Abstract

Many randomized algorithms can be derandomized efficiently using either the method of conditional expectations or probability spaces with low (almost-) independence. A series of papers, beginning with work by Luby (1988) and continuing with Berger & Rompel (1991) and Chari et al. (1994), showed that these techniques can be combined to give deterministic parallel algorithms for combinatorial optimization problems involving sums of w-juntas. We improve these algorithms through derandomized variable partitioning. This reduces the processor complexity to essentially independent of w while the running time is reduced from exponential in w to linear in w. For example, we improve the time complexity of an algorithm of Berger & Rompel (1991) for rainbow hypergraph coloring by a factor of approximately $\log^2 n$ and the processor complexity by a factor of approximately $m^{\ln 2}$. As a major application of this, we give an NC algorithm for the Lovász Local Lemma. Previous NC algorithms, including the seminal algorithm of Moser & Tardos (2010) and the work of Chandrasekaran et. al (2013), required that (essentially) the bad-events could span only $O(\log n)$ variables; we relax this to allowing $\mathrm{polylog}(n)$ variables. As two applications of our new algorithm, we give algorithms for defective vertex coloring and domatic graph partition. One main sub-problem encountered in these algorithms is to generate a probability space which can "fool" a given list of $GF(2)$ Fourier characters. Schulman (1992) gave an NC algorithm for this; we dramatically improve its efficiency to near-optimal time and processor complexity and code dimension. This leads to a new algorithm to solve the heavy-codeword problem, introduced by Naor & Naor (1993), with a near-linear processor complexity $(mn)^{1+o(1)}$.