4-1-1 Kamikodanaka, Nakahara-ku, Kawasaki 211-8588, JAPAN iwane@jp.fujitsu.com

Takuya Matsuzaki, Norico Arai National Institute of Informatics takuya-matsuzaki@nii.ac.jp, arai@nii.ac.jp

MS37

Constructing a Single Cell in a Cylindrical Algebraic Decomposition

In a 2012 paper, Jovanovic and de Moura present NLSAT, an novel algorithm that uses Conflict-Driven Clause Learning (CDCL)-style search to decide the satisfiability of systems of polynomial equalities and inequalities over the real numbers. An essential step in this algorithm is to take an assignment of real values that do not satisfy the original system, and generalize it to a larger region in which all points fail to satisfy the original system. They do this by constructing a single cell in a Cylindrical Algebraic Decomposition (CAD) defined from a set of polynomials namely the cell that contains the unsatisfying assignment. This motivates the consideration of a new problem: Given a set P of polynomials in n variables and a point α in ndimensional real space, how can we efficiently construct a representation of the cell containing α from the CAD defined (with respect to some projection operator) by the set P. This talk will present an algorithm for this problem.

Christopher Brown United States Naval Academy wcbrown@usna.edu

MS37

Sparse Interpolation and Error-Correcting Coding

Interpolation of univariate polynomials from values, with several of the values erroneous, is used to construct the Reed-Solomon error correcting code. The Berlekamp/Welch decoder recovers a polynomial of degree < d from values at d + 2E distinct points, where $k \leq E$ values are faulty, that for unkown k at unknown locations. With Matthew Comer and Clement Pernet [ISSAC 2012] we have considered recovery of a t-sparse polynomial of unknown sparsity t, with a given bound $T \geq t$. Our decoding procedure requires (2E + 1)2T values, while an error-

correcting code is possible for evaluations at 2T + 2E distinct positive real points. We use our decoding procedure for numeric sparse interpolation with outlier errors. In my talk I will discuss the hardness of the $\Omega(ET)$ lower bound of evaluations for the task of correcting errors in exact and numeric sparse interpolation.

Erich Kaltofen

North Carolina State University Mathematics Department kaltofen@math.ncsu.edu

MS37

Hybrid Methods for Exact Geometric Computation

Classical methods for exact computation with geometric objects such as algebraic curves and surfaces mainly suffer from the high computational cost for the considered symbolic operations. For instance, most of the existing methods for computing the topology of a planar algebraic curve $C = V(f), f \in Z[x, y]$, need to compute the entire subresultant sequence of the defining polynomial fand its derivative f_y , and the latter computation has to be considered even in the case, where the given curve has a very simple geometry, or where we are only interested in the topology of the curve restricted to a certain local neighborhood. This non-adaptive behavior partially also explains why such exact methods based on a heavy algebraic machinery have nearly not been used in algorithms from Computational Geometry. In this talk, we summarize some recent results on the development of symbolicnumerical algorithms to compute arrangements of planar algebraic curves. The novel algorithms need a considerably less amount of purely symbolic operations, whereas most computations are based on approximate but certified arithmetic. In addition, the remaining symbolic computations are outsourced to a highly efficient implementation on graphics card. Exhaustive benchmarks show that the corresponding implementations do not only outperform existing methods by magnitudes, but they are even comparable to purely numerical approaches with respect to running time. Furthermore, a theoretical analysis gives evidence that the novel algorithms achieve record bounds for the bit complexity of the considered problem which seems to be almost optimal. Finally, we outline how to combine the latter mentioned methods with a purely numerical subdivision approach. We expect that such a hybrid method will eventually yield excellent local behavior as well.

Michael Sagraloff MPII, Germany msagralo@mpi-inf.mpg.de

MS37

New Algorithms for Computing Roadmaps in Smooth Bounded Real Algebraic Sets

We consider the problem of constructing roadmaps of real algebraic sets. This problem was introduced by Canny to answer connectivity questions and solve motion planning problems. Canny's algorithm is recursive, but decreases the dimension only by one through each recursive call. In this talk, we present a divide-and-conquer version of Canny's algorithm, where the dimension is halved through each recursive call.

Mohab Safey El Din UPMC, Univ Paris 6 Mohab.Safey@lip6.fr

Eric Schost

Computer Science Department Univ. of Western Ontario eschost@uwo.ca

MS38

Nontrivial Bounds for Steady State Concentrations

We develop tools from computational algebraic geometry for the study of biochemical reaction networks with massaction kinetics. If there exists a *positive* conservation relation involving a given species, one gets a *trivial* upper bound for its concentration all along the trajectory. Under general conditions, we get better *nontrivial* upper bounds at steady state, regardless of explicit analytic expressions or the occurrence of multistationarity.

<u>Alicia Dickenstein</u>, Mercedes Pérez Millán Universidad de Buenos Aires alidick@dm.uba.ar, mercedes.s.pm@gmail.com

MS38

Zero-Eigenvalue Turing Instability in General Chemical Reaction Networks

We describe a necessary condition for zero-eigenvalue Turing instability, i.e., Turing instability arising from a real eigenvalue changing sign from negative to positive, for general chemical reaction networks modeled with mass-action kinetics. The reaction mechanisms are represented by the species-reaction graph (SR graph). If the SR graph satisfies certain conditions, similar to the conditions for ruling out multiple equilibria in spatially homogeneous differential equations systems, then the corresponding mass-action reaction-diffusion system cannot exhibit zero-eigenvalue Turing instability for any parameter values.

Maya Mincheva Northern Illinois University mincheva@math.niu.edu

Gheorghe Craciun Department of Mathematics, University of Wisconsin-Madison craciun@math.wisc.edu

MS38

Ruling out Hopf Bifurcations in Systems of Interacting Elements

We describe a new graphical approach to the question of whether dynamical systems modeling systems of interacting elements admit Hopf bifurcations. The techniques make use of the spectral properties of additive compound matrices: in particular, we show that a condition on the cycles of a labelled digraph (called the DSR^[2] graph) rules out the possibility of nonreal eigenvalues of the Jacobian matrix passing through the imaginary axis. The connection with the DSR graph and relevant biological examples will also be discussed.

<u>Casian Pantea</u> Imperial College London c.pantea@imperial.ac.uk

Murad Banaji

University of Portsmouth murad.banaji@port.ac.uk

David Angeli Imperial College d.angeli@imperial.ac.uk

MS38

Enzymatic Networks and Toric Steady States

A chemical reaction system has toric steady states if its steady state ideal is binomial. In these systems, some questions can be answered easily (for example, the existence of positive steady states and multistationarity). It was shown in Pérez Millán et al. (2012) that the network for multisite phosphorylation in a sequential and distributive mechanism has toric steady states. We show here a broader family of enzymatic networks with this property.

<u>Mercedes Perez Millan</u>, Alicia Dickenstein Universidad de Buenos Aires mpmillan@dm.uba.ar, alidick@dm.uba.ar

MS39

A-Hypergeometric Systems and Estimation Problems in Statistics

Nakayama et al. (2011) proposed the holonomic gradient descent (HGD) which is a new method in statistics to make a maximal likelihood estimate for a given unnormalized probability distribution. We give new algorithms of transforming a given A-hypergeometric system into a Pfaffian system which the HGD requires as an input. We expect that our algorithms yield a new class of an exponential family of probability distributions for which we can apply the HGD efficiently.

Takayuki Hibi Osaka University / JST CREST hibi@math.sci.osaka-u.ac.jp

Kenta Nishiyama Shizuoka Prefecture University k-nishiyama@u-shizuoka-ken.ac.jp

Nobuki Takayama Kobe University takayama@math.kobe-u.ac.jp

MS39

Holonomic Gradient Descent in Directional Statistics

Abstract not available at time of publication.

<u>Tomonari Sei</u> Keio University sei@math.keio.ac.jp

MS39

Holonomic Gradient Method for Multivariate Normal Distribution Theory

Abstract not available at time of publication.

<u>Akimichi Takemura</u> Tokyo University takemura@stat.t.u-tokyo.ac.jp

MS39 A-Hypergeometric Systems

Abstract not available at time of publication.

<u>Uli Walther</u> Department of Mathematics Purdue University walther@math.purdue.edu

MS40

Solving Polynomial Systems in Noether Position with Puiseux Series

A system of polynomials is said to be in Noether position when we may set the first d coordinates to zero and obtain a smaller system, which defines a zero dimensional solution set. Such smaller systems are called initial form systems. We obtain the initial form systems by identifying special exponent vectors, called pretropisms. Pretropisms are the analogues to the points in the tropical prevariety of the system. Each d dimensional cone of pretropisms we find, leads to an initial form system and identifies which coordinates may be set to zero. The d dimensional cone of pretropisms may become the leading exponents of the Puiseux series of a d dimensional solution set and the regular solutions of the initial form system the leading coefficients, provided that we can find a second term in the Puiseux series. In this talk we show how to obtain the second terms in the series. We first show how to identify the exponents of the second terms and then we show how the coefficients of the second terms can be obtained by solving smaller linear systems. Once the second terms are obtained, more terms can be computed by iterating the same method that was used for the second terms.

Danko Adrovic, Jan Verschelde Department of Mathematics, Statistics and Computer Science University of Illinois at Chicago adrovic@math.uic.edu, jan@math.uic.edu

MS40

Application of Numerical Algebraic Geometry to Geometric Data Analysis

It is desirable to represent large data sets with smaller ones. One naive approach is to take the arithmetic mean of the data set, though this is not always sufficient. In this talk, I will define a subspace mean that is unitarily invariant. Computing this mean comes with a price; it is the solution to a challenging optimization problem. I will reformulate this as a polynomial system and discuss solution strategies via numerical algebraic geometry.

<u>Brent R. Davis</u> Colorado State University bdavis1@rams.colostate.edu

Daniel J. Bates Colorado State University Department of Mathematics bates@math.colostate.edu

Chris Peterson, Michael Kirby, Justin Marks Colorado State University $peterson@math.colostate.edu, \ kirby@math.colostate.edu, marks@math.colostate.edu$

MS40

Bifurcation Analysis and Continuation Methods in Celestial Mechanics

The computation at the heart of many mathematical models of physical systems is the solution of a parameterdependent system of nonlinear equations. Many such problems give rise to families of solutions. The computation of such solution families and their singularities, e.g., folds, bifurcation points, etc., is the domain of numerical parameter continuation algorithms. In this talk we will apply such methods to periodic trajectories in the Circular Restricted Three Body Problem (CR3BP) from celestial mechanics. In particular, we will focus on computing families of periodic orbits arising from Hopf bifurcations, commonly referred to as Libration points. We will show how to compute families of periodic solutions of conservative systems with two-point boundary value problem continuation software, including detection of bifurcations and corresponding branch switching.

Randy Paffenroth

Numerica randy.paffenroth@numerica.us

MS40

Polynomial Systems and Algebraic Number Fields

Let F(z) be a polynomial system with coefficients in an algebraic number field A. In this talk we discuss work with Chris Peterson (Colorado State Univ.) and Tim McCoy (Notre Dame Univ.) to numerically compute the irreducible decomposition of the solution set V(F) over A.

<u>Andrew Sommese</u> Dept. of Applied & Computational Mathematics & Statistics University of Notre Dame sommese@nd.edu

MS41

Arithmetic of Jacobians over Function Fields

By work of Shioda and Katsura, a Fermat variety admits a dominant rational map from a product of lower dimensional Fermat varieties. A generalization of their work allows one to explicitly construct surfaces, dominated by products of curves, with this property retained under extension of the base field. We describe this generalization and applications to the arithmetic of Jacobians over function fields.

Lisa Berger SUNY Stony Brook lbrgr@math.sunysb.edu

MS41

The Local-global Principle for Divisibility in the Cohomology of Elliptic Curves

The Grunwald-Wang theorem implies that a rational number is an n-th power if and only if it is an n-th power everywhere locally. When the multiplicative group is replaced by a general commutative algebraic group this local-global principle for divisibility by n can fail. I will discuss recent work in which we show that for elliptic curves the localglobal principle for divisibility by n can fail if and only if n is divisible by 4 or 9. Time permitting I will also discuss analogous results for divisibility in the Weil-Châtelet group and other higher cohomology groups of elliptic curves.

<u>Brendan Creutz</u> University of Sydney School of Mathematics and Statistics B.Creutz@maths.usyd.edu.au

MS41

Effective One-Dimensional Infrastructure in Function Fields of Arbitrary Degree

The distinguished principal ideals in the maximal order of a global field with two infinite places form the *infrastructure* of the field. This is "almost" a group under ideal composition, only barely failing associativity. The infrastructure can be used to significantly speed up computation of the regulator, the ideal class number and (in the case of function fields) the degree zero divisor class number of the field. This is joint work with Adrian Tang.

<u>Renate Scheidler</u> University of Calgary rscheidl@ucalgary.ca

MS41

Canonical Heights in Arithmetic Geometry and Arithmetic Dynamics

Let $f: X \to X$ be a dominant rational self-map of a smooth projective variety defined over $\overline{\mathbf{Q}}$. The dynamical degree δ_f of f measures the dynamical complexity of the iterates f^n of f. The arithmetic degree $\alpha_f(x) =$ $\lim h_X (f^n(x))^{1/n}$ of a point $x \in X(\overline{\mathbf{Q}})$ similarly measures the arithmetic complexity of the f-orbit $\mathcal{O}_{\{}(\S)$ of x. I will discuss the inequality $\alpha_f(x) \leq \delta_f$, the conjecture that $\alpha_f(x)$ is an algebraic integer, with a proof if f is a morphism, and the conjecture that if $\mathcal{O}_{\{}(\S)$ is Zariski dense, then $\alpha_f(x) = \delta_f$, with a proof when X is an abelian variety. (Joint work with Shu Kawaguchi)

Joseph H. Silverman Brown University jhs@math.brown.edu

MS41

Computing L-Series of Low Genus Curves

David Harvey recently proposed a new algorithm that computes the zeta function of a hyperelliptic curve of genus g at all primes $p \leq N$, with an average running time (per prime) is polynomial in both g and $\log p$. I will describe joint work with Harvey on the practical implementation of this algorithm for $g \leq 3$, and also discuss extensions of the algorithm to handle some non-hyperelliptic cases.

Andrew V. Sutherland MIT drew@math.mit.edu

David Harvey University of New South Wales d.harvey@unsw.edu.au

MS42

On Polar Coding with Algebraic Geometric Kernels

Arikan developed polar coding as a breakthrough method for an explicit construction of symmetric capacity achieving codes for binary DMCs with low encoding and decoding complexity. Recently, Reed-Solomon and BCH codes have been considered as kernels of polar codes. We implement more general constructions with algebraic geometry codes as kernels, specifically codes from maximal and optimal function fields. In addition, we demonstrate that multipoint code kernels arise naturally from shortening those associated with one-point codes.

Sarah Anderson

Clemson University sarah5@clemson.edu

MS42

Smooth Models for the Deligne-Lusztig Curves

The curves of Deligne-Lusztig type come in three families: The Hermitian curves, the Suzuki curves, and the Ree curves. They are defined over a finite field and have many rational points and a large automorphism group, making them attractive for various applications. The curves have natural abstract definitions and they have known singular models that yield descriptions of their function field. Some applications and further questions about the curves require a nonsingular model. We provide natural explicit smooth embeddings for the Suzuki curves in P^4 and for the Ree curves in P^{13} . We use these to describe new properties of the curves.

Iwan Duursma

University of Illinois at Urbana-Champaign Department of Mathematics duursma@math.uiuc.edu

Abdulla Eid University of Illinois eid1@illinois.edu

MS42

On q-Ary Polar Coding

We discuss polar coding for q-ary channels, where q is a power of a prime. In polar coding, independent copies of a discrete memoryless channel are synthesized into a second set of channels so that asymptotically (in block length) all channels, except for a vanishing fraction, are either noiseless or almost pure noise. In this talk, we address the use of algebraic geometry codes in this setting.

<u>Gretchen L. Matthews</u> Clemson University

gmatthe@clemson.edu

MS42 List Decoding of Subspace Codes

Subspace codes are defined to be subsets of the projective geometry over a finite field. If all elements of such a code have the same dimension, such a code is called a constant dimension code. These codes are useful for different applications, e.g. random linear network coding. List decoding is the concept of finding not only the closest codeword to a received word but the list of codewords inside a ball of a given radius around the received word. In this talk we show how a list decoding algorithm of constant dimension codes can be derived in the well-known Plcker embedding of the Grassmannian.

Anna-Lena Trautmann University of Zurich anna-lena.trautmann@math.uzh.ch

Joachim Rosenthal Institut für Mathematik Universität Zürich rosenthal@math.uzh.ch

MS42

The Minimum Distance and the Index of Nilpotency

Let C be a [n, k, d]-linear code. Let A be a $k \times n$ generating matrix for C. Assuming that A has no proportional columns, nor zero columns, we think of the columns of A as the homogeneous coordinates of n distinct points in P^{k-1} . Under the extra condition that any k - 1 of these points span a hyperplane, we find a formula that relates the minimum distance d to the index of nilpotency of the ideal of a fat-points scheme with support the coatoms of the arrangement of dual hyperplanes to these points.

<u>Stefan Tohaneanu</u> University of Western Ontario stohanea@uwo.ca

MS43

Object Image Correspondence for Algebraic Curves under Projections.

We present a novel algorithm for deciding whether a given planar curve is an image of a given spatial curve, obtained by a central or a parallel projection. The motivation comes from the problem of establishing a correspondence between an object and an image, taken by a camera with unknown position and parameters. The computational advantage of our algorithm, in comparison to algorithms based on the straightforward approach, lies in a significant reduction of a number of real parameters that need to be eliminated to solve this problem.

Joseph Burdis, <u>Irina Kogan</u>, Hoon Hong North Carolina <u>State University</u> joe.burdis@gmail.com, iakogan@ncsu.edu, hong@ncsu.edu

MS43

Simplified Morse Skeletons from Digital Images

The closely related techniques of Morse theory and persistent homology provide a rigorous framework for generalizing and unifying various tools from image analysis that address the problems of segmentation and shape analysis (e.g. skeletonization, watersheds, and *ad hoc* simplification). We present the latest foundations and efficient implementations of discrete morse theory based algorithms for building simplified skeletons from real x-ray micro-CT data of rock cores and other samples with intricate threedimensional structure.

