Abstract

We study a family of closely-related distributed graph problems, which we call *degree splitting*, where roughly speaking the objective is to partition (or orient) the edges such that each node’s degree is split almost uniformly. Our findings lead to answers for a number of problems, a sampling of which includes: (1) We present a polylog \( n \) round deterministic algorithm for \( (2\Delta - 1) \cdot (1 + o(1)) \) edge-coloring, where \( \Delta \) denotes the maximum degree. Modulo the \( 1 + o(1) \) factor, this settles one of the long-standing open problems of the area from the 1990’s (see e.g. Panconesi and Srinivasan [PODC’92]). Indeed, a weaker requirement of \( (2\Delta - 1) \cdot \text{polylog}(\Delta) \) edge-coloring in polylog \( n \) rounds was asked for in the 4th open question in the *Distributed Graph Coloring* book by Barenboim and Elkin. (2) We show that *sinkless orientation*—i.e., orienting edges such that each node has at least one outgoing edge—on \( \Delta \)-regular graphs can be solved in \( O(\log \Delta \log n) \) rounds randomized and in \( O(\log \Delta \ n) \) rounds deterministically. These prove the corresponding lower bounds by Brandt et al. [STOC’16] and Chang, Kopelowitz, and Pettie [FOCS’16] to be tight. Moreover, these show that sinkless orientation exhibits an exponential separation between its randomized and deterministic complexities, akin to the results of Chang et al. for \( \Delta \)-coloring \( \Delta \)-regular trees. (3) We present a randomized \( O(\log^4 n) \) round algorithm for orienting \( a \)-arboricity graphs with maximum out-degree \( a(1 + \epsilon) \). This can be also turned into a decomposition into \( a(1 + \epsilon) \) forests when \( a = \Omega(\log n) \) and into \( a(1 + \epsilon) \) pseudo-forests when \( a = o(\log n) \). Obtaining an efficient distributed decomposition into less than \( 2a \) forests was stated as the 10th open problem in the book by Barenboim and Elkin.