Abstract

For a simple (unbiased) random walk on a connected graph with \( n \) vertices, the cover time (the expected number of steps it takes to visit all vertices) is at most \( O(n^3) \). We consider locally biased random walks, in which the probability of traversing an edge depends on the degrees of its endpoints. We confirm a conjecture of Abdullah, Cooper and Draief [2015] that the min-degree local bias rule ensures a cover time of \( O(n^2) \). For this we formulate and prove the following lemma about spanning trees. Let \( R(e) \) denote for edge \( e \) the minimum degree among its two endpoints. We say that a weight function \( W \) for the edges is feasible if it is nonnegative, dominated by \( R \) (for every edge \( W(e) \leq R(e) \)) and the sum over all edges of the ratios \( W(e)/R(e) \) equals \( n - 1 \). For example, in trees \( W(e) = R(e) \), and in regular graphs the sum of edge weights is \( d(n - 1) \).

Lemma: for every feasible \( W \), the minimum weight spanning tree has total weight \( O(n) \). For regular graphs, a similar lemma was proved by Kahn, Linial, Nisan and Saks [1989].