Vanessa Robins, Olaf Delgado-Friedrichs, Adrian Sheppard

Australian National University Vanessa.Robins@anu.edu.au, olaf.delgado@googlemail.com, adrian.sheppard@anu.edu.au

MS43

Object/Image Equations for Object Recognition, Shape Analysis, and Statistics.

We will discuss two fundamental problems related to object recognition. Our focus will be on point features under weak perspective projection, since this is a case with a complete solution. The first problem involves the geometric constraints (object-image equations) that must hold between a set of object feature points (object configuration) and any image of those points under a weak perspective projection. These constraints are formulated in an invariant way, so that object pose, image orientation, or the choice of coordinates used to express the feature point locations either on the object or in the image are irrelevant. These constraints turn out to be expressions in the so-called shape coordinates on the moduli space of configurations. The second problem concerns the notion of a shape space and shape statistics. These spaces have certain natural metrics that allow for statistical analysis when faced with noisy feature data.

Peter F. Stiller

Texas A&M University stiller@math.tamu.edu

MS43

Using Gaussian-Weighted Graph Laplacian in Geometric Shape Processing

The Laplace-Beltrami operator of a given manifold is a fundamental object encoding the intrinsic geometry of the underlying manifold, and has been widely used in graphics and machine learning applications. In practice, the shape of interest is often available only through discrete approximations, such as a set of points sampled from it. The Gaussian-weighted graph Laplacian is a popular discretization of the Laplace-Beltrami operator for such point clouds data. In this talk, I will talk about our recent work where we use the Gaussian-weighted graph Lapacian for two shape analysis applications. The first one is to reconstruct a singular surface (a collection of possibly intersecting surface patches with non-smooth creases) from a set of points sampled from it. The second one is about to re-distribute points on a hidden domain to achieve a certain prescribed target probabilistic distribution (such as to make the points distribution uniform). The development of these applications leverages some theoretical understanding of the Gaussian-weighted graph Laplace operator.

Yusu Wang

The Ohio State University, Columbus, Ohio yusu@cse.ohio-state.edu

MS44

On Waring's Problem for Systems of Skew-Symmetric Forms

This talk explores a problem of finding the smallest positive integer s(m,n,k) such that (m + 1) generic skewsymmetric (k + 1)-forms in (n + 1) variables as linear combinations of the same s(m,n,k) decomposable skewsymmetric (k + 1)-forms. In this talk, we will describe how objects in algebraic geometry can be associated to systems of skew-symmetric forms and discuss algebraic and geometric approaches to establish the existence of triples (m, n, k), where s(m, n, k) is more than expected.

<u>Hirotachi Abo</u>, Jia Wan University of Idaho abo@uidaho.edu, wan2235@vandals.uidaho.edu

MS44

Ranks and Nuclear Norms of Tensors

Unfortunately, it is often difficult to determine the rank of a higher order tensors. A common simplification is convex relaxation: instead of the rank of a tensor, we may consider its nuclear norm. For many tensors, for which we do not know the rank, we can determine the nuclear norm. Examples are: the matrix multiplication tensor, the determinant, permanent, and the multiplication tensor in group algebras. We also will generalize the notion of the Singular Value Decomposition (at least for some tensors) and find the singular values of some tensors of interest.

Harm Derksen University of Michigan hderksen@umich.edu

MS44 Higher Secants of Sato's Grassmannian

Sato's Grassmannian from integrable systems is an inverse limit of all finite-dimensional Grassmannians. I will discuss progress on a project to determine whether its higher secant varieties are defined in bounded degree.

<u>Jan Draisma</u>

Eindhoven University of Technology, The Netherlands Department of Mathematics and Computer Science j.draisma@tue.nl

MS44

Equations of Secant Varieties

It is a classical result of Mumford that in a sufficiently ample embedding, any variety is defined by determinantal equations. There is a lot of interest in the corresponding questions for secant varieties. Namely, if $X \subset P^n$ and if we are given an integer r_0 , can we find another integer d_0 such for all $d \ge d_0$ and for all $r \le r_0$ we have that $\sigma_r(X_d)$ is determinantal. We use the notation X_d to denote $\nu_d(X)$, the image of X under the d-th Veronese re-embedding. We will talk about results of Buczynski-Ginsensky-Landsberg and Buszynska-Buczynski which reduce the problem to the case of $X = P^n$ but show that the problem for $X = P^n$ is more complicated than perhaps was thought.

Adam Ginensky University of Chicago adam.ginensky@yahoo.com

$\mathbf{MS44}$

Writing Forms as Sums of Higher Powers of Lower Degree Forms

Complex forms in n variables of degree kd can always be written as a sum of k-th powers of forms of degree d. We discuss various questions related to minimal representations of this kind, especially when n = 2 and when $k \geq 3$

and $d \geq 2$.

Bruce Reznick

University of Illinois, Urbana-Champaign reznick@math.uiuc.edu

MS45

Combinatorial Degree Bound for Toric Ideals of Hypergraphs

Associated to any hypergraph is a toric ideal encoding the algebraic relations among its edges. We study these ideals and the combinatorics of their minimal generators, and derive general degree bounds for both uniform and nonuniform hypergraphs. As an application, we show that the defining ideal of the tangential variety is generated by quadratics and cubics in cumulant coordinates.

Elizabeth Gross

University of Illinois at Chicago egross7@uic.edu

Sonja Petrovic Department of Statistics Pennsylvania State University petrovic@psu.edu

MS45

Numerical Computations and Galois Groups in Schubert Calculus

Schubert calculus is an important class of geometric problems involving linear spaces meeting other fixed but general linear spaces. Problems in Schubert calculus can be modeled by systems of polynomial equations. Thus, we can use numerical methods to find the solutions to these geometrical problems. We present a Macaulay2 implementation of numerical algorithms that solve Schubert problems. These algorithms are based on the geometric Pieri and Littlewood-Richardson homotopies. We use our implementation to study Galois groups of Schubert problems. This work is partially joint with Anton Leykin and Frank Sottile.

Abraham Martin del Campo IST Austria abraham.mc@ist.ac.at

MS45

Comparison of Symbolic and Ordinary Powers of Ideals

Given an ideal I in a ring R, we can define the t-th symbolic powers $I^{(t)}$ by evaluating elements of R to the order t at each associated prime of I. Comparison symbolic and ordinary powers has attracted considerable attention in the past decades. We will show some uniform bounds, computable examples and conjectures in this area.

Wenbo Niu

Purdue University niu6@math.purdue.edu

MS45

Hyperdeterminants of Polynomials

Hyperdeterminants were brought into a modern light by Gel'fand, Kapranov, and Zelevinsky. Inspired by their work, we study the hyperdeterminant applied to a polynomial (interpreted as a symmetric tensor). The hyperdeterminant of a polynomial factors into several irreducible factors with multiplicities. These factors along with their degrees and their multiplicities are identified geometrically (via projective duals of Chow and Segre-Veronese varieties) and have a nice combinatorial interpretation.

Luke Oeding

University of California Berkeley Università degli Studi di Firenze lukeoeding@gmail.com

MS45

Software for Computing Multiplier Ideals

Multiplier ideals are interesting for a range of applications in birational geometry, commutative algebra, and algebraic statistics. Unfortunately they are usually hard to compute. Shibuta's algorithm, implemented in a Macaulay2 package by Christine Berkesch and Anton Leykin, can compute multiplier ideals of modestly sized examples. I describe a Macaulay2 package being developed that uses combinatorial algorithms for particular classes of ideals including monomial ideals, monomial curves, and determinantal ideals, allowing computations of somewhat larger examples.

Zach Teitler Boise State University zteitler@boisestate.edu

MS46

Toward a Rigorous Variation of Coppersmith's Algorithm on Three Variables

Coppersmith's technique for finding small roots on a polynomial $p_1(x, y)$ works by constructing $p_2(x, y)$ independent from p_1 , sharing the same root and then solving the system $\{p_1 = 0, p_2 = 0\}$. For $p_1(x, y, z)$, the generalization, which finds p_2 and p_3 independent from p_1 , is heuristic since the system $\{p_1 = 0, p_2 = 0, p_3 = 0\}$ can sometimes not be solved. In this talk, we propose a rigorous variation of Coppersmith's algorithm providing algebraically independent polynomials that makes use of Gröbner bases and which can be generalized for multivariate polynomials.

<u>Aurélie Bauer</u> ANSSI, France aurelie.bauer@ssi.gouv.fr

MS46

Polynomial Analogues of Coppersmith's Method, with Applications to List-decoding of Errorcorrecting Codes

I will show how the analogue of Coppersmith's method over the ring of polynomials gives a "lattice-based' algorithm for list-decoding several families of important errorcorrecting codes, including Reed-Solomon codes in the univariate case and Parvaresh-Vardy codes in the multivariate case. This approach yields algorithms that are extremely fast both in theory and in practice. Interestingly, in the coding-theory application it is possible to construct codes so that the heuristic assumptions commonly used in multivariate Coppersmith-type methods over the integers are not necessary. I will discuss joint work with Henry Cohn and with Casey Devet and Ian Goldberg.

Nadia Heninger

Princeton University nadiah@cs.princeton.edu

MS46

The Heuristic Coppersmith Technique from a Computer Algebra Point of View

In this talk we introduce objects and algorithms coming from computational commutative algebra that let us describe precisely why the Coppersmith's technique becomes heuristic in more than two variables. We then present how, in some cases, one can circumvent this heuristic.

<u>Guénaël Renault</u>

Université Pierre et Marie Curie, France guenael.renault@lip6.fr

MS46

An Introduction to Coppersmith's Theorem and its Applications

In this talk, we give an overview of how Don Coppersmith uses lattice reduction techniques to compute small roots of univariate polynomials in Z/nZ efficiently even when the factorization of n is unknown, and mention some applications to number theoretic problems and cryptanalysis. As an introduction to the other talks of the minisymposium, we also briefly discuss the difficulties that arise when trying to directly apply similar techniques in higher dimensions.

<u>Mehdi Tibouchi</u> NTT, Japan mehdi.tibouchi@normalesup.org

MS47

Implicitization by Interpolation: Symbolic and Numerical Methods

We apply symbolic or numerical methods in order to compute a matrix kernel and determine the coefficients of the implicit equation knowing its support. We propose solutions for the multidimensional kernel case, that occurs if the predicted support is a superset of the actual. We compare different approaches for constructing a real or complex matrix in Maple and SAGE. In our experiments we have used classical algebraic curves and surfaces as well as NURBS.

Tatjana Kalinka

National and Kapodistrian University of Athens tatjana.kalinka@gmail.com

MS47

Coping with Singular Isolated Zeros of Polynomial Systems Using Symbolic perturbations

We present techniques for extracting information from an isolated singular zero of a polynomial ideal. We work on the dual polynomial ring and derive a description of the multiplicity structure in terms of local differential conditions. The algorithm is based on matrix-kernel computations, which can be carried out numerically, starting from an approximation of the zero in question. Based on this algorithm, we derive a deflation technique, by augmenting the original system using a number of new parameters, so that the multiplicity is obviated: the deflated system interpolates the initial root with multiplicity one; consequently, it can be used in Newton-based iterative schemes in order to refine the approximate root with quadratic convergence.

Angelos Mantzaflaris RICAM Austrian Academy of Sciences Angelos.Manzaflaris@oeaw.ac.at

Bernard Mourrain INRIA Sophia Antipolis Bernard.Mourrain@inria.fr

MS47

Vanishing Ideals of Limited Precision Points

In this talk we address the problem of characterizing a set X of limited precision points of \mathbb{R}^n from a symbolicnumerical point of view. In contrast to other approaches of the recent literature we define the numerical vanishing ideal associated to X as a zero-dimensional complete intersection generated by polynomials of low degree and whose exact zero-set contains a small perturbation of the input points.

<u>Maria-Laura Torrente</u> University of Genova torrente@dima.unige.it

MS47 Interpolation and Ehrhart Theory

We explore the interplay between interpolation and Ehrhart theory. The computation of Ehrhart polynomials is a difficult problem with many applications. The use of interpolation techniques for the computation of Ehrhart polynomials seems natural and potentially important in practice. Changing the point of view, we investigate how certain interpolation problems can be interpreted in Ehrhart theory. Although, in general, Ehrhart polynomials cannot be computed efficiently, such a connection would provide insight and possible algorithmic advantages.

Zafeirakis Zafeirakopoulos Johannes Kepler University zafeirak@risc.uni-linz.ac.at

MS48

Sheaves, Cosheaves and Applications

This talk outlines a unified approach to persistence, network coding, and sensor networks. In all of these applications, data is parameterized over a cell complex. The extraction and analysis of qualitative features of data is mediated through the use of constructible (co)sheaf theory. A generalized version of a barcode is introduced to aid in visualization and interpretation. Finally, a new homology theory that is sensitive to the embedding of data is presented.

Justin Curry University of Pennsylvania curry.justin@gmail.com

MS48

Generalized Interleavings and Weak Laws of Large Numbers for 2-D Persistent Homology

Implicit in the results of the 2009 paper "Persistence-Based

Clustering in Riemannian Manifolds" by Chazal, Guibas, Oudot, and Skraba is a (loose) analogue of the weak law of large numbers for 1-D persistent homology. I'll introduce (J_1, J_2) -interleavings, which are generalized interleavings defined on multidimensional filtrations and persistence modules. I'll then explain how (J_1, J_2) -interleavings can be used to formulate and prove 2-D adaptations of the "weak law of large numbers for persistent homology."

<u>Michael Lesnick</u> Stanford University mlesnick@ias.edu

MS48

A Continuous Mean for Sets of Persistence Diagrams

Recent progress has shown that the abstract space of persistence diagrams is Polish, and found necessary and sufficient conditions for a set to be compact. To understand the average of a set of diagrams, Fréchet means were used, a construction which is possible for any metric space. The Fréchet mean is a set, not an element. Mileyko et al showed that for well behaved distributions, the Fréchet mean is non-empty. However, there are simple counterexamples showing that the mean is non-unique, which creates issues when we are interested in continuous, time-varying systems. In order to solve this problem, we present a generalization of the Fréchet mean which is continuous for sets of continuously varying persistence diagrams.

<u>Elizabeth Munch</u> Duke University liz@elizabethmunch.com

MS48

Measuring the Stability of Intersections to C^0 Perturbations

Consider two smooth submanifolds M and N of X. The two intersect transversally if, at each point x in the intersection $M \cap N$, the tangent space of M at x plus the tangent space of N at x is the tangent space of X at x. Transverse intersections are interesting because they are stable to infinitesimally small or, in other words, C^{∞} perturbations. Motivated by the philosophy of science, we would like to quantify the stability of the intersection. In this talk, I will present the relatively new idea of the well group. The well group measures the stability of the homology of $M \cap N$ with respect to C^0 perturbations of the embedding of M into X. I will show that the well group is the abstraction of the persistent homology group.

<u>Amit Patel</u> Rutgers University amit@akpatel.org

MS48

Kernel Distance for Geometric Inference

We study the geometric inference from a noisy point cloud using the kernel distance. Recently Chazal, Cohen-Steiner, and Mergot have introduced distance to a measure, which is a distance-like function that is robust to perturbations and noise on the data. Their work and its extensions represents the state-of-the-art in geometric inference on noisy point cloud data. The point of this talk is to show that the kernel distance can be used in place of the distance to a measure. We prove that the kernel distance possesses very similar properties to the distance to a measure of Chazal et.al., but also has several advantages. (i) Its inference has a very naturally interpretation as the superlevel set of a kernel density estimate. (ii) The kernel distance is Lipschitz with respect to the outlier parameter s. Similar properties are not know for the distance to a measure. (iii) The kernel distance has a small coreset representation, which permits efficient inferences on tens of millions of points.

Bei Wang, Jeff Phillips University of Utah beiwang@sci.utah.edu, jeffp@cs.utah.edu

MS49

Discriminants and Applications

Discriminant is a key tool when examining well-constrained systems, including the case of one univariate polynomial. We present some useful properties and applications in order to motivate their study. We focus on exploiting the sparseness of polynomials via the theory of Newton polytopes and sparse elimination. We also discuss a multiplicativity formula of the discriminant in the case of two polynomials with fixed supports when one of them factors.

Alicia Dickenstein Universidad de Buenos Aires alidick@dm.uba.ar

Ioannis Z. Emiris National and Kapodistrian University of Athens emiris@di.uoa.gr

Anna Karasoulou

University of Athens akarasou@di.uoa.gr

MS49

Khovanskii-Rolle Continuation for Real Solutions

Over 30 years ago, Askold Khovanskii used a multivariate generalization of Rolle's Theorem to give a bound on the number of positive solutions to a system of polynomial equations that depends only on the number of terms appearing in the equations, and not on their degree. Reflecting this dependence, this class of bounds are called fewnomial bounds. Khovanskii's proof was revisited by Bihan and Sottile, who gave a refined bound that is asymptotically sharp, in a certain sense. Subsequently, we observed that Khovanskii's use of Rolle's Theorem can form the basis of a continuation algorithm for numerically computing the real solutions to a system of fewnomials. This Khovanskii-Rolle algorithm finds the real solutions by following real arcs, and avoids computing complex solutions to the fewnomial system. Consequently its complexity depends upon the fewnomial bound and not on the number of complex solutions. This talk will explain this Khovanskii-Rolle algorithm and describe some technical challenges that are encountered in its implementation. This represents joint work with Dan Bates.

<u>Frank Sottile</u> Texas A&M University sottile@math.tamu.edu

Daniel J. Bates Colorado State University Department of Mathematics bates@math.colostate.edu

MS49

Computing Critical Points with Gröbner Bases: Complexity and Applications

Computing critical points of a polynomial map q restricted to an algebraic variety V is a topical issue in several algorithms in real algebraic geometry and in global minimization problems. In this talk, we present new complexity bounds for computing such critical points with symbolic algorithms. Let f_1, \ldots, f_p be polynomials in $\mathbf{Q}[\mathbf{x}_1, \ldots, \mathbf{x}_n]$ defining a variety V and I be the ideal vanishing on the critical points of g restricted to V. The ideal I is generated by f_1, \ldots, f_p and by the maximal minors of the Jacobian matrix of (f_1, \ldots, f_p, g) . Under genericity assumptions on the coefficients of f_1, \ldots, f_p, g , we show new upper bounds on the arithmetic complexity of computing a Gröbner basis of I, which reflects the behavior observed in practice. These bounds are obtained by taking into account structural properties of determinantal ideals generated by maximal minors. In particular, these bounds are polynomial in the number of critical points for several families of parameters.

