Abstract

We describe a \((1 + \varepsilon)\)-approximation algorithm for finding the minimum distortion embedding of an \(n\)-point metric space \(X\) into the shortest path metric space of a weighted graph \(G\) with \(m\) vertices. The running time of our algorithm is

\[
m^{O(1)} \cdot n^{O(\omega)} \cdot (\delta_{opt} \Delta)^{\omega \cdot (1/\varepsilon)^{\lambda+2}} \cdot \lambda \cdot (O(\delta_{opt}))^{2^\lambda}
\]

parametrized by the values of the minimum distortion, \(\delta_{opt}\), the spread, \(\Delta\), of the points of \(X\), the treewidth, \(\omega\), of \(G\), and the doubling dimension, \(\lambda\), of \(G\). In particular, our result implies a PTAS provided an \(X\) with polynomial spread, and the doubling dimension of \(G\), the treewidth of \(G\), and \(\delta_{opt}\), are all constant. For example, if \(X\) has a polynomial spread and \(\delta_{opt}\) is a constant, we obtain PTAS’s for embedding \(X\) into the following spaces: the line, a cycle, a tree of bounded doubling dimension, and a \(k\)-outer planar graph of bounded doubling dimension (for a constant \(k\)).