Abstract

We consider the problem of online Min-cost Perfect Matching with Delays (MPMD) recently introduced by Emek et al, (STOC 2016). This problem is defined on an underlying n-point metric space. An adversary presents real-time requests online at points of the metric space, and the algorithm is required to match them, possibly after keeping them waiting for some time. The cost incurred is the sum of the distances between matched pairs of requests (the connection cost), and the sum of the waiting times of the requests (the delay cost). We prove the first logarithmic upper bound and the first polylogarithmic lower bound on the randomized competitive ratio of this problem. We present an algorithm with a competitive ratio of $O(\log n)$, which improves the upper bound of $O(\log^2 n + \log \Delta)$ of Emek et al, by removing the dependence on Δ, the aspect ratio of the metric space (which can be unbounded as a function of n). The core of our algorithm is a deterministic algorithm for MPMD on metrics induced by edge-weighted trees of height h, whose cost is guaranteed to be at most $O(1)$ times the connection cost plus $O(h)$ times the delay cost of every feasible solution. The reduction from MPMD on arbitrary metrics to MPMD on trees is achieved using the result on embedding n-point metric spaces into distributions over weighted hierarchically separated trees of height $O(\log n)$, with distortion $O(\log n)$. We also prove a lower bound of $\Omega(\sqrt{\log n})$ on the competitive ratio of any randomized algorithm. This is the first lower bound which increases with n, and is attained on the metric of n equally spaced points on a line.