Pierre-Jean Spaenlehauer <u>UPMC</u>, Univ. Paris 6 pierre-jean.spaenlehauer@m4x.org

Jean-Charles Faugère Salsa project, INRIA Paris-Rocquencourt jean-charles.faugere@inria.fr

Mohab Safey El Din UPMC, Univ Paris 6 Mohab.Safey@lip6.fr

MS49

An Algorithm to Compute the Dimension of Real Algebraic Sets

We present a new probabilistic algorithm for computing the real dimension of a real solution set $S \subset^n$ of s polynomials of degree bounded by D in n variables.

Mohab Safey El Din UPMC, Univ Paris 6 Mohab.Safey@lip6.fr

Elias Tsigaridas Inria Paris-Rocquencourt elias.tsigaridas@inria.fr

MS50

Introduction and Overview of the Topic Area 'Formulas in Interpolation'

This workshop will explore the use of interpolation in both Symbolic and Numerical Algebraic Geometry. From closed formulas for both Cauchy and osculatory rational interpolation to more sophisticated applications including Birkhoff interpolation and its multivariate versions. We will present some of recent results and challenges.

<u>Carlos D'Andrea</u> Universitat de Barcelona cdandrea@ub.edu

Polynomial Algebra for Birkhoff Interpolants

We introduce a unifying formulation of a number of related problems which can all be solved using a contour integral formula. Each of these problems requires finding a non-trivial linear combination of some of the values of a function f, and its first and higher derivatives, at a number of data points. This linear combination is re- quired to have zero value when f is a polynomial of up to a specific degree p. Examples of this type of problem include Lagrange, Hermite and Hermite-Birkhoff interpolation; fixed-denominator rational interpolation; and various numerical quadrature and differentiation formulae. Other applications include the estimation of missing data and root-finding.

John C. Butcher Department of Mathematics The University of Auckland butcher@math.auckland.ac.nz

Rob M. Corless Department of Applied Mathematics University of Western Ontario rcorless@uwo.ca

Laureano Gonzalez-Vega Universidad de Cantabria Dpto. Matematicas laureano.gonzalez@unican.es

Azar Shakoori Universidad de Cantabria, Spain azar.shakoori@gmail.com

MS50

An Algorithm for Evaluation and Interpolation in Higher Dimensions

We propose efficient new algorithms for multi-dimensional multi-point evaluation and interpolation on certain subsets of tensor product grids. These point-sets naturally occur in the design of efficient multiplication algorithms for finite-dimensional algebras of the form $K[x_1, \ldots, x_n]/I$, where I is generated by monomials.

Joris van der Hoeven LIX, Ecole polytechnique vdhoeven@lix.polytechnique.fr

Eric Schost Computer Science Department Univ. of Western Ontario eschost@uwo.ca

MS50

Sylvester Double Sums and Divided Differences

Abstract not available at time of publication.

Aviva Szpirglas Université de Poitier aviva@math.univ-poitiers.fr

MS51 A Bayesian Information Criterion for Singular

Models

For singular models, such as reduced rank regression or mixture models, the Bayesian Information Criterion (BIC) may fail to reflect the large-sample behavior of the marginal likelihood. This talk will suggest a practical extension of BIC that uses recently developed large-sample theory for singular marginal likelihood integrals to resolve this problem.

<u>Mathias Drton</u> Department of Statistics

University of Washington md5@uw.edu

Martyn Plummer International Agency for Research on Cancer plummerm@iarc.fr

MS51

Singular Learning Theory and Causal Inference

Many algorithms for inferring causality are based on partial correlation testing. Partial correlations define hypersurfaces in the parameter space of a directed Gaussian graphical model. The volumes obtained by bounding partial correlations play an important role for the performance of causal inference algorithms and the quantification of bias in causal inference. We develop an asymptotic theory for computing these volumes. Our analysis involves computing the singular loci of the partial correlation hypersurfaces and using the method of real log canonical thresholds.

Shaowei Lin A*STAR lins@i2r.a-star.edu.sg

<u>Caroline Uhler</u> Institute of Science and Technology Austria caroline.uhler@ist.ac.at

Bernd Sturmfels University of California , Berkeley bernd@Math.Berkeley.EDU

Peter Buehlmann ETH Zurich peter.buehlmann@stat.math.ethz.ch

MS51

Resolution of Singularities and Statistical Model Evaluation

It has been difficult to evaluate statistical models which do not satisfy regularity condition, since asymptotic theory of likelihood function near singularities has been left unknown. In this presentation, we show that resolution of singularities provides us the standard representation of the parameter space, by which we can make the universal statistical estimation theorem. As a result, statistical concepts, AIC and BIC in regular models, are generalized onto singular models.

<u>Sumio Watanabe</u> Department of Computational Intelligence and Systems Science Tokyo Institute of Technology, Japan swatanab@dis.titech.ac.jp

Asymptotic Inference for Gaussian Hidden Tree Models

Hidden tree models are Bayesian networks on rooted trees with all inner nodes representing data which are unobserved. Models of this type can be for example found in phylogenetics and they are complicated both from algebraic and statistical point of view. The likelihood function is multimodal, estimates of the parameters lie often on the boundary of the parameter space. The fact that the model is not identifiable rises many additional issues. In this talk we show how singular learning theory can be used to get a better insight into asymptotic analysis of these models in the case when studied system is multivariate Gaussian.

<u>Piotr Zwiernik</u> Mittag-Leffler Institute Stockholm, Sweden piotr.zwiernik@gmail.com

MS52

Numerical Methods for Highly Structured Polynomial Systems Coming from Magnetism

Abstract not available at time of publication.

Daniel J. Bates Colorado State University Department of Mathematics bates@math.colostate.edu

MS52

Applying Fiber Products to Polynomial Maps and the Planar Pentad

Fiber product techniques in numerical algebraic geometry can be used to determine the parameters over which a polynomial system has solution set with dimension greater than the expected dimension. For parameterized polynomial systems, the dimension of the (complex) solution set is the same for almost all choices of parameters. Fiber products can identify exceptional sets of parameter values where the dimension of the solution set is greater than expected. An application of these techniques arises in Kinematics, were parameters in polynomial systems correspond to physical parameters of mechanisms. An exceptional set of parameters corresponds to mechanism configurations that have more mobility than is expected. Fiber product techniques require finding solutions to polynomial systems of increasing size. We will discuss algorithms for avoiding or limiting the growth of these systems, in particular we will consider applying these techniques to the Planar Pentad.

Eric Hanson

Department of Mathematics Colorado State University hanson@math.colostate.edu

Dan Bates Colorado State University bates@math.colostate.edu

Jon Hauenstein North Carolina State University hauenstein@ncsu.edu

Charles Wampler General Motors Research Laboratories, Enterprise Systems Lab 30500 Mound Road, Warren, MI 48090-9055, USA charles.w.wampler@gm.com

MS52

Numerical Determination of Witness Points on Real Solution Components of Polynomial and Differential Polynomial Equations

In this talk we extend complex homotopy methods to finding witness points on the irreducible components of real varieties. In particular we compute such witness points as the isolated real solutions of a constrained optimization problem by a homotopy mixed volume based solver over C. First a random hyperplane characterized by its random normal vector is chosen. Witness points are computed which are at the intersection of this hyperplane with components. Other witness points are the regular local critical points of the distance from the plane to components. A method is given for constructing regular witness points on components, in the case that the critical points are singular. The method is applicable to systems satisfying certain regularity conditions. Illustrative examples are given. We show that the method can be used in the consistent initialization phase of a popular method due to Pryce and Pantelides for preprocessing such differential algebraic equations for numerical solution.

Wenyuan Wu

Chongqing Institute of Green and Intelligent Technology Chinese Academy of Sciences wuwenyuan@cigit.ac.cn

Greg Reid Applied Math Department University of Western Ontario reid@uwo.ca

MS53

Shellability and Freeness of Continuous Splines

We provide an example of a shellable polyhedral complex \mathcal{P} in \mathbb{R}^2 such that the module of continuous splines $C^0(\hat{\mathcal{P}})$ is not a free module over the polynomial ring in three variables, answering a question raised in [Schenck, Equivariant Chow Cohomology of Non-Simplicial Toric Varieties, 2012].

Michael DiPasquale University of Ilinois dipasqu1@illinois.edu

MS53

Computational Topology and Visualization

In this talk, we outline various challenges and problems that arise in high performance scientific visualization. In particular, we concern ourselves with the fidelity of the output. Inspired by applications, we discuss methods of ensuring topological fidelity for curves and surfaces with curvature adaptive sampling.

Lance E. Miller University of Utah lance.e.miller@gmail.com

Splines on Polyhedral Complexes

We discuss recent results on splines on polyhedral complexes, including results on freeness of the spline module, combinatorial and geometric aspects, and bounds for degrees of generators. The polyhedral case is quite different from the simplicial case, and I will illustrate with several examples the subtle points that arise.

Hal Schenck

Mathematics Department University of Illinois schenck@math.uiuc.edu

$\mathbf{MS53}$

Syzygies and Singularities of Tensor Product Surfaces

This project is motivated by questions in geometric modeling, where one would like to understand the implicit equation and singular locus of a parametric surface. Consider the polynomial ring R = k[s, t, u, v] over an algebraically closed field k. Regard R as a bigraded k-algebra, in which s, t have degree (1, 0) and u, v have degree (0, 1). We consider a class of surfaces S in P^3 parametrized by four bihomogeneous polynomials in R. Such surfaces are called tensor product surfaces. We classify all possible minimal free resolutions of the ideal given by the parametrization functions and we relate the syzygies to the singularities of the projective surface S. These resolutions play a key role in determining the implicit equation for S.

<u>Alexandra Seceleanu</u> University of Nebraska aseceleanu2@math.unl.edu

Hal Schenck Mathematics Department University of Illinois schenck@math.uiuc.edu

Javid Validashti University of Illinois at Urbana-Champaign jvalidas@illinois.edu

MS53

The Schenck-Stiller Conjecture on the Dimension of Splines

Let Δ be a triangulation of an open topological disk in \mathbb{R}^2 . Let $r \geq 0$ be an integer. Let $C_d^r(\Delta)$ be the space of polynomial splines on Δ of smoothness r and degree d. If $d \geq 3r + 2$ (or $d \geq 3r + 1$ for generic Δ) Alfeld and Schumaker give a formula for the dimension of this vector space that depends on the combinatorics and the local geometric data of Δ . Schenck and Stiller conjectured that this formula holds for $d \geq 2r + 1$, for any d and r. There has been little progress into proving this conjecture, and I will talk about an example that verifies the conjecture and also that makes it tight. We also investigate the validity of the conjecture when pseudo edges (edges that touch the boundary) are added to any triangulation known to satisfy Schenck-Stiller conjecture.

<u>Stefan Tohaneanu</u> University of Western Ontario stohanea@uwo.ca

MS54

Quantum Algorithms for the Subset-sum Problem

This talk introduces a subset-sum algorithm with heuristic asymptotic cost exponent below 0.25. The new algorithm combines the 2010 Howgrave-Graham–Joux subsetsum algorithm with a new streamlined approach to quantum walks.

Daniel Bernstein

University of Illinois at Chicago djb@math.uic.edu

MS54

McBits: fast constant-time code-based cryptography

This talk presents extremely fast algorithms for code-based public-key cryptogra- phy, including full protection against timing attacks. For example, our software achieves a reciprocal throughput of just 36615 cycles per decryption at a 80-bit security level on a single Ivy Bridge core. These algorithms rely on an additive FFT for fast root computation, a transposed additive FFT for fast syndrome computation, and a sorting network to avoid cache-timing attacks.

Tung Chou

Technische Universiteit Eindhoven The Netherlands blueprint@crypto.tw

Daniel Bernstein University of Illinois at Chicago djb@math.uic.edu

Peter Schwabe Radboud University Nijmegen The Netherlands peter@cryptojedi.org

MS54

Solving the Shortest Vector Problem in Lattices Faster Using Quantum Search

By applying Grovers quantum search algorithm to the lattice algorithms of Micciancio and Voulgaris, Nguyen and Vidick, Wang et al., and Pujol and Stehlé, we obtain improved asymptotic quantum results for solving the shortest vector problem. With quantum computers we can provably find a shortest vector in time $2^{1.799n+o(n)}$, improving upon the classical time complexity of $2^{2.465n+o(n)}$ of Pujol and Stehlé and the $2^{2n+o(n)}$ of Micciancio and Voulgaris, while heuristically we expect to find a shortest vector in time $2^{0.312n+o(n)}$, improving upon the classical time complexity of $2^{0.384n+o(n)}$ of Wang et al. These quantum complexities will be an important guide for the selection of parameters for post-quantum cryptosystems based on the hardness of the shortest vector problem.

Thijs Laarhoven Technische Universiteit Eindhoven t.m.m.laarhoven@tue.nl

Michele Mosca University of Waterloo michele.mosca@uwaterloo.ca

Joop van de Pol

University of Bristol joop.vandepol@bristol.ac.uk

$\mathbf{MS55}$

The Geometry of Lightlike Surfaces in Minkowski Space

We investigate the geometric properties of lightlike surfaces in the Minkowski space $R^{2,1}$, using Cartan's method of moving frames to compute a complete set of local invariants for such surfaces. Using these invariants, we give a complete local classification of lightlike surfaces of constant type in $R^{2,1}$ and construct new examples of such surfaces.

<u>Jeanne Clelland</u> University of Colorado Boulder jeanne.clelland@colorado.edu

Brian Carlsen University of Colorado brian.carlsen@colorado.edu

MS55

Moving Frames and Flows for Curves in Centroaffine Space

Centroaffine geometry in ³ is the study of invariant properties of objects with respect to the action of SL(3) without translations. Along curves free of inflection points, one can construct a unique SL(3)-valued moving frame, an invariant arclength, and fundamental differential invariants (the analogues of Euclidean curvature and torsion). Given a non-stretching invariant curve flow, the passage from the velocity components (relative to the moving frame) to the time derivatives of the curvatures gives a skew adjoint differential operator \mathcal{P} . We use this to construct a double hierarchy of mutually commuting bi-Hamiltonian curve flows, whose curvature evolution equations turn out to be a version of the Boussinesq hierarchy. A subsequence of linear combinations of flows in this hierarchy preserve curves lying along a cone through the origin, and induce the Kaup-Kuperschmidt hierarchy at the level of curvature.

Thomas Ivey College of Charleston iveyt@cofc.edu

Annalisa M. Calini College of Charleston Department of Mathematics calinia@cofc.edu

Gloria Mari Beffa University of Wisconsin, Madison maribeff@math.wisc.edu

MS55

Cohomology of Variational Bicomplexes Invariant under a Pseudo-Group Action

I will describe some new techniques based on the moving frames method for computing the cohomology of variational bicomplexes invariant under a pseudo-group action. Moving frames can be used to produce complete sets of differential invariants and invariant coframes on jet bundles and to analyze their algebraic structure, thus providing a basic tool for studying invariant bicomplexes. Surprisingly, the moving frames method often allows one to compute their local cohomology by purely algebraic means.

Juha Pohjanpelto Oregon State University juha@math.oregonstate.edu

MS55

Symbols, Tableaux, and Pseudogroups (with Sage)

A century ago, the most profound discovery in both algebra and geometry was the classification of Lie groups via representation theory. Lie pseudogroups generalize Lie groups and lie behind all moving-frame and equivalence problems; however, there has been no real progress in their classification. In this talk, I will discuss a program-in-progress for using refined knowledge of the characteristic variety to (I hope) unveil some structure within the category of Lie pseudogroups. An important aspect of this project is the development of effective, open-source computational tools to study examples, so I will discuss how Lie pseudogroups, symbols, tableaux, and characteristic varieties can be studied with a new Sage package.

Abraham D. Smith

Fordham University abrahamdavidsmith@gmail.com

MS55

Recursive Moving Frames

In the standard implementation of the equivariant moving frame method, differential invariants of a Lie pseudogroup action are constructed by first prolonging the action to the infinite submanifold jet space and then normalizing the pseudo-group parameters. Typically, this procedure yields unwieldy expressions that limit the method's practical scope. In the recursive approach the prolonged action is computed incrementally, and pseudo-group parameters are normalized as they appear to control, as much as possible, the expression swell.

Francis Valiquette Dalhousie University francisv@mathstat.dal.ca

MS56

Rank of Tensors via Secant Varieties and Fat Points

This talk explores the problem of finding the dimension of the s-secant variety of X, where X is a Segre variety or a Segre-Veronese variety. The method for computing such a dimension is essentially based on Terracini's Lemma. Starting from that viewpoint, a projective scheme Z that is the union of fat points (and in some cases fat linear spaces) can be built. The scheme Z has the property that its Hilbert function in a suitable degree gives the dimension of the secant variety.

Maria Virginia Catalisano University of Genoa catalisano@diptem.unige.it

The Common Lines Variety

A key step in the reconstruction of 3D molecular structures in cryo-EM microscopy is recovering the original imaging directions from noisy 2D cryo-EM images. In 2009 Shkolnisky and Singer discovered an algorithm that can successfully recover cryo-EM imaging directions even from images corrupted with substantial error. In 2011, Hadani and Singer provided a rigorous mathematical explanation for both the algorithm's correctness and its robustness to noise. This algorithm centers on constructing a certain linear operator, the *common lines operator*. We study the algebraic variety of all possible common lines operators, provide defining equations, and explore applications of this common lines variety to denoising of experimental cryo-EM data.

David M. Dynerman University of Wisconsin dynerman@math.wisc.edu

$\mathbf{MS56}$

The Waring Rank of the Vandermonde Determinant

The Waring rank for a homogeneous polynomial is the smallest number of linear forms needed to write the polynomial as a sum of pure powers of linear forms. We compute the rank of the Vandermonde determinant using the lower bound of Ranestad and Schreyer and classical facts from invariant theory. Our approach generalizes to the defining polynomial for any real reflection arrangement and the reflection multi-arrangements for some complex reflection groups. In particular we recover some but not all of the known results on the Waring rank of monomials.

<u>Alexander Woo</u> University of Idaho awoo@uidaho.edu

Zach Teitler Boise State University zteitler@boisestate.edu

MS57

Merging Divisorial with Colored Fans

There are two generalizations of toric varieties: First, embeddings of spherical homogeneous spaces can be represented by colored fans. Second, lower-dimensional torus actions on algebraic varieties can be described by so-called polyhedral divisors. They are hybrid objects between algebraic and discrete geometry. We are going to merge both concepts by using the Tits fibration and a special torus action on spherical varieties. This will be based on the wonderful compactification of associated sober spherical varieties.

