Abstract

In this paper, we exploit linear programming duality in the online setting, where input arrives on the fly, from the unique perspective of designing lower bounds (i.e., hardness results) on the competitive ratio. In particular, we provide a systematic method (as opposed to ad hoc case analysis that is typically done) for obtaining online deterministic and randomized lower bounds on the competitive ratio for a wide variety of problems. We show the usefulness of our approach by providing new, tight hardness results for three diverse online problems: the Vector Bin Packing problem, Ad-auctions (and various online matching problems), and the Capital Investment problem. Our methods are sufficiently general that they can also be used to reconstruct existing lower bounds. Our approach is in stark contrast to previous works, which exploit linear programming duality to obtain positive results, often via the useful primal-dual scheme. We design a general recipe with the opposite aim of obtaining negative results via duality. The general idea behind our approach is to construct a parameterized family of primal linear programs based on a candidate collection of input sequences for proving the lower bound, where the objective function corresponds to optimizing the competitive ratio. Solving the parameterized family of primal linear programs optimally would yield a valid lower bound, but is a challenging task and limits the tools that can be applied, since analysis must be done precisely and exact optimality needs to be proved. To this end, we consider the corresponding parameterized family of dual linear programs and provide feasible solutions, where the objective function yields a lower bound on the competitive ratio. This opens up additional doors for analysis, including some of the techniques we employ (e.g., continuous analysis, differential equations, etc.), as we need not be so careful about exact optimality. We are confident that our methods can be successfully applied to produce many more lower bounds for a wide array of online problems.