Abstract

Distance-hereditary graphs form an important class of graphs, from the theoretical point of view, due to the fact that they are the totally decomposable graphs for the split-decomposition. The previous best enumerative result for these graphs is from Nakano et al. (J. Comp. Sci. Tech., 2007), who have proven that the number of distance-hereditary graphs on $n$ vertices is bounded by $2^{\lceil3.59n\rceil}$.

In this paper, using classical tools of enumerative combinatorics, we improve on this result by providing an exact enumeration and full asymptotic of distance-hereditary graphs, which allows to show that the number of distance-hereditary graphs on $n$ vertices is tightly bounded by $(7.24975\ldots)^n$—opening the perspective such graphs could be encoded on $3n$ bits. We also provide the exact enumeration and full asymptotics of an important subclass, the 3-leaf power graphs. Our work illustrates the power of revisiting graph decomposition results through the framework of analytic combinatorics.