<u>Klaus Altmann</u> FU Berlin altmann@math.fu-berlin.de

Valentina Kiritchenko HSE Moscow vkiritch@hse.ru

Lars Petersen DB Frankfurt petersen@math.fu-berlin.de

MS57

Computations with Mori Dream Spaces

The concept of a Mori dream space is a natural generalization of that of a toric variety. Linking suitable combinatorial data with the Cox ring leads to a complete encoding of Mori dream spaces. This can be used for explicit computations, e.g. of geometric invariants. In the talk we give a survey on current results and present a software package.

Jürgen Hausen

Universität Tübingen juergen.hausen@uni-tuebingen.de

MS57

Using Polyhedral Divisors in Algebraic Geometry

Polyhedral divisors generalize the concept of toric varieties in order to understand varieties with the action of a lower dimensional torus. This talk will give a brief introduction into the concept of polyhedral divisors as well as elaborate on the broad field of applications. Special emphasis will be put on the implementation in SINGULAR and POLYMAKE.

Lars Kastner FU Berlin

kastner@math.fu-berlin.de

MS57

Computing Cox Rings

We present a technique to compute Cox rings using toric ambient modifications. We apply this method to compute the Cox rings of certain (singular) del Pezzo surfaces, Gorenstein log del Pezzo surfaces of Picard number one and blow ups of projective space. The algorithms are available as software packages.

Simon Keicher

Universität Tübingen keicher@mail.mathematik.uni-tuebingen.de

MS57

New Developments in Singular and Application to the Computation of the Git Fan

This talk provides a quick overview over newly introduced features in SINGULAR on one running example. The features include:

- a toolbox for parallelizing interpreter level code
- custom data types and overload of existing operators on interpreter level
- an interface for integrating custom C++ code
- an interface to polymake for convex geometry

This talk reflects the work of multiple authors in and outside the SINGULAR team. No prior knowledge is necessary for understanding it.

<u>Yue Ren</u> TU Kaiserslautern ren@mathematik.uni-kl.de

Lie Markov Models with Prescribed Symmetry

Recent work has discussed the importance of multiplicative closure for the Markov models used in phylogenetics. A sufficient condition for multiplicative closure is ensured if the set of rate-matrices of the model form a Lie algebra. These models are Lie Markov models and they include groupbased and equivariant models. We discuss how to generate Lie Markov models by demanding certain symmetries under nucleotide permutations and explore the geometric embedding of the cone of stochastic rate matrices within the ambient space.

<u>Jess Fernández-Sánchez</u> Universitat Politècnica de Catalunya jesus.fernandez.sanchez@upc.edu

Peter D. Jarvis, Jeremy Sumner University of Tasmania Peter.Jarvis@utas.edu.au, jsumner@utas.edu.au

MS58

Tensor Rank and Toric Structure

Markov process depends on a number of unknown parameters. By varying the parameters we obtain different probability distributions. This gives us a parametrisation of an algebraic variety. I will present relations between toric geometry and phylogenetic models. In particular, I will focus on the general Markov model, represented by the secant variety of the Segre embedding. This relates to the problem of determining the rank of a tensor.

<u>Mateusz Michalek</u> Max Planck Institute mateusz.michalek@ujf-grenoble.fr

MS58

Invariant Based Phylogenetic Reconstruction: Opportunities and Obstacles

Invariant based algorithms for phylogenetic reconstruction were widely dismissed by practicing biologists because invariants were perceived to have limited accuracy in constructing trees. Advances in algebraic geometry have provided a deeper understanding of phylogenetic invariants. These advances have led to a theoretical framework for understanding phylogenetic reconstruction. Here we describe an effort to translate these theoretical advances to effective phylogenetic reconstruction software usable by practicing biologists.

Joseph P. Rusinko Winthrop University rusinkoj@winthrop.edu

MS58

Nonparametric Estimation of Phylogenetic Tree Distributions

As population-based models of gene trees such as coalescent have been developed to more accurately model distributions of gene trees across genomes, meanwhile detection of horizontal gene transfers and discordances among gene trees have become important problems in phylogenetics. Here we focus on the problem of *discordance* among gene trees, and the distribution on gene trees as a whole. We

view "typical' gene trees as samples from some distribution f that generates gene trees as independent samples. We also suppose there may be rare outlier gene trees which are not "typical,' and are samples from some other distribution f' very different from f. Given the tremendous amount of ongoing effort to develop better parametric models for gene tree distributions, here we take a nonparametric approach. One advantage of nonparametric estimation is that modeling decisions and assumptions are avoided. In contrast to parametric models such as coalescents, using a nonparametric approach avoids issues such as model misspecification which might potentially confound detection of outlier trees. While most of methods to detect outliers apply statistical methods over an Euclidean (vector) space, using a Markov Chain Monte Carlo technique, we propose to develop statistical methods to estimate the distribution of trees over the space of trees which is not an Euclidean space.

Grady Weyenberg University of Kentucky grady.weyenberg@uky.edu

Peter Huggins CMU phuggins@andrew.cmu.edu

Christopher Schardl, Daniel Howe, <u>Ruriko Yoshida</u> University of Kentucky chris.schardl@uky.edu, daniel.howe@uky.edu, ruriko.yoshida@uky.edu

MS59

Recovering a Sparse Polynomial Model from Data with Noise and Outliers

We consider black-boxes that produce values with possibly noise and outliers. We aim to recover a sparse model from a small number of probes from this black-box. Such problems include recovering a sparse polynomial, possibly in a shifted bases, or producing a sparse multiple of the interpolating polynomial or rational fraction. We used minimisation techniques from linear algebra or numerical analysis (e.g. compressed sensing).

Brice B. Boyer North Carolina State University bbboyer@ncsu.edu

Erich L Kaltofen, Matthew Comer NCSU kaltofen@ncsu.edu, mcommer@ncsu.edu

MS59

Numerical Reconstruction of Polytopes from Directional Moments

In the shape-from-moments problem, we are interested in retrieving the vertices of polytopes in any dimension. Previous algorithms used formulas in the complex plane with Prony or pencil methods to recover only polygons. A new formula allows us to link directional moments and projected vertices. With the help of algebraic and numerical methods, we develop an algorithm to recover the whole set of vertices.

<u>Mathieu Collowald</u> Inria Sophia-Antipolis France

mathieu. collowald@inria.fr

Annie Cuyt University of Antwerp Dept of Mathematics and Computer Science annie.cuyt@ua.ac.be

Evelyne Hubert INRIA Méditerranée evelyne.hubert@inria.fr

Wen-shin Lee Antwerp Universiteit wenshin.lee@ua.ac.be

MS59

Sparse Models, Interpolation and Polynomials – A Summary

In my presentation, I will summarize what I have observed and learned from the talks in our minisymposium. I will also give my own current view of the subject of sparse model construction and fitting and approximate interpolation.

<u>Erich Kaltofen</u> North Carolina State University Mathematics Department kaltofen@math.ncsu.edu

MS59

Sparse Interpolation and Signal Processing

In signal processing the interest in sparsity has exploded. The fact that signals can be reconstructed from undersampled data opens up a whole new range of possibilities. At the same time, research in sparse models has existed for decades. In computer algebra, sparse representations are developed to improve computational performance. Recent achievements allow input and output representations in finite precision. Based on these, we develop new techniques for signal processing. Selected applications are presented.

Wen-shin Lee University of Antwerp wen-shin.lee@ua.ac.be

Annie Cuyt University of Antwerp Dept of Mathematics and Computer Science annie.cuyt@ua.ac.be

MS59

Sparse Multivariate Function Recovery From Values with Noise and Outlier Errors

Error-correcting decoding is generalized to multivariate sparse rational function recovery from evaluations that can be numerically inaccurate and where several evaluations can have severe errors ("outliers'). The generalization of the Berlekamp-Welch decoder to exact Cauchy interpolation of univariate rational functions from values with faults is by Kaltofen and Pernet in 2012. We give a different univariate solution based on structured linear algebra that yields a stable decoder with floating point arithmetic. Our multivariate polynomial and rational function interpolation algorithm combines Zippel's symbolic sparse polynomial interpolation technique [Ph.D. Thesis MIT 1979] with the numeric algorithm by Kaltofen, Yang, and Zhi [Proc. SNC 2007], and removes outliers ("cleans up data') through techniques from error correcting codes. Our multivariate algorithm can build a sparse model from a number of evaluations that is linear in the sparsity of the model.

Erich Kaltofen North Carolina State University Mathematics Department kaltofen@math.ncsu.edu

Zhengfeng Yang

Shanghai Key Laboratory of Trustworthy Computing, SEI East China Normal University, Shanghai 200062, China zfyang@sei.ecnu.edu.cn

MS60

Statistics for Points on Curves over Finite Fields

We will look at the distribution of the zeroes of the zeta functions of Artin-Schreier covers over a fixed finite field of characteristic p as the genus grows. We will focus on two cases: the p-rank zero locus and the ordinary locus.

<u>Alina Bucur</u>

University of California at San Diego alina@math.ucsd.edu

Chantal David Concordia University cdavid@mathstat.concordia.ca

Brooke Feigon The City College of New York CUNY bfeigon@ccny.cuny.edu

Matilde Lalin University of Montreal mlalin@dms.umontreal.ca

MS60

Random Matrices and Cohen-Lenstra Distributions in Function Fields

The Cohen-Lenstra-Martinet heuristics predict the frequency with which a fixed finite abelian group appears as a class group of certain extensions of a given number field. Recently, Malle found numerical data suggesting that these heuristics fail when the base field contains unexpected roots of unity. He then proposed a modified frequency, correcting this failure. I will explain a random matrix heuristic that leads to a function field version of Malle's conjecture (and generalizations of it).

Derek Garton

Northwestern University derek@math.northwestern.edu

MS60

Degenerations and Non-algebraically Closed Fields

Abstract not available at time of publication.

Brian Osserman

University of California, Davis osserman@math.ucdavis.edu;mailto:osserman@math.ucd

The Ekedahl-Oort type of Supersingular Curves

A curve X defined over a finite field is supersingular if its Jacobian is isogenous to a product of supersingular elliptic curves. Equivalently, the Newton polygon of the L-function has all slopes equal to 1/2. The Ekedahl-Oort type is another invariant of a curve defined in positive characteristic p. It characterizes the p-torsion group scheme of the Jacobian of X or, equivalently, the structure of the Dieudonne module or the de Rham cohomology. In this talk, I discuss results about the Ekedahl-Oort type of supersingular curves.

<u>Rachel Pries</u> Colorado State University pries@math.colostate.edu

MS60

Counting Dihedral Function Fields

Research into the construction of certain low degree function fields has surged in recent years, in part due to the cryptographic significance of elliptic and hyperelliptic curves. However there is comparatively little data available for higher degree function fields, leaving open many questions about the number of function fields of fixed degree and given discriminant. We will present a Kummer theoretic algorithm for tabulating a complete list of dihedral function fields of bounded genus over a fixed finite field. This method also reveals how one can (heuristically) compute the expected number of such fields. We will present this heuristic and show how it compares to the actual data.

<u>Colin J. Weir</u> University of Calgary cjweir@ucalgary.ca

MS61

Order-Degree Bounds for Recurrence and Differential Operators

Desingularization is the problem of finding a left multiple of a given Ore operator in which some factor of the leading coefficient of the original operator is removed. An orderdegree curve for a given Ore operator is a curve in the (r, d)-plane such that for all points (r, d) above this curve, there exists a left multiple of order r and degree d of the given operator. We give a new algorithm for a desingularization result by Abramov and van Hoeij for the shift case, and show how desingularization implies order-degree curves which are extremely accurate in examples.

Shaoshi Chen Dept. of Mathematics / NCSU schen21@ncsu.edu

<u>Maximilian Jaroschek</u> RISC / Johannes Kepler University Linz, Austria maximilian.jaroschek@risc.jku.at

Manuel Kauers RISC, JKU Linz manuel.kauers@risc.jku.at

Michael Singer North Carolina State University singer@math.ncsu.edu

MS61

Valuations of Sequences: Examples in Search of a Theory

Let p be a prime and $x \in \mathbf{N}$. The p-adic valuation of x, denoted by ${}^{\circ}\nu_p(x)$, is the highest power of x that divides x. A classical theorem of Legendre shows that the p-adic valuation of factorials, ${}^{\circ}\nu_p(n!)$, can be expressed in terms of the digits of n in base p. From here it is simple to recover basic results on arithmetic properties of factorials and binomial coefficients. The talk describes a variety of examples of sequences ${}^{\circ}\nu_p(x_n)$, where $\{x_n : n \in \mathbf{N}\}$ has some combinatorial or number-theoretical interest. These sequences of valuations are represented by a binary tree structure and (conjectured) properties of these trees will be presented. The examples include Stirling numbers of the second kind, the ASM sequence counting Alternating Sign Matrices and also partial sums of elementary functions.

<u>VIctor Moll</u> Tulane University vhm@tulane.edu

MS61

Symbolic Summation for Combinatorial and Physical Problems

We illustrate a symbolic summation toolbox based on an enhanced difference field theory of Karr's $\Pi\Sigma$ fields. In this setting we can apply Z's creative telescoping to compute recurrence relations for definite sums whose summands are given in terms of indefinite nested sums and products. Furthermore, given such a recurrence, one can compute all solutions that are expressible in terms of indefinite nested sums and products (such solutions are also called d'Alembertian solutions, a subclass of Liouvillian solutions). With this machinery we will discover (and prove) new identities related to combinatorics and number theory. In addition, we report on a cooperation with J. Blümlein (DESY, German Electron-Synchrotron) how these summation tools can be utilized to simplify multi-sums, coming from massive 3-loop Feynman integrals, to expressions in terms of indefinite nested sums and products.

<u>Carsten Schneider</u> RISC / Johannes Kepler University Linz, Austria cschneid@risc.jku.at

MS61

Open Combinatorial Problems Arising from New Sequences in the OEIS

The On-Line Encyclopedia of Integer Sequences is a database of over 200,000 number sequences. In the past 49 years it has evolved from punched cards to a wiki. I will describe the present version, illustrating it with a number of unsolved combinatorial problems arising from recent submissions.

<u>Neil Sloane</u> AT&T Shannon Labs Room C233 njasloane@gmail.com

Refined Bounds on Connected Components of Sign Conditions on a Variety II

Let R be a real closed field and $Q_1, \ldots, Q_\ell \in R[X_1, \ldots, X_k]$ such that for each $i, 1 \leq i \leq \ell$, $\deg(Q_i) = d_i$. For $1 \leq i \leq \ell$, denote by $\mathcal{Q}_{\rangle} = \{\mathcal{Q}_{\infty}, \ldots, \mathcal{Q}_{\rangle}\}$, V_i the real variety defined by \mathcal{Q}_{\rangle} , and k_i the real dimension of V_i (by convention $V_0 = \mathbb{R}^k$ and $k_0 = k$). Suppose also that for $2 \leq i \leq \ell$, $d_i \geq (k - k_{i-2} + 1)d_{i-1}$, and that $\ell \leq k$. We prove that the number of semi-algebraically connected components of V_ℓ is bounded by $O(k)^{2k} \left(\prod_{1 \leq j < \ell} d_j^{k_{j-1}-k_j}\right) d_\ell^{k_{\ell-1}}$. Additionally, if $\mathcal{P} \subset \mathbb{R}[X_1, \ldots, X_k]$ is a finite family of polynomials with $\deg(P) \leq d$ for all $P \in \mathcal{P}$, $\operatorname{card} \mathcal{P} = f$, and $d \geq (k - k_{\ell-1} + 1)d_\ell$, we prove that the number of semi-algebraically connected components of the realizations of all realizable sign conditions of the family \mathcal{P} restricted to V_ℓ is bounded by $O(k)^{2k}(sd)^{k_\ell} d_1^{k_1-k_2} \cdots d_{\ell-1}^{k_{\ell-2}-k_{\ell-1}} d_\ell^{k_{\ell-1}-k_\ell}$.

Sal P. Barone

Purdue University, USA sbarone@live.com

Saugata Basu Purdue University sbasu@math.purdue.edu

MS62

The geometry of the TDOAbased localization

The localization of a radiant source is a common issue to many scientific and technological research areas. We solved the existence and uniqueness problem for planar source localization based on the measurements of the time differences of arrival (TDOAs) of the source signal to three distinct receivers. In geometric terms, one has to characterize a particular algebraic double cover of the real plane, that we studied using multilinear algebra and (real) algebraic geometry techniques.

Marco Compagnoni Dipartimento di Matematica "Francesco Brioschi" Politecnico di Milano marco.compagnoni@polimi.it

Roberto Notari Dipartimento di Matematica Politecnico di Milano roberto.notari@polimi.it

Fabio Antonacci, Augusto Sarti Dipartimento di Elettronica ed Informazione Politecnico di Milano antonacc@elet.polimi.it, sarti@elet.polimi.it

MS62

Applications of Real Numerical Algebraic Geometry

Large-scale polynomial systems arise in many applications including biology, chemistry, engineering, and physics. Often in these applications, only a few real isolated solutions and real connected components are of interest. Given a witness set for an irreducible algebraic set X, this talk will explore using various numerical algebraic geometric algorithms to compute the real connected components of the real points of X. This talk will conclude by demonstrating the algorithms on several of these applications.

<u>Jonathan Hauenstein</u>

North Carolina State University jdhauens@ncsu.edu

Charles Wampler General Motors Research Laboratories, Enterprise

Systems Lab 30500 Mound Road, Warren, MI 48090-9055, USA charles.w.wampler@gm.com

MS62

Some Applications of Cylindrical Algebraic Decomposition

Cylindrical Algebraic Decomposition (CAD) can be applied to determine the truth value of a given system of quantified polynomial inequalities. Gerhold and Kauers introduced a method based on CAD to prove inequalities on expressions that are defined by (linear or nonlinear) systems of recurrences with polynomial coefficients. Special functions as well as many combinatorial objects are within this class. In this talk we present some of our recent applications of the method.

