Abstract

In the (classical) Secretary Problem, one has to hire the best among n candidates. The candidates are interviewed, one at a time, at a uniformly random order, and one has to decide on the spot, whether to hire a candidate or continue interviewing. It is well known that the best candidate can be hired with a probability of $1/e$ (Dynkin, 1963). Recent works extend this problem to settings in which multiple candidates can be hired, subject to some constraint. Here, one wishes to hire a set of candidates maximizing a given objective set function. Almost all extensions considered in the literature assume the objective set function is either linear or submodular. Unfortunately, real world functions might not have either of these properties. Consider, for example, a scenario where one hires researchers for a project. Indeed, it can be that some researchers can substitute others for that matter. However, it can also be that some combinations of researchers result in synergy (see, e.g., Woolley et al., Science 2010, for a study on collective intelligence). The first phenomenon can be modeled by a submoudlar set function, while the latter cannot. In this work, we study the secretary problem with an *arbitrary* non-negative monotone valuation function, subject to a general matroid constraint. One can prove that, generally, only very poor results can be obtained for this class of objective functions. We tackle this hardness by combining the following: (1) Parametrizing our algorithms by the *supermodular degree* of the objective function (defined by Feige and Izsak, ITCS 2013), which, roughly speaking, measures the distance of a function from being submodular. (2) Suggesting an (arguably) natural model that permits approximation guarantees that are *polynomial* in the supermodular degree (as opposed to the standard model which allows only *exponential* guarantees). Our algorithms learn the input by running a non-trivial estimation algorithm on a portion of it whose size depends on the supermodular degree. We also provide better approximation guarantees for the special case of a uniform matroid constraint. To the best of our knowledge, our results represent the first algorithms for a secretary problem handling arbitrary non-negative monotone valuation functions.