Abstract

A very challenging problem from statistical physics is computing the partition function of the ferromagnetic Ising model, even in the relatively simple case of no applied field. In this case, the partition function can be written as a function of the subgraphs of the underlying graph in which all vertices have even degree. In their seminal work, Jerrum and Sinclair showed that this quantity can be approximated by a rapidly converging Markov chain on all subgraphs. However, their chain frequently leaves the space of even subgraphs. Our aim is to devise and analyze a new class of Markov chains that do not leave this space, in the hopes of finding a faster sampling algorithm. We define Markov chains by viewing the even subgraphs as a vector space (often called the cycle space) whose transitions are defined by the addition of basis elements. The rate of convergence depends on the basis chosen, and our analysis proceeds by dividing bases into two types, short and long. The classical single-site update Markov chain known as Glauber dynamics is a special case of our short cycle basis Markov chains. We show that for any graph and any long basis, there is a temperature for which the corresponding Markov chain requires an exponential time to mix. Moreover, we show that for d-dimensional grids with $d \geq 2$—those of the most physical importance—all fundamental bases (a natural class of bases derived from spanning trees) are long. For the 2-dimensional grid on the torus, we show that there is a temperature for which the Markov chain requires exponential time for any chosen basis.