Veronika Pillwein

Research Institute for Symbolic Computation (RISC) Johannes Kepler Univ. Linz veronika.pillwein@risc.jku.at

MS62

Safety Verification of Cyber-Physical Systems Using the Theory of Reals

We describe computational techniques for verification and synthesis of cyber-physical systems. These techniques invariably rely on reasoning over the theory of reals. Recently, the formal methods community has made some progress in integrating and improving algorithms based on cylindrical algebraic decomposition for reasoning about semialgebraic sets within the larger verification context. We will survey these recent advances in scaling reasoning over reals to large formulas.

<u>Ashish Tiwari</u> SRI International, USA tiwari@csl.sri.com

MS63

Subresultants, Sylvester Sums and the Rational Interpolation Problem

This talk presents a solution for the classical univariate rational interpolation problem by means of (univariate) subresultants. In the case of Cauchy interpolation (interpolation without multiplicities), explicit formulas for the solution in terms of symmetric functions of the input data are recovered, generalizing the well-known formulas for Lagrange interpolation. In the case of the osculatory rational interpolation (interpolation with multiplicities), determinantal expressions in terms of the input data are presented.

Carlos D'Andrea Universitat de Barcelona cdandrea@ub.edu <u>Teresa Krick</u> Universidad de Buenos Aires krick@dm.uba.ar

Agnes Szanto North Carolina State University aszanto@ncsu.edu

MS63

Divided Differences and Combinatorial Nullstellensatz

Alon's combinatorial Nullstellensatz, and in particular the resulting nonvanishing criterion is one of the most powerful algebraic tools in combinatorics, with a diverse array of applications. In this paper we extend the nonvanishing theorem in two directions. We prove a version allowing multiple points. Also, we establish a variant which is valid over arbitrary commutative rings, not merely over subrings of fields. For the proofs we use the technique of divided differences to finite fields. We extend some of the known applications of the original nonvanishing theorem to a setting allowing multiplicities, including the theorem of Alon and Füredi on the hyperplane coverings of discrete cubes.

<u>Géza Kós</u>, Lajos Rónyai Computer and Automation Research Institute Hungarian Academy of Sciences kosgeza@sztaki.hu, ronyai@sztaki.hu

Tamás Mészáros Dept. of Mathematics, Central European University Budapest tmeszaros87@gmail.com

MS63

Fraction-Free Polynomial Arithmetic with Interpolation Bases

In this talk we consider the problem of fraction free computation working with polynomials represented in nonstandard bases. Examples of these non-standard bases include Lagrange, Newton, and various orthogonal polynomial bases. Operations covered include rational interpolation, polynomial gcd and matrix gcd problems.

George Labahn School of Computer Science University of Waterloo glabahn@scg.uwaterloo.ca

MS63

Interpolation and Walks on the Hilbert Scheme

The interpolation problem at a given set of points can be interpreted as the construction of an explicit representation of idempotents in some quotient algebra. A natural question is how this construction changes when the points are moving. We will first describe a generalization of the univariate Weierstrass method which explicitly relates the perturbation of a complete intersection system defining a certain point set to the perturbation of the point set. Next we will consider the description of the variety of polynomial systems defining a given number of points counted with multiplicities, called the punctual Hilbert Scheme. We will give equations of degree 2 related to the commutation relations of border bases, describe its tangent space at a given border basis and discuss some limit problems in this interpolation process.

<u>Bernard Mourrain</u> GALAAD INRIA Sophia Antipolis Bernard.Mourrain@sophia.inria.fr

MS64 Calabi-Yau 3-folds. Collect them all!

Viable models of particle physics from string theory require compactification of 6 spatial dimensions onto a manifold with specific properties put forward by Calabi and Yau, including a complex structure and vanishing first Chern class. These conditions suggest that we might find a large, and more importantly, calculable class of such manifolds existing as hypersurfaces in a 4d complex toric variety and consequently encoded in a class of 4d reflexive lattice polytopes. Kreuzer and Skarke have painstakingly enumerated all such reflexive polytopes, and have made strides in extracting the desired Calabi-Yau 3-fold geometries. I discuss these results, a few ensuing difficulties, and my own current efforts to obtain a more complete exposition using the computer package SAGE.

Ross Altman

Northeastern University altman.ro@husky.neu.edu

MS64

Numerical Algebraic Geometry and Potential Energy Landscapes

The surface drawn by a potential energy function, which is usually a multivariate nonlinear function, is called the potential energy landscape (PEL) of the given Physical/Chemical system. The stationary points of the PEL, where the gradient of the potential vanishes, are used to explore many important Physical and Chemical properties of the system. Recently, we have employed the numerical algebraic geometry (NAG) method to study the stationary points of the PELs of various models arising from Physics and Chemistry and have discovered their many interesting characteristics. In this talk, I will mention some of these results after giving a very brief introduction to the NAG method. I will then go on discussing our latest adventure: exploring the PELs of random potentials with NAG, which will address not only one of the classic problems in Algebraic Geometry but will also find numerous applications in different areas such as String Theory, Statistical Physics, Neural Networks, etc.

Dhagash Mehta Department of Physics Syracuse University dbmehta@syr.edu

MS64

Algebraic Geometry and the Search for Calabi-Yau Manifolds with Large Volume Vacua

We describe an efficient, construction independent, algorithmic test to determine whether Calabi–Yau threefolds admit a structure compatible with the Large Volume moduli stabilization scenario of type IIB superstring theory. Using the algorithm, we scan complete intersection and toric hypersurface Calabi-Yau threefolds with small Picard numbers, and deduce that 418 among 4434 manifolds have a Large Volume Limit with a single large four-cycle. We describe major extensions to this survey, which are currently underway.

<u>Brent Nelson</u> Dept of Physics, Northeastern University b.nelson@neu.edu

$\mathbf{MS64}$

Supersymmetric Hidden Sectors for Heterotic Standard Models

Abstract not available at time of publication.

Burt Ovrut

Dept of Physics, University of Pennsylvenia ovrut@elcapitan.hep.upenn.edu

MS64

Integrand Reduction of High-loop Scattering Amplitudes via Computational Algebraic Geometry

Integrand reduction is a powerful method for loop scattering amplitudes computation. We present an algebraic geometry approach for Integrand-level reduction, which is valid for any renormalizable theories and arbitrary loop orders, in principle. The integrand form of amplitudes can be determined by Groebner basis method and the unitarity structure is systematically studied by algebraic geometry. We also show some explicit two-loop and three-loop N=0,1,2,4 (super)-Yang-Mills amplitudes examples, based on our methods.

Yang Zhang Niels Bohr Institute, Denmark zhang@nbi.dk

MS65

Paramotopy: Parallel Parameter Homotopy Software Through Bertini

Many real-world problems involve parameterized systems of polynomials. The quick solution of such systems for large sets of parameter samples will enable the scientist to scan for regions of interest in parameter space, collect data for conjectures, and other applications. Paramotopy is a program which enables efficient solution of parameterized systems via parameter homotopy, leaning heavily on Bertini as the numerical solver. It uses parallel computing techniques, provides easy back-end access to processing of data, and can automatically re-solve the system at any trouble points. This talk will discuss some features and applications of Paramotopy.

Daniel A. Brake Colorado State University brake@math.colostate.edu

MS65

Applications of Homotopy Method to Nonlinear Pdes

This talk will cover some recent progress on numerical methods to solve systems of nonlinear partial differential equations (PDEs) arising from biology and physics. This new approach, which is used to compute multiple solutions and singularity of nonlinear PDEs, makes use of polynomial systems (with thousands of variables) arising by discretization. Examples from hyperbolic systems will be used to demonstrate the ideas.

<u>Wenrui Hao</u>

Dept. of Applied and Comp. Mathematics and Statistics University of Notre Dame whao@nd.edu

MS65

Chebyshev Method for a Free Boundary Problem Modeling Tumor Growth

Abstract not available at time of publication.

<u>Oliver Kernell</u> University of Notre Dame otkernell@gmail.com

MS65

Numerical Algebraic Geometry in Algebraic Statistics

Maximum likelihood estimation is a fundamental computational task in statistics and involves beautiful geometry. We discuss this task for determinantal varieties (matrices with rank constraints) and show how numerical algebraic geometry can be used to maximize the likelihood function. Our computational results with the software Bertini led to surprising conjectures and duality theorems. This is joint work with Jan Draisma, Jon Hauenstein, and Bernd Sturmfels.

Jose Rodriguez

Department of Mathematics University of California, Berkeley jo.ro@berkeley.edu

MS65 A Web Interface for PHCpack

Our web interface for PHCpack allows users to submit polynomial systems through a browser, after signing up via an automated email process. Through a password protected account, the user has access the solutions of the submitted systems stored on our server. Solved problems may serve as start systems for similar problems. The web server is connected to a remote computational server which manages the job queue for the distributed solving of polynomial systems. The user administration is done with MySQL and the servers are implemented in Python.

Jan Verschelde Department of Mathematics, Statistics and Computer Science University of Illinois at Chicago jan@math.uic.edu

Xiangcheng Yu

Department of Mathematics, Statistics, and Computer Science University of Illinois at Chicago xyu30@uic.edu

MS66

Kolmogorov's Problem on the Class of Multiply Monotone Functions

In this talk we give necessary and sufficient conditions for the system of positive numbers $M_{k_1}, M_{k_2}, ..., M_{k_d}, 0 \leq$ $k_1 < ... < k_d \leq r$, to guarantee the existence of an *r*-monotone function defined on the negative half-line _ and such that $||x^{(k_i)}||_{\infty} = M_{k_i}$, i = 1, 2, ..., d.

Yuliya Babenko Kennesaw State University ybabenko@kennesaw.edu

Vladislav Babenko, Oleg Kovalenko Dnepropetrovsk National University Dnepropetrovsk, Ukraine babenko.vladislav@gmail.com, olegkovalenko90@gmail.com

MS66

Toric Degenerations of (irrational) Bezier Patches

A Bezier surface is intuitively governed by control points; in particular, the surface lies within the convex hull of its control points. This convex hull is often indicated by drawing some edges between the control points, the resulting structure is called a control mesh. In this talk, we give a precise mathematical description of how these meshes "control" the Bezier surface. We will present our results in the general context of (irrational) toric Bezier patches.

Luis D. Garcia-Puente Sam Houston State University ldg005@shsu.edu

Frank Sottile Texas A&M University sottile@math.tamu.edu

Chungang Zhu Dalian University of Science and Technology cgzhu@dlut.edu.cn

Elisa Postinghel Institute of Mathematics of the Polish Academy of Sciences elisa.postinghel@gmail.com

Nelly Villamizar Johann Radon Institute for Computational and Applied Mathema nelly.villamizar@ricam.oeaw.ac.at

MS66

Towards an Algebra for Rational Curves and Surfaces in Two and Three Dimensions

The complex numbers allow us to multiply complex valued rational curves and surface patches in the plane. By spherical inversion (perspective projection) this approach generates an algebra for curves and surface patches on the sphere. Similarly, the quaternions allow us to multiply (with some restrictions) rational curves and surfaces in 3dimensions. The goal of this talk is to explore the geometry accompanying this algebra for rational curves and surfaces in two and three dimensions.

<u>Ron Goldman</u> Rice University Computer Science rng@rice.edu

MS66

Wachspress Varieties

Wachspress Varieties Eugene Wachspress introduced rational barycentric coordinates for convex polygons in \mathbb{R}^2 , which Warren generalized to all convex polytopes in \mathbb{R}^d . These Wachspress barycentric coordinates are the unique rational barycentric coordinates of minimal degree. The Wachspress coordinates of a polytope P in \mathbb{R}^d define a map of projective *d*-space to a projective space spanned by the vertices of P whose image is a Wachspress variety. In this talk, I will describe the Wachspress variety and its defining ideal, which is the ideal of algebraic relations among the Wachspress coordinates. This is joint work with Corey Irving, Hal Schenck, and Greg Smith.

<u>Frank Sottile</u> Texas A&M University sottile@math.tamu.edu

Corey Irving Santa Clara University cirving@scu.edu

Henry Schenck University of Illinois schenck@math.uiuc.edu

Gregory G. Smith Queen's University ggsmith@mast.queensu.ca

MS66

Representation of Surface Pencil with A Common Line of Curvature

Developable surface and line of curvature play important roles in geometric modeling and pratical applications. By utilizing the Frenet frame, we derive the necessary and sufficient condition for the parametric surface pencils possessing the given curve as the line of curvature, and present the conditions for the resulting surfaces are developable.

Chungang Zhu

Dalian University of Science and Technology cgzhu@dlut.edu.cn

Caiyun Li, Renhong Wang Dalian University of Technology licaiyundlut@yahoo.cn, renhong@dlut.edu.cn

MS67

Graph Based Codes for Flash Memories

Finite geometries have been useful in several coding applications, from early Write-Once-Memory (WOM) codes for data storage to the design of LDPC error-correcting codes. However, modern data storage techniques impose several conflicting design constraints that make the direct use of finite geometry LDPC codes harder to apply. In this talk we will discuss how graph based codes can be used in coding for non-volatile memories, and suggest how to modify algebraic codes (such as finite geometry codes) for this application.

Christine A. Kelley University of Nebraska ckelley2@math.unl.edu Kathryn Haymaker University of Nebraska-Lincoln s-khaymak1@math.unl.edu

MS67

GF(q)-Linear Codes over $GF(q^t)$

We will examine the classical theorems for linear codes and see if and how they apply to GF(q)-linear codes over $GF(q^t)$, We will discuss some of the following: the MacWilliams Identities, the Gleason polynomials, the Gleason-Pierce Theorem, Mass Formulas, the Balance Principle, the Singleton Bound, and MDS codes.

Cary Huffman

Loyola University Chicago whuffma@luc.edu

MS67

Geometric Perspective on Alternant Codes

Alternant codes are a large class of codes containing many of the most well-known constructions, such as BCH-codes and Goppa codes. Many other constructions containing best known codes exist, but there is no known way to guarantee optimal constructions. We will present a geometric description of alternant codes and show how this description can lead to some new bounds and ideas for constructing good codes.

Kyle Marshall University of Zurich kyle.marshall@math.uzh.ch

MS67

On the Number of Constacyclic Codes on a Class of Local Finite Frobenius Rings

The study of cyclic and constacyclic codes over various alphabets has been of considerable interest in Coding Theory. These alphabets include finite fields, Galois rings, and finite chain rings. In this talk, the number of constacyclic codes the alphabet of which is a finite local non-chain Frobenius ring with a maximal ideal of nilpotency index 3, is determined, where the length of the codes is relatively prime to the characteristic of the residue field of the ring. Results from Commutative Algebra including the length of a module, Nakayama's Lemma, and Galois extension of a finite local ring, are helpful in this discussion.

Horacio Tapia-Recillas UAM Mexico htr@xanum.uam.mx

MS68

Calculations in the Lie Invariant Calculus of Variations - the General Case

In recent works, the authors considered Lagrangians invariant under a Lie group action, in the case where the independent variables are themselves invariant. Using a moving frame for the Lie group action, we showed how to calculate the Euler Lagrange equations for the invariants and to obtain the space of conservation laws in terms of a vector of invariants and the adjoint representation of the frame. In this talk, we show how our calculations extend to cases where the independent variables participate in the action. We take for our main expository example the standard linear action of SL(2) on the two independent variables. This choice is motivated by potential applications to variational fluid problems which conserve potential vorticity.

Elizabeth Mansfield University of Kent e.l.mansfield@kent.ac.uk

<u>Tania Goncalves</u> Universidade Federal de So Carlos tmng@kentforlife.net

MS68

Symbolic Computation of Lax Pairs of Systems of Partial Difference Equations Using Consistency Around the Cube

A three-step method due to Bobenko & Suris and Nijhoff to derive Lax pairs for scalar partial difference equations is extended to systems which are defined on a quadrilateral and consistent around the cube. Lax pairs will be presented for several systems including the integrable 2component potential Korteweg-de Vries lattice system, as well as nonlinear Schrodinger and Boussinesq-type lattice systems. Previously unknown Lax pairs will be presented for systems of partial difference equations recently derived by Hietarinta. The method is algorithmic and is being implemented in Mathematica.

Willy A. Hereman

Department of Applied Mathematics and Statistics Colorado School of Mines whereman@mines.edu

Terry Bridgman Dept Appl Mathematics & Statistics, Colorado School of Mines tbridgma@mines.edu

MS68

Induced Curvature Flow and Integrability

The operator providing the induced curvature flow for a geometric flow can be computed algorithmically. The method relies on the construction of a complete set of syzygies for a generating set of differential invariants of the geometry and differential elimination. With the Maple packages AIDA [Algebraic Invariants and their Differential Algebras] and diffalg to re-examine geometries in the plane. By removing the non-stretching condition, an exhaustive search provides new generalisations of the usual integrable equations as induced curvature flow. This is work in progress.

Evelyne Hubert INRIA Méditerranée evelyne.hubert@inria.fr

Peter van der Kamp University of La Trobe p.vanderkamp@latrobe.edu.au

MS68

Pseudo-spherical Twisted Columns

The aim of this presentation is to explain the global geometric properties of singular helicoids with Gaussian curvature K=-1. We will show that, for every positive integer nthere is a one-parameter family of pseudo-spherical singular helicoids which are topologically embedded cylinders with the shape of twisted columns. Numerical routines to detect and visualize the pseudo-spherical twisted columns are given. The geometrical problem while being rather simple from the conceptual point of view, presents a certain computational complexity which is caused by the presence of singularities. The theoretical framework of our approach is based on the method of moving frame while the problems of computational and numerical nature have been dealt with the use of the software Mathematica 9. The interest of the result lies in the fact that the known examples of pseudo-spherical surfaces that are topologically embedded cylinders are very sporadic and substantially boil down to the Beltrami pseudo-sphere and the pseudo-spherical surfaces of rotation of cylindrical type, whose origin dates back to the early days of the theory. Another reason that gives meaning to our result must be sought in possible applications to the study of the "modulated wave solutions" of certain mathematical models which have been recently used in the theory of non-linear elasticity.

Emilio Musso

Dipartimento di Matematica emilio.musso@polito.it

MS68

Holomorphic Differentials and Laguerre deformation of surfaces

We present a local characterization of L-minimal surfaces and Laguerre deformations (T-transforms) of L-minimal isothermic surfaces in terms of the holomorphicity of a quartic and a quadratic differential form. This result is then discussed in relation with the differential geometry of the Laguerre Gauss map of surfaces, viewed as a spacelike immersion in Minkowski spacetime. As an application, we will show that the Lawson correspondence between certain constant mean curvature surfaces in different space forms and the analogous correspondence between certain constant mean curvature spacelike surfaces in different Lorentzian space forms can be viewed as special cases of the Laguerre deformation of L-isothermic surfaces. Our approach uses the method of moving frames. This is based on joint work with E. Musso.

Lorenzo Nicolodi Università degli Studi di Parma lorenzo.nicolodi@unipr.it

MS69

Real Rank of Real Symmetric Tensors

I will explain the ways in which symmetric real tensor decomposition differs from the complex case and present recent results. Open question abound in the real case.

Greg Blekherman Georgia Institute of Technology greg@math.gatech.edu

MS69

Tensor Network Decompositions

Abstract not available at time of publication.

Jason Morton Pennsylvania State University morton@math.psu.edu

MS69

Eigenvectors of Tensors and Waring Decomposition.

In recent work with Ottaviani, we have developed algorithms for symmetric tensor decomposition. I will explain these algorithms, their connection to algebraic geometry, and possible applications.

Luke Oeding University of California Berkeley Università degli Studi di Firenze lukeoeding@gmail.com

Giorgio Ottaviani University of Firenze ottavian@math.unifi.it

MS69

Algorithms for Tensor Decomposition via Numerical Homotopy and Optimization

This talk will present consider several numerical algorithms for tensor decompositions and tensor rank. The algorithms are based in numerical homotopy and in semi-definite programming but arise from the point of view of algebraic geometry.

Chris Peterson

Colorado State University peterson@math.colostate.edu

MS69

Computational Complexity and Linear Preservers

We study the relation between computational complexity and linear preservers of various problems. We first investigate this relation for determinant, immanants, and α permenant. Then we consider the same question for rank and non-negative rank, p-norms for different values of p, and positive definiteness and complete positivity.

<u>Ke Ye</u>, Lek-Heng Lim University of Chicago kye@math.uchicago.edu, lekheng@galton.uchicago.edu

MS70

Bernstein-Sato Ideals

For any topological space, one can define the cohomolog jump loci. These are natural strata in the space of local systems of rank one on the space. When the space is given by polynomials, i.e. a quasi-projective variety, the cohomology jump loci have rigid arithmetic structure. This is a result obtained together with Botong Wang. We will address the practical question of how to compute the cohomology jump loci, the conjectural answer being: via Bernstein-Sato ideals. This generalizes the computation of Milnor monodromy eigenvalues of a hypersurface.

<u>Nero Budur</u> Notre Dame nbudur@nd.edu

MS70

Modularity and the Reciprocal Plane

Let \mathcal{A} be a collection of n linear hyperplanes in K^{ℓ} , where

K is an algebraically closed field. The Orlik-Terao algebra of \mathcal{A} is the subalgebra $\mathbb{R}(\mathcal{A})$ of the rational functions generated by reciprocals of linear forms vanishing on hyperplanes of \mathcal{A} . It determines an irreducible projective variety $Y(\mathcal{A})$, the reciprocal plane. We show that a flat X of \mathcal{A} is modular if and only if $\mathbb{R}(\mathcal{A})$ is a split extension of the Orlik-Terao algebra of the subarrangement \mathcal{A}_X . This provides another refinement of Stanley's modular factorization theorem and a new characterization of modularity. We note that if \mathcal{A} is supersolvable, then $\mathbb{R}(\mathcal{A})$ is Koszul, and characterize when $\mathbb{R}(\mathcal{A})$ is a complete intersection.

Graham Denham

University of Western Ontario gdenham@uwo.ca

MS70

F-jumping Numbers of Homogeneous Polynomials

This is joint work with Luis Núñez Betancourt, Emily Witt, and Wenliang Zhang. F-jumping numbers are important invariants of singularities in characteristic p > 0, and may be thought of as positive characteristic analogs of jumping exponents in characteristic zero. The aim of this work is to investigate certain arithmetic properties of these invariants; in particular, we will present a description of F-pure thresholds of homogeneous polynomials with an isolated singluarity.

<u>Daniel Hernandez</u> University of Minnesota dhernan@umn.edu

Luis C. Núñez-Betancourt University of Michigan luisnub@umich.edu

Emily E. Witt University of Minnesota ewitt@umn.edu

Wenliang Zhang University of Nebraska wzhang15@unl.edu

MS70

A Stopping Condition to Compute Test Ideals and F-Jumping Numbers

We will discuss a stopping condition that gives algorithms to compute test ideals and F-jumping numbers, which play an important role in the study of singularities in positive characteristic. In addition, we will see properties of F-pure thresholds that are a consequence of this condition.

Luis C. Núñez-Betancourt University of Michigan luisnub@umich.edu

Daniel Hernandez, Emily E. Witt University of Minnesota dhernan@umn.edu, ewitt@umn.edu

MS70

An Algorithm to Find Annihilators of Artinian Modules Compatible with a Frobenius Map

I will discuss an algorithm to find radical annihilators of

submodules, compatible with a Frobenius map, of an Artinian module over a ring of formal power series over a field of characteristic p. This is a joint work with Mordechai Katzman.

Wenliang Zhang University of Nebraska wzhang15@unl.edu

Mordechai Katzman University of Sheffield m.katzman@@sheffield.ac.uk

MS71

Boolean Matrices with Prescribed Row and Column Sums, Associated Partition Functions and Hyperbolic Polynomials

Hyperbolic (and their generalizations) polynomials were recently used to improve several lower bounds in combinatorics and geometry. These new lower bounds can be used to obtain poly-time algorithms to approximate many hard interesting quantities, notably the mixed volume, within simply exponential multiplicative error. The talk will describe one refinement of the lower bounds and its application to the approximation of partition functions associated with Boolean matrices with prescribed row and column sums. In a way, we will describe a natural class of $n \times n$ nonnegative matrices which allows a poly-time deterministic algorithm to approximate the permanent with the factor $const\sqrt{n}$. Besides, we will explain how the celebrated Gale-Ryzer Theorem follows immediately from the hyperbolic framework.

<u>Leonid Gurvits</u> Los Alamos National Laboratory gurvits@lanl.gov

MS71

Norm-constrained Determinantal Representations of Multivariable Polynomials

For every multivariable polynomial p, with p(0) = 1, we construct a determinantal representation

$$p = \det(I - KZ),$$

where Z is a diagonal matrix with coordinate variables on the diagonal and K is a complex square matrix. When norm constraints on K are imposed, we give connections to the multivariable von Neumann inequality, Agler denominators, and stability. We show that if a multivariable polynomial q, q(0) = 0, satisfies the von Neumann inequality, then 1 - q admits a determinantal representation with K a contraction. On the other hand, every determinantal representation with a contractive K gives rise to a rational inner function in the Schur-Agler class.

Dmitry Kaliuzhnyi-Verbovetskyi

Drexel University dmitryk@math.drexel.edu

MS71

Determinantal Representations of Singular Hypersurfaces in $\mathbf{P}^{\mathbf{n}}$

A (global) determinantal representation of projective hypersurface X in $\mathbf{P}^{\mathbf{n}}$ is a matrix whose entries are linear forms in homogeneous coordinates and whose deter-

minant defines the hypersurface. We study the properties of such representations for singular (possibly reducible or non-reduced) hypersurfaces. In particular, we obtain the decomposability criteria for determinantal representations of globally reducible hypersurfaces. Further, we classify the determinantal representations in terms of the corresponding kernel sheaves on X. Finally, we extend the results to the case of symmetric/self-adjoint representations, with implications to hyperbolic polynomials and generalized Lax conjecture.

Dmitry Kerner

Ben Gurion University dmitry.kerner@gmail.com

MS71

Primal-Dual Algorithms for Optimization over Hyperbolicity Cones

Hyperbolicity cones are rich in structure, yet it remains unclear whether the structure can be utilized to design optimization algorithms that generally are more efficient than interior-point methods. However, as we will discuss, when a warm-start is available – that is, when an approximate optimal solution is available (by, say, having already solved a problem differing only slightly in the data) – then the boundary structure of the hyperbolicity cone most certainly can be efficiently harnessed. Unsurprisingly, the most effective harnessing makes use of duality, but only duality structure arising from the hyperbolicity cone's boundary in a neighborhood of the warm-start. Far more challenging is understanding the global structure of the dual cones of (general) hyperbolicity cones. We present steps in this direction, and discuss their relevance for designing primal-dual algorithms beyond the warm-start setting.

James M. Renegar Cornell University School of ORIE renegar@orie.cornell.edu

MS71

Hyperbolic Cone Programming: Structure and Interior-Point Algorithms

I will first discuss some structural and geometric properties of convex cones which are hyperbolicity cones of some hyperbolic polynomial. Then, I will present some interiorpoint algorithms and their theoretical features on the hyperbolic cone programming problems. This talk is based on joint work with Tor Myklebust.

<u>Levent Tuncel</u> University of Waterloo Dept. of Combinatorics and Optimization ltuncel@math.uwaterloo.ca

MS72

Linear Algebra with Errors: On the Complexity of the Learning with Errors Problem

The Learning With Errors problem (LWE) - a generalisation of the LPN problem to larger moduli - enjoys widespread appreciation from the cryptographic community because it allows for the construction of a wide range of cryptographic primitives and comes with convincing security arguments. However, the concrete hardness of concrete LWE instances is not that well understood. The best known algorithm for LWE is an adaptation of the BKW algorithm to the larger moduli case. In this work we investigate this adaptation in detail, refine it and provide concrete costs for solving LWE instances from the literature.

<u>Martin Albrecht</u> Université Pierre et Marie Curie Paris, France martinralbrecht@googlemail.com

MS72

Fast Matrix Decomposition in F2

An efficient algorithm to perform a block decomposition (and so to compute the rank) of large dense rectangular matrices with entries in \mathbf{F}_2 is presented. Depending on the way the matrix is stored, the operations acting on rows or block of consecutive columns (stored as one integer) should be preferred. In this paper, an algorithm that completely avoids the column permutations is given. In particular, a block decomposition is presented and its running times are compared with the ones adopted into SAGE.

Enrico Bertolazzi

Dipartimento di Ingegneria Industriale Facoltà di Ingegneria - Università di Trento, Italy enrico.bertolazzi@unitn.it

Anna Rimoldi Dipartimento di Matematica Università di Trento anna.rimoldi@gmail.com

MS72

Relaxed Hensel Lifting for Dense, Sparse and Structured Linear System Solving

This talk is about linear system solving on integers or polynomials via lifting techniques. We study the Hensel lifting technique using relaxed algorithms and take a careful look at the adaptation of this method to different matrix representations (dense, structured, sparse...). We compare our results to the concurrent lifting technique (Newton iteration) and show improvements both in theory and practice.

Romain Lebreton LIRMM Univ. Montpellier 2, France lebreton@lirmm.fr

MS72

Rational Linear Solvers and Local Smith Forms and How They Apply to Homology Computation

Simplicial Homology is determined by the integer Smith normal forms of boundary matrices, which are (1,0,-1) matrices that are most often very sparse. It is important to be efficient in both time and space on Smith form computation. I will report on some recent developments concerning space efficiency and show what remains open. I will discuss the use of block blackbox methods, important for effective parallel implementations.

David Saunders

Dept of Computer and Information Sciences University of Delaware, Newark saunders@cis.udel.edu

p-Adic Height Pairings and Integral Points on Hyperelliptic Curves

We give a formula for the component at p of the p-adic height pairing of a divisor of degree 0 on a hyperelliptic curve in terms of iterated p-adic integrals. We use this to give a Chabauty-like method for computing p-adic approximations to integral points on such curves when the Mordell-Weil rank of the Jacobian equals the genus.

Jennifer Balakrishnan Harvard University jen@math.harvard.edu

Amnon Besser University of Oxford amnon.besser@maths.ox.ac.uk

J. Steffen Mueller Universitaet Hamburg jan.steffen.mueller@uni-hamburg.de

MS73

Real Hyperelliptic Curves of Genus 2

We will present optimized explicit formulas for the arithmetic of real hyperelliptic curves of genus 2 in both affine and projective coordinates. We will compare the results to imaginary curves and timings of cryptographic protocols in both cases.

<u>Stefan Erickson</u> Colorado College Stefan.Erickson@ColoradoCollege.edu

MS73

A Lutz-Nagell Theorem for Hyperelliptic Curves

We'll present a version of the Lutz-Nagell Theorem for curves C defined by $y^2 = f(x)$, where f is a monic polynomial of degree 2g + 1 defined over the ring of integers of a number field or its non-archimedean completions. Namely, if J is the Jacobian of C, and ϕ is the Abel-Jacobi map of C into J sending the point at infinity on C to the origin of J, then if P = (a, b) is a rational point of C such that $\phi(P)$ is torsion in J, then a and b are integral if the order n of Pis not a prime power, and we can bound the denominators of a and b if n is. When a and b are integral, we'll discuss criteria for when b^2 divides the discriminant of f.

<u>David Grant</u> University of Colorado grant@colorado.edu

MS73

Improved Scalar Multiplication on Hyperelliptic Curves

Scalar multiplication, adding a divisor class to itself n times, is a central operation of elliptic and hyperelliptic curve cryptosystems. In this talk, we describe recent results and on-going efforts to improve this operation using ideas involving double-base expansions of the scalar, including an algorithm for anomalous binary hyperelliptic curves that requires a sublinear number of divisor additions in the size of the scalar. This is joint work with H.

Labrande and S. Lindner.

<u>Michael Jacobson</u> University of Calgary jacobs@cpsc.ucalgary.ca

MS73

Effective Chabauty for symmetric Powers of Curves

While we know by Faltings' theorem that curves of genus at least 2 have finitely many rational points, his theorem is not effective. In 1985, R. Coleman showed that Chabauty's method, which works when the Mordell-Weil rank of the Jacobian of the curve is small, can be used to give a good effective bound on the number of rational points of curves of genus g > 1. We draw ideas from tropical geometry to show that we can also give an effective bound on the size of the finite component of Sym^dX, where X is a curve that satisfies certain Chabauty-type condition, hence showing that Chabauty's method can be used for certain higher-dimensional varieties.

Jennifer Park

Massachusetts Institute of Technology jmypark@math.mit.edu

MS74

Evasion Paths in Mobile Sensor Networks

Suppose disk-shaped sensors wander in a planar domain. A sensor doesn't know its location but does know which sensors it overlaps. We say that an evasion path exists in this sensor network if a moving evader can avoid detection. Vin de Silva and Robert Ghrist give a necessary condition, depending only on the time-varying connectivity graph of the sensors, for an evasion path to exist. Can we sharpen this result? We consider an example where the existence of an evasion path depends not only on the network's connectivity data but also on its embedding. We also study the space of evasion paths using a generalization of the unstable J. F. Adams spectral sequence to diagrams of spaces.

Henry Adams, Gunnar E. Carlsson Stanford University

 $henrya @math.stanford.edu, \ gunnar @math.stanford.edu \\$

MS74

Spaces of Shapes: Creating Moduli Spaces of Chemical Compounds for Drug Discovery

Abstract not available at time of publication.

Anthony Bak Stanford University bak@math.stanford.edu

MS74

Homological Algebra over Semirings for Optimization

Some optimization dualities (e.g. MFMC) generalize to a Poincare Duality for sheaves of semimodules. We detail how a generalized homological algebra for semimodules over general semirings yields new methods for solving stochastic optimization problems. No familiarity with semimodule theory will be assumed.

Sanjeevi Krishnan University of Pennsylvania sanjeevi.krishnan@gmail.com

MS74

Multicore Homology

We design and implement a framework for parallel computation of homology of cellular spaces over field coefficients, by decomposing the space. Theoretically, we show that optimal decomposition into local pieces is NP-Hard. In practice, we achieve roughly an 8x speedup of homology computation on a 3-dimensional complex with about 10 million simplices using 11 cores.

Ryan Lewis Stanford University me@ryanlewis.net

MS74

Inferring Dynamics with Perisistence

Dynamical systems are often easier to analyze if we have a good parametrization. For periodic or recurrent systems, this is the map to a circle. The first part of the talk will describe how persistent cohomology can help construct a natural parametrization of certain dynamical systems. The second part will go into specific cases how this information augments traditional invariants such as the Conley index and allows us to infer properties of a dynamical system.

<u>Primoz Skraba</u> Jozef Stefan Institute Ljubljana, Slovenia primoz.skraba@ijs.si

MS75

Algebraic Geometry of Partially Nested Canalyzing Functions

A Boolean function is canalyzing if some variable has the property that taking a specified input determines the output. If this variable does *not* take that input, then the output is a function on n - 1 variables. We may ask if it too is canalyzing, and so on. In this talk I will discuss the *nested canalyzing depth* of a function, which measures how many times we can recursively pick off canalyzing variables in this manner.

Qijun He Clemson University qhe@clemson.edu

MS75

Hodge-Kodaira Decomposition of Evolving Neural Networks

Although the network topology of the brain can be very important, the conventional network analyses are often limited to locally defined variables such as degrees. Those intrinsically local variables cannot capture global recurrent structures, which are frequently observed in neural networks. For example, a network that alters its coupling strengths under STDP learning rule has tendency to make paths and loops of sequential firings. We applied the Hodge-Kodaira decomposition of graph flows to evolving neural network models in order to count the number of global loops as a topological measure of network structures. The dimension of each flow not only reflected the known bifurcation diagrams but also discovered the inhomogeneity of the chaotic region.

Keiji Miura Tohoku University miura@ecei.tohoku.ac.jp

Takaaki Aoki Kagawa University takaaki.aoki.work@gmail.com

MS75

Algebraic Geometry in the Life and Physical Sciences: Past, Present, and Future

We will give an overview of how algebraic geometry has impacted the life and physical sciences in recent years. We will also discuss highlights from a workshop at the Mathematical Biosciences Institute that immediately preceded this conference on novel ways to integrate discrete and algebraic methods into undergraduate education. We will conclude by sharing some surprising and upcoming research areas.

Brandilyn Stigler

Southern Methodist University bstigler@smu.edu

Matthew Macauley Mathematical Sciences Clemson University macaule@clemson.edu

MS75

Reverse Engineering Functional Networks of the Human Brain within the Polynomial Dynamical Systems Framework

In this talk we will present an application example of a reverse engineering method within the framework of polynomial dynamical systems. The objective is to unravel the interaction patters among functional brain regions identified by task-based functional magnetic resonance imaging (fMRI) time series data. These interaction patterns are used to construct functional networks of the human brain to enable a comparative study across groups of individuals.

<u>Paola Vera-Licona</u> Institut Curie paola.vera-licona@curie.fr

MS75

Detection of De Novo Copy Number Variants from Whole Exome Sequencing Data

Exome sequencing is becoming a standard tool for mapping Mendelian disease-causing variations, yet the exome data are often more noisy. Here we describe a novel method for the detection of de novo copy number variants from such data. We developed a likelihood model combined with a hidden Markov model to detect the variants jointly from trio samples. Our model outperforms other methods that call variants separately.

Mingfu Zhu, Yongzhuang Liu

Duke University mingfu.zhu@duke.edu, yz.liu@duke.edu

MS76

Reducing Implicitization to Interpolation via Support Prediction

We reduce implicitization of parametric curves and (hyper)surfaces to computing the nullspace of a numeric matrix, given a superset of the support of the implicit equation. For predicting the support we compute the Newton polytope of the implicit polynomial, via sparse resultant theory. We shall discuss both the support prediction and the interpolation steps of the method, highlight their connections and describe an application of the method to computing discriminants.

<u>Christos Konaxis</u> ACMAC, University of Crete ckonaxis@acmac.uoc.gr

Ioannis Z. Emiris National and Kapodistrian University of Athens emiris@di.uoa.gr

Tatjana Kalinka University of Athens kalinkat@di.uoa.gr

Thang Luu Ba Hanoi National Univ. of Education thanglb@di.uoa.gr

MS76

Remarks on Nagata's Conjecture

Abstract not available at time of publication.

<u>Rick Miranda</u> Colorado State University rick.miranda@colostate.edu

MS76

Polynomial Interpolation and Sums of Powers

The affine space of polynomials of degree $\leq d$ in n variables, with assigned values of any number of general linear combinations of first partial derivatives, has the expected dimension if $d \neq 2$ with five exceptional cases. Four among these cases appear in Alexander-Hirschowitz theorem, and correspond to the assignment of *all* the first partial derivatives. If d = 2 the exceptional cases are described. This problem is related to the decomposition of a polynomial as sum of powers of linear forms.

Giorgio Ottaviani University of Firenze ottavian@math.unifi.it

Chiara Brambilla Politecnico delle Marche, Ancona brambilla@dipmat.univpm.it

MS76

Subresultants in Multiple Roots and Connections

to Hermite-Birkhoff Interpolation

This is a joint work with Carlos D'Andrea and Teresa Krick. The topic is a continuation of our previous work on Poisson-like formulas for univariate and multivariate subresultants in terms of the roots of a polynomial system, as well as our work on the relationship between univariate subresultants and Sylvester double sums, provided that all these roots were simple. As it was pointed out by an anonymous referee, it would be interesting to work out these results for the case of multiple roots. This work is a first attempt in that direction. We successfully extended Poisson-like formulas for univariate and multivariate subresultants in the presence of multiple roots. We also obtained closed formulas for subresultants in roots in the univariate setting for some non-trivial extremal cases. However, it is still open how to generalize Sylvester double sums in the multiple roots case. In the talk we will also emphasize the relationship of this problem to Hermite-Birkhoff interpolation formulas.

Carlos D'Andrea Universitat de Barcelona cdandrea@ub.edu

Teresa Krick Universidad de Buenos Aires teresa.krick@gmail.com

Agnes Szanto North Carolina State University aszanto@ncsu.edu

MS77 Invariant Theory for Matrix Product States

Abstract not available at time of publication.

<u>Jacob Biamonte</u> ISI Torino jacob.biamonte@qubit.org

MS77

Tensor Networks

Abstract not available at time of publication.

Jason Morton Pennsylvania State University morton@math.psu.edu

MS77

Contracting Tensor Networks

Abstract not available at time of publication.

Jacob Turner Pennsylvania State University turner@math.psu.edu

MS78 Title Not Available at Time of Publication

Abstract not available at time of publication.

Amihay Hanany Theoretical Physics Division, Imperial College London a.hanany@imperial.ac.uk

Vector Bundles, Calabi-Yau Threefolds and the Heterotic String

We review some recent progress in explicitly constructing stable vector bundles, inspired by the search for the standard model within the context of heterotic string compactifications, on Calabi-Yau threefolds. In due course, we will see the necessity of computational and algorithmic geometry on a large scale.

Yang-Hui He

University of Oxford City University of London yang-hui.he.1@city.ac.uk

MS78

Generalized T-duality, String Theory and the Real World'

We examine the notion of T-duality in string theory from a phase space point of view and then emphasize the concept of Born's reciprocity, a natural generalization of T-duality, which posits the more general duality between spacetime and momentum space. We propose that Born's reciprocity demands a dynamical momentum space, and thus a dynamical phase space, opening up an exciting possibility for new dualities in string theory and new connections to the real world. [This work is done together with Laurent Freidel (Perimeter Institute) and Rob Leigh (Urbana).

Djordje Minic Virginia Tech dminic@vt.edu

MS78

New Moduli Spaces of Brane Tilings on Riemann Surfaces

Brane tilings are bipartite periodic graphs on the torus which represent one of the largest known classes of 4d N=1supersymmetric gauge theories. A brief review of brane tilings will lead to a discussion on recent developments of the subject. These include the addition of boundaries as well as the variation of the genus of the Riemann surface of the bipartite graphs. The talk will give an overview of the physical significance of these extensions to brane tilings.

Rak-Kyeong Seong

Theoretical Physics Division, Imperial College London rak-kyeong.seong05@imperial.ac.uk

MS79

Linear Obstructions for Linear Systems in Pⁿ

The linear system L of degree-d hypersurfaces of $\mathbf{P}^{\mathbf{n}}$ with prescribed general multiple points is said to be special if the conditions imposed by the multiple points are not linearly independent. This phenomenon occurs when the multiplicities force L to contain in its base locus, besides the multiple points, also higher dimensional cycles. We will discuss the case when such cycles are linear and introduce the notion of linear speciality. It takes into account the obstructions given by multiple linear base cycles and provides a geometric interpretation of algebraic conjectures by Fröberg and Iarrobino on the Hilbert function of ideals generated by powers of general linear forms. Institute of Mathematics of the Polish Academy of Sciences elisa.postinghel@gmail.com

Chiara Brambilla Politecnico delle Marche, Ancona brambilla@dipmat.univpm.it

Olivia Dumitrescu Department of Mathematics University of California, Davis dolivia@math.ucdavis.edu

MS79

Some Algebraic Problems in Polynomial Interpolation

We will discuss some problems in multivariate polynomial interpolation and their relationship to questions in algebraic geometry; in particular to questions regarding subspace arrangements and irreducibility of border schemes.

Boris Shekhtman Department of Mathematics and Statistics University of South Florida shekhtma@usf.edu

MS79

Special Positions of Body-and-cad Frameworks

A body-and-cad framework consists of a finite set of fulldimensional rigid bodies and geometric constraints among them. These frameworks are the combinatorial models of structures arising in computer-aided design software. Being able to detect constraints that specify a minimally rigid body-and-cad framework has the potential to improve user feedback. Following work of White and Whiteley for body-and-bar frameworks, it is possible to characterize generic minimal infinitesimal rigidity by the nonvanishing of a polynomial, the "pure condition," which can be written as a bracket polynomial in the coordinate ring of a Grassmannian. This talk will focus on how understanding the relationship between the combinatorics of the framework, the irreducible factors of the pure condition, and the Grassmann-Cayley factorization of the bracket polynomial can yield insight into the identification and behavior of special positions.

<u>Jessica Sidman</u>, Ruimin Cai Mount Holyoke College jsidman@mtholyoke.edu, cai22r@mtholyoke.edu

James Farre University of Texas, Austin jamesrfarre@gmail.com

Audrey Lee-St.John Mount Holyoke College astjohn@mtholyoke.edu

Louis Theran Freie Universitat Berlin theran@math.fu-berlin.de

MS79

Classification of Planar Pythagorean Hodograph

Quintics

We describe PH quintic curves using two parameters, which determine their global shape up to Euclidean similarities. Based on this description we give examples of all possible kinds of PH quintics with respect to the distribution of their real singularities.

Zbynek Sir

Charles University in Prague Czech Republic zbynek.sir@gmail.com

MS80

Partitions of Frobenius Rings Induced by the Homogeneous Weight

The values of the homogeneous weight on a finite commutative Frobenius ring are determined. This is used to investigate the partition induced by this weight and its dual partition under character-theoretic dualization. A characterization of the rings is given for which the induced partition is reflexive (i.e., coincides with its bidual) or even self-dual.

Heide Gluesing-Luerssen University of Kentucky heide.gl@uky.edu

MS80

Efficient Representation for the Trace Zero Subgroup via Rational Functions

The trace zero subgroup is a subgroup of the divisor class group of a (hyper)elliptic curve, defined over a finite field. Being able to represent the elements of this group as compactly (i.e., using as few bits) as possible is relevant for the cryptographic applications. In this talk, I will discuss a new method for representing the elements of the trace zero subgroup. The method uses as representation the coefficients of certain rational functions.

<u>Elisa Gorla</u> University of N

University of Neuchatel elisa.gorla@unine.ch

Maike Massierer University of Basel and University of Neuchatel maike.massierer@unibas.ch

MS80

Partial Spreads in Network Coding

As in the novel approach by R. Kötter and F. R. Kschischang, we study network codes as families of k-dimensional linear spaces over a finite field. Following an idea in finite projective geometry, we introduce a class of network codes which we call *partial spread codes*. Partial spread codes naturally generalize the known family of spread codes. We provide an easy description of such codes, discuss their maximality, and explain how to decode them efficiently.

Alberto Ravagnani, Elisa Gorla University of Neuchatel alberto.ravagnani@unine.ch, elisa.gorla@unine.ch

MS80

An MQ/Code Cryptosystem Proposal

We describe a new trap-door (and PKC) proposal. The proposal is "multivariate quadratic' (relies on the hardness of solving systems of quadratic equations); it is also codebased, and uses the code-scrambling technique of McEliece (1978). However, in the new proposal, the error-correcting code is not revealed in the public key, which protects against the leading attacks on McEliece's method. The proposal motivates an open question in coding theory.

<u>Leonard Schulman</u> Caltech schulman@caltech.edu

Jonathan Hall Michigan State University jhall@math.msu.edu

MS80

Colorability of Hypergraphs Using Commutative Algebra

A hypergraph is properly k-colorable if each vertex can be colored by one of k colors such that no edge is monochromatic. The talk will present a complete algebraic characterization of the 2-colorability of uniform hypergraphs and discuss a decomposition result for hypergraph 2colorings. We will also discuss extensions of our results to k-colorability of uniform hypergraphs.

<u>Lubos Thoma</u>, Michael Krul Department of Mathematics University of Rhode Island thoma@math.uri.edu, krul@math.uri.edu

MS82

Moment Matrices and Applications

Moment matrices play an important role in inverse problems. Examples are sparse interpolation, pole detection, frequency estimation, etc. In many applications, the targeted problem is associated with a structured moment matrix. We employ tools from numerical linear algebra to investigate the numerical behavior and conditioning of the targeted problem. We present some selected applications and demonstrate how the techniques can be generalized to a multivariate problem statement.

Annie Cuyt

University of Antwerp Dept of Mathematics and Computer Science annie.cuyt@ua.ac.be

Wen-shin Lee University of Antwerp wen-shin.lee@ua.ac.be

MS82

Truncated Moment Problems, Extensions, and Positivity

We discuss two interrelated abstract solutions to the multivariable Truncated Moment Problem (TMP). One solution shows that an *n*-dimensional sequence $y^{(2d)}$ of degree 2d admits a representing measure if and only if the moment matrix $M_d(y)$ has a positive flat extension M_{d+k} for some $k \geq 0$. A second solution shows that y has a representing measure if and only if the corresponding Riesz functional $L_y : \mathbf{R}[x]_{2d} \longrightarrow \mathbf{R}$ admits a positive extension $\widetilde{L} : \mathbf{R}[x]_{2d+2} \Longrightarrow \mathbf{R}$. The difficulty in both approaches is to develop concrete conditions for the desired extensions. In the case n = 2, d = 3, TMP for $y^{(6)}$ remains largely unsolved, but we show that if $M_3 \succeq 0$ and rank $M_3 \leq 6$, then L_y is positive; consequently, $y^{(5)}$ has a representing measure and $y^{(6)}$ admits approximate representing measures.

Lawrence A. Fialkow SUNY New Paltz fialkowl@newpaltz.edu

MS82

A Semidefinite Approach to the K_i Cover Problem

We apply theta body relaxations to the K_i cover problem and use this to show polynomial time solvability for certain classes of graphs. In particular, we give an effective relaxation where all K_i -p-hole facets are valid, addressing an open question of Conforti, Corneil and Mahjoub. For the triangle free problem, we also provide an integrality gap for the first relaxation step as well as negative convergence results for the complete graph.

<u>Joao Gouveia</u> Universidade de Coimbra jgouveia@mat.uc.pt

James Pfeiffer University of Washington jpfeiff@math.washington.edu

MS82

Quadratic Forms, Flat Extensions and Applications

We describe the properties of a quadratic form associated to a linear form over a polynomial ring and show how to recover its decomposition in terms of evaluations and differentials. Using a general flat extension property, we describe a method for computing such decompositions from moment matrices. Applications illustrate this approach, including tensor decomposition problems, the solution of exponential polynomials or the construction of cubature rules.

<u>Bernard Mourrain</u> INRIA Sophia Antipolis Bernard.Mourrain@inria.fr

MS82 Shape from Moments

Abstract not available at time of publication.

Dmitrii Pasechnik NTU Singapore dima@ntu.edu.sg

MS83

Torus Invariants and Binomial D-Modules

Binomial D-modules are given by a binomial ideal and homogeneity operators. Combinatorial tools from toric geometry have been successful at analyzing many aspects of binomial D-modules, which carry a torus action. We will consider how to interpret taking invariants of D-modules with torus actions, with the goal of gaining a new understanding of the classical hypergeometric systems studied by, among others, Gauss, Appell, and Lauricella.

<u>Christine Berkesch</u> Duke University cberkesc@gmail.com

Laura Felicia Matusevich Texas A&M University laura@math.tamu.edu

Uli Walther Department of Mathematics Purdue University walther@math.purdue.edu

MS83

Transformations of Hypergeometric Functions

We show how some classical transformation identities involving hypergeometric functions arise from symmetries of the underlying polytope. This provides us with an algorithm to search for such identities in more general situations.

<u>Laura Matusevich</u> Texas A&M University laura@math.tamu.edu

Jens Forsgaard Stockholm University Sweden jensf@math.su.se

MS83

A Charaterization of F-jumping Numbers

In their study of test ideals and F-jumping numbers, Blickle, Mustata and Smith introduce the *D*-modules M_{α} . We prove that the simplicity of these modules determine if α is an F-jumping number and describe an algorithm to decide whether α is an F-jumping number. This is joint work with Luis Núñez-Betancourt.

Felipe Perez University of Michigan juanfp@umich.edu

MS83

Differential Operators and Invariant Theory

I will describe the role played by differential operators in classical invariant theory.

Will Traves

US Naval Academy traves@usna.edu

MS83

Explicit formulas for F-pure Thresholds and Roots of Bernstein-Sato Polynomials

The *F*-pure threshold is an invariant of singularities in characteristic p > 0. Mustață, Takagi, and Watanabe

proved that if a special type of formula for an F-pure threshold can be found, roots of the Bernstein-Sato polynomial (in characteristic zero) can be recovered; Budur, Mustață, and Saito found such formulas for monomial ideals. We find (quite subtle) explicit formulas for F-pure thresholds of certain classes of *polynomials*, and investigate their implications. This is joint work with Daniel Hernández.

Emily E. Witt University of Minnesota ewitt@umn.edu

Daniel Hernández Mathematical Sciences Research Institute dhernan@umn.edu

MS84

Combinatorics of Hyperbolic Polynomials

I will talk about hyperbolic polynomials in combinatorics. In particular their relevance to the (a)symmetric simple exclusion process, and its rich combinatorial structure.

<u>Petter Branden</u> KTH Stockholm petterbranden@gmail.com

MS84

Hyperbolicity Cones and Projections of Spectrahedra

The Generalized Lax Conjecture states that every hyperbolicity cone is a section of the cone of positive semidefinite matrices. This central open question has been verified for various classes of hyperbolic polynomials and, keeping the balance, has been disproved for various strengthenings. In this talk I will discuss a weaker version, the Projected Lax Conjecture: Every hyperbolicity cone is the projection of a spectrahedral cone. While this is certainly weaker than the Generalized Lax Conjecture, I will show that it holds for the large (dense) class of smooth hyperbolic polynomials. This is joint work with Tim Netzer.

Raman Sanyal Freie Universität Berlin sanyal@math.fu-berlin.de

MS84

Determinantal Representations of Projective Hyperbolic Curves

A real projective curve C in the *d*-dimensional projective space is called hyperbolic with respect to a real d-2dimensional projective subspace \mathcal{V} (we assume that C and \mathcal{V} do not intersect) if for every real hyperplane \mathcal{H} through \mathcal{V} , C intersects \mathcal{H} in real points only. In this talk we describe a Livšic-type determinantal representations of a complex projective curve, which is given by an element of $\bigwedge^2 \mathbf{C}^{d+1} \otimes \mathbf{C}^{n \times n}$, and show that similarly to the well known case of plane curves (d = 2), a hyperbolic curve always admits a real symmetric Livšic-type determinantal representation satisfying a positivity condition that gives a witness to hyperbolicity.

Eli Shamovich

Department of Mathematics Ben Gurion University of the Negev shamovic@gmail.com

MS84

Hyperbolic Polynomials, Interlacers, and Sums of Squares

Hyperbolic polynomials are real polynomials whose real hypersurfaces are nested ovals, the inner most of which is convex. These polynomials appear in many areas of mathematics, including optimization, combinatorics and differential equations. I'll give an introduction to this topic and discuss the special connection between hyperbolic polynomials and their interlacing polynomials (whose real ovals interlace the those of the original). This will let us relate the inner oval of a hyperbolic hypersurface to the cone of nonnegative polynomials and, sometimes, to sums of squares. An important example will be the basis generating polynomial of a matroid. This is joint work with Mario Kummer and Daniel Plaumann.

Cynthia Vinzant University of Michigan Dept. of Mathematics vinzant@umich.edu

MS84

Stable Polynomials and Sums of Squares in Matroid Theory

A multivariate complex polynomial is "stable' provided that it never vanishes when all variables take values with positive imaginary part. Early last decade the relevance of stable polynomials to matroid theory (and conversely) came to light. (This comes from an abstraction of electrical network theory.) Brändén subsequently characterized real multiaffine stable polynomials by means of a system of parameterized inequalities, forging another connection with semidefinite optimization. Wei and I supplemented Brändén's criterion with another one that is easier to check, and used it to verify the stable (HPP) property for seven small matroids, including the Vámos matroid. Brändén then used this to settle a conjecture of Helton and Vinnikov. I will sketch these developments, concentrating on my work with Wei and my plans for extending it.

David Wagner

University of Waterloo, Canada Dept. of Combin. and Opt. dgwagner@uwaterloo.ca

MS85

Accelerating Block Wiedemann Implementation in LinBox

Block Wiedemann algorithm allows to solve sparse linear system Ax = b in quadratic time. The key ingredient is to compute the minimal matrix generator of the sequence $S = \{U^T A^i V\}_{i=1..l}$ with matrices U, V having much less columns then A. In this talk, I will discuss the practical efficiency of such approach within the LinBox library. I will show the benefit of using a sparse U in order to compute S faster. I will also present improvements on minimal matrix generator calculation that allow to use fast polynomial matrix multiplication while preserving the original Coppersmith approach or using an online (relaxed) computation model.

Pascal Giorgi LIRMM Univ. Montpellier 2, France pascal.giorgi@lirmm.fr

MS85

On the Complexity of Multivariate Interpolation with Multiplicities and of Simultaneous Polynomial Approximations

The first step in Guruswami and Sudan's list-decoding algorithm amounts to bivariate interpolation with prescribed multiplicities and degree constraints. Essentially two approaches have been proposed so far: one involves polynomial lattices, the other solves so-called (extended) key equations. Focusing on the latter approach, we extend it to the multivariate case and show how to reduce this problem to a problem of simultaneous polynomial approximations, which can be solved thanks to existing fast structured linear solvers.

Vincent Neiger LIP, ENS-Lyon, France vincent.neiger@ens-lyon.fr

Muhammad F.I. Chowdhury ORCCA, Western University, London ON, Canada mchowdh3@uwo.ca

Claude-Pierre Jeannerod INRIA Rhone-Alpes ENS Lyon, France Claude-Pierre.Jeannerod@ens-lyon.fr

Eric Schost Computer Science Department Univ. of Western Ontario eschost@uwo.ca

Gilles Villard CNRS, École Normale Supérieure de Lyon, U. Lyon Gilles.Villard@ens-lyon.fr

MS85

Simultaneous Computation of Row and Column Rank Profiles

We present a new full pivoting strategy in Gaussian elimination, generating a PLUQ decomposition, that makes it possible to recover at the same time both row and column rank profiles of the input matrix and of any of its leading sub-matrices. We propose a rank-sensitive quad-recursive algorithm that computes the latter PLUQ decomposition of an $m \times n$ matrix of rank r in $O(mnr^{\omega-2})$ field operations, with ω the exponent of matrix multiplication. Over a word size finite field, this algorithm also improves the practical efficiency of previously known implementations.

<u>Clément Pernet</u> Université de Grenoble clement.pernet@imag.fr

MS85

Lattice Reduction of Polynomial Matrices

Compared the analogous problem on integer matrices, lattice reduction of polynomial matrices is considered to be easy: in this talk I'll give a survey of existing algorithms for lattice reduction of polynomial matrices that demonstrate this. I'll also discuss some related results, including derandomization of previously Las Vegas algorithms, techniques for handling matrices of arbitrary shape and rank, and algorithms for normalizing a reduced basis to arrive at the canonical Popov form.

Arne Storjohann D. Cheriton School of C.S. Univ of Waterloo, ON, Canada astorjoh@uwaterloo.ca

MS85

Parallel Exact Gaussian Elimination of Rank Deficient Matrices

This work deals with the parallelization of Gaussian elimination triangular matrix decomposition algorithms over a finite field Z/pZ, even in case of rank deficient matrices. Indeed these kinds of computation are known and extensively investigated in numerical computation and the cutting of the matrix is done in a static regular way. But in exact computation, particularly in rank deficient case the cutting proves to be more difficult and the size of blocs is determined dynamically during the computation. Furthermore, since the dimension of blocs is varying, these adaptive applications have workloads that are unpredictable and change during the computation. Such applications require dynamic load balancers that adjust the decomposition as the computation proceeds. We studied different approaches : OpenMP tasks parallelization, pragma omp for loops and tasks parallelization with KAAPI that have dynamic data flow synchronizations. In this work we show also the advantages of the latter by comparing our implementations with KAAPI and with OpenMP.

Ziad Sultan

INRIA/LIG-MOAIS Grenoble Univ. ziad.sultan@imag.fr

MS86

An Algorithm for Computing Degrees of Parametrizations of Elliptic Curves by Shimura Curves

An elliptic curve, E, over \mathbf{Q} of conductor N has a modular parameterization from the modular curve $X_0(N)$ to E. The degree of this map is called the modular degree. Generalizing to number fields, we no longer always have modular curves. Takahashi and Ribet use the Jaquet-Langlands correspondence to parameterize elliptic curves over \mathbf{Q} by Shimura curves. I will examine how this generalizes to modular abelian varieties over number fields.

Alyson Deines

University of Washington adeines@math.washington.edu

MS86

Genus 2 Curves with Good Reduction Away from p = 3

In 1995, Smart classified all curves over Q of genus 2 which have good reduction at primes except p = 2. Chris Rasmussen and I are working to extend Smart's work to classify genus 2 curves with good reduction for all primes except for 3 (or 5). This project connects several interesting problems and techniques, including solving S-unit equations and using the LLL-lattice basis reduction algorithm to reduce the size of a very large search space.

Beth Malmskog Colorado College beth.malmskog@gmail.com

Chris Rasmussen Wesleyan University crasmussen@wesleyan.edu

MS86

Enumerating Abelian Varieties using Matrix Groups

The Frobenius endomorphism of an abelian variety A/F_q acts as a symplectic similitude on the torsion subgroups $A[\ell^n](\overline{F}_q)$. In 2003, Gekeler used an equidistribution assumption on the elements of $\operatorname{GL}_2(Z/\ell^r)$ to show that the number of elliptic curves with certain characteristics is related, via results of Sato-Tate and the class number, to the Euler factors of the *L*-function of a quadratic imaginary field. By computing the sizes of conjugacy classes of Frobenius elements in the groups GSp_{2g} and applying a theorem of Everett Howe, we extend Gekeler's heuristic to abelian varieties with complex multiplication.

<u>Cassie L. Williams</u> James Madison University willi5cl@jmu.edu

MS86

2-Adic Images of Galois Representations Associated to Elliptic Curves over Q

We give a classification of all possible 2-adic images of Galois representations associated to elliptic curves over Q. To this end, we compute the 'relevant' tower of 2-power level modular curves, develop new techniques to compute their equations, and classify rational points on these curves.

David Zureick-Brown Emory dzb@mathcs.emory.edu

Jeremy Rouse Wake Forest rouseja@wfu.edu

MS87

The Neural Ring: An Algebraic Tool for Analyzing Neural Codes

Neurons in the brain represent stimuli via neural codes. An important problem confronted by the brain is to infer properties of represented stimulus spaces using only the intrinsic structure of neural codes. How does the brain do this? To address this question we define the neural ring, an algebraic object that encodes the full combinatorial data of a neural code, and show how it can be used to extract relevant features from the code.

<u>Carina Curto</u> Department of Mathematics University of Nebraska-Lincoln ccurto2@math.unl.edu

Vladimir Itskov Department of Mathematics University of Nebraska, Lincoln vitskov2@math.unl.edu

Alan Veliz-Cuba University of Nebraska Lincoln aveliz-cuba2@unl.edu

Nora Youngs University of Nebraska-Lincoln s-nyoungs1@math.unl.edu

MS87

Data Characterization and Identification for Network Inference

Recently polynomial dynamical systems (PDSs) have been used as models of gene regulatory networks (GRNs). An advantage of using PDSs as models is that the set of all models that "fit" a given data set for a GRN admits an algebraic structure, similar to a vector space. However, few results other than a criterion by L. Robbiano have been published on the amount and type of data necessary for identifying PDS models and there are no methods for generating the specific data sets which would unambiguously identify the network. In this talk, necessary and sufficient criteria will be presented for a data set to uniquely identify PDSs of GRNs using tools from computational algebra and algebraic geometry.

<u>Elena S. Dimitrova</u> Clemson University edimit@clemson.edu

Brandilyn Stigler Southern Methodist University bstigler@smu.edu

MS87

Exact Hypothesis Tests for Biological Network Data

We develop methodology for exact statistical tests of hypotheses for models of biological network dynamics. The methodology formulates Markovian exponential families with a dispersion parameter phi, then uses sequential importance sampling to compute expectations within basins of attraction and within level sets of a sufficient statistic. Comparisons of hypotheses can be done conditional on basins of attraction. Examples use recent data on cancer and stomatal regulation.

Ian Dinwoodie, Kruti Pandya Portland State University ihd@pdx.edu, kpandya@pdx.edu

MS87

Geometric Approach to Learning Bayesian Networks with Applications to Biology

This talk will cover descriptions of probabilistic conditional independence (CI) models and learning graphical models which has applications in biology (epistasis, gene regulatory networks, protein signaling, systems biology), Markov random processes, probabilistic reasoning, artificial intelligence and more. Given observed data, the goal is to find the CI structure which best explains the data. I will motivate the problem with some interesting biological applications. Next, I will quickly overview graphical approaches to the description of CI structures. Then, I will describe a superior algebraic description of CI structures introduced by Studeny et al. which has many elegant properties, suitable for applications of linear programming methods. The remainder of the talk will be devoted to linear optimization approaches to learning Bayesian networks, which are special graphical models.

David Haws

IBM T.J. Watson Research Center dhaws@us.ibm.com

MS87

Encoding Simplicial Complexes by Neural Networks

Networks of neurons in the brain encode memories via their synaptic connections, yet the relationship between network connectivity and encoded memory patterns is still poorly understood, especially in cases where the set of patterns is highly structured. Motivated by well-studied neural codes, we tackle the problem of encoding simplicial complexes in feedforward and recurrent networks. We find surprisingly strong results using various geometric and combinatorial tools, including the inverse nerve construction and Cayley-Menger determinants.

<u>Vladimir Itskov</u> Department of Mathematics University of Nebraska, Lincoln vitskov2@math.unl.edu

Carina Curto Department of Mathematics University of Nebraska-Lincoln ccurto2@math.unl.edu

Chad Giusti University of Nebraska-Lincoln cgiusti2@unl.edu

PP1

Mallows Mixture Model and Its Vanishing Ideal

The Mallows model is simple statistical model for ranked data. We study a mixture model based on the original Mallows model with two mixture components. We examine this model from and algebraic geometry standpoint. We then fully characterize the degree 1 and degree 2 generators of the vanishing ideal for this mixture model when there are two mixture components.

<u>Brandon W. Bock</u>, Seth Sullivant North Carolina State University bwbock@ncsu.edu, smsulli2@ncsu.edu

PP1

Geometrical Algorithms for Civil Helicopter (CH) Rotor-Blades Instantaneous Rotation Center Determination in Deformable/Turbulence Conditions Using Numerical Reuleaux Method (MRM)

Civil Helicopter (CH) rotor blades are usually manufactured/designed with deformable materials. Their movement is governed mainly by four angles, rotor, flapping, lagging and torsional ones. In severe turbulence/windy conditions structural dynamics of blades could become deformed and/or damaged (blade flapping/lagging bending moment and torsional deflections angles could appear and be significant, we do not analyze torsion). Then, there are changes in the IRC of rotor.Dynamically, this IRC biased position creates a variation chain in the Aerodynamics conditions of CH.We carry out a primary approximation to analyze geometrically/numerically the IRC variation with the NRM.Initial algorithms are developed with numerical simulations. Aerodynamics consequences derived from this data are physically analyzed. F Casesnoves MSc MD. ASME (American Society of Mechanical Engineering. Individual Researcher Membership) References (Selected) Theory and Primary Computational Simulations of the Numerical Reuleaux Method (NRM), Casesnoves, F. International and Journal of Mathematics Computation (http://www.ceser.in/ceserp/index.php/ijmc/issue/view/119).Volume 13,Issue Number D11.Year 2011.

Francisco Casesnoves

American Mathematical Society (Individual Researcher Member) mathematics88@gmx.com

PP1

Cad Numerical Vertebral Surface Refinements and Geometrical Data Development for Surgical Devices Design

Lumbar Vertebra CAD constitute an useful method both for anatomical study of bone shape/structure and surgical instrumentation design. We present a polynomial discretization and refinement derived from previous rougher polynomials fits of vertebral surface.Non-Linear Optimization Methods/specific-software were used for the objective.So we can get a more accurate and real surface representation. It is shown a group of geometricalmathematical formuli that can be obtained from refinement data.Industrial applications of these techniques are related to manufacturing data/design of lumbar spine instrumentation and artificial implants. F Casesnoves MSC MD ASME (Individual Researcher Affiliation) Refs:'Computational Simulations of Vertebral Body for Optimal Instrumentation Design'. Casesnoves, F. ASME JOURNAL OF MEDICAL DEVICES.May 23,2012. J. Med. Devices/June 2012/Volume 6/Issue 2/021014 (11 pages)http://dx.doi.org/10.1115/1.4006670

Francisco Casesnoves

American Mathematical Society (Individual Researcher Member) mathematics88@gmx.com

PP1

Distance-Based Phylogenetic Algorithms Around a Polytomy

Distance-based phylogenetic algorithms map an arbitrary dissimilarity map representing biological data to a tree metric. The set of all dissimilarity maps is a Euclidean space containing as a subset the space of all tree metrics, which is a polyhedral fan and a tropical variety. We study the behavior of distance-based tree reconstruction algorithms such as UPGMA and Neighbor-Joining near a polytomy using the polyhedral geometry of the subdivision of Euclidean space induced by the algorithm.

<u>Ruth E. Davidson</u>, Seth Sullivant North Carolina State University redavids@ncsu.edu, smsulli2@ncsu.edu

PP1

Perturbed Regeneration for Finding Singular Solutions

Homotopy continuation is used to find solutions to systems of polynomial equations. Numerical algebraic geometry, a relatively young and quickly growing field, utilizes continuation methods to find and classify isolated and positivedimensional solutions. We will present some developments on approximating singular solutions efficiently. Our technique is especially useful for certain large problems. In particular, perturbed regeneration will find such solutions without the cost of deflation, which is necessary in standard regeneration to regularize them.

<u>Brent R. Davis</u> Colorado State University

bdavis1@rams.colostate.edu

Daniel J. Bates Colorado State University Department of Mathematics bates@math.colostate.edu

Chris Peterson Colorado State University peterson@math.colostate.edu

Eric Hanson Department of Mathematics Colorado State University hanson@math.colostate.edu

David Eklund Colorado State University daek@kth.se

PP1

A Fractal Model of Time

In traditional quantum mechanics, time is considered to be an observable; no time operator has been established. One of the most familiar results of quantum mechanics is the quantization of energy. Inherent in Planck's constant, with its units of time multiplied by energy, lies the concept of the quantization of time. For a certain class of state functions, time can be quantized in time-energy quanta, based upon the existence of a quantum-mechanical time operator. This time operator would be a function of the Hamiltonian, the energy operator, and yield four results for each energy level. The applications of this model include high energy fusion and cosmology; however, our results are entirely theoretical, and have to be confirmed with a real system, such as a hydrogen atom. The question of whether time can be quantized or no is more than academic. If such quantization does indeed exist, it might serve as a basis for a unified field theory.

Jorge Diaz-Castro

University of P.R. School of Law University of P.R. at Rio Piedras transcripciones@mail.com

$\mathbf{PP1}$

A Set of Polynomial Systems from Population Biology

An important problem in population biology is determin-

ing equilibria of haplotype frequencies. It concerns a square parametrized polynomial system of dimension N equal to a population's number of haplotypes. Bertini, with a linearproduct formulation, captures the generic solutions without paths diverging. The disparity between the linearproduct count and the total degree, $2^N - 1$ and 3^N , respectively, grows rapidly with N. Often most haplotypes are lethal; sparse linear algebra provides an efficient approach to solving such systems.

<u>Jesse W. Drendel</u> Colorado State University jesse.drendel@gmail.com

PP1

Parameterized Polynomial Systems and Numerical Algebraic Geometry

Given a parameterized system of polynomial equations, it is natural to consider how different choices of parameters affect the zero set. This poster will highlight some results relating to this question and methods involving parameterized polynomial systems. Specifically, we will consider using Numerical Algebraic Geometry (NAG) to study parameterized systems. We will give a high-level overview of perturbed regeneration, fiber products for finding exceptional parameter values, and solving systems over (semi)algebraic sets of parameters.

Eric Hanson

Department of Mathematics Colorado State University hanson@math.colostate.edu

Dan Bates Colorado State University bates@math.colostate.edu

PP1

A Hybrid Numerical-Symbolic Algorithm for Computing the Solutions of Fewnomial Systems

Gale duality is an isomorphism between solutions of a polynomial system and those of its Gale dual system of master functions. This transformation, known as the Gale transform, is completely symbolic. The solutions of the master functions can be computed (by previously known methods) and an unwrapping algorithm can be applied to obtain the solutions of the original fewnomial system. This poster focuses on the construction and optimization of the dual system.

Matthew Niemerg, Dan Bates

Colorado State University niemerg@math.colostate.edu, bates@math.colostate.edu,

Jon Hauenstein North Carolina State University hauenstein@ncsu.edu

Frank Sottile Texas A&M University sottile@hilbert.math.tamu.edu

PP1

Machine Learning for Phylogenetic Invariants

We build on Eriksson and Yaos work, which used machine

learning to optimize the power of phylogenetic invariants to reconstruct evolutionary trees. While previous work focused on selecting a good set of invariants for the construction of quartet trees, we extend this work to trees with more taxa. In addition to the invariants, we include inequalities arising from the study of the real points on the phylogenetic variety into the metric learning algorithm.

Hannah M. Swan, Joseph P. Rusinko, Emili Price Winthrop University swanh2@winthrop.edu, rusinkoj@winthrop.edu, pricee4@winthrop.edu