Abstract

The Firefighter problem and a variant of it, known as Resource Minimization for Fire Containment (RMFC), are natural models for optimal inhibition of harmful spreading processes. Despite considerable progress on several fronts, the approximability of these problems is still badly understood. This is the case even when the underlying graph is a tree, which is one of the most-studied graph structures in this context and the focus of this paper. In their simplest version, a fire spreads from one fixed vertex step by step from burning to adjacent non-burning vertices, and at each time step B many non-burning vertices can be protected from catching fire. The Firefighter problem asks, for a given B, to maximize the number of vertices that will not catch fire, whereas RMFC (on a tree) asks to find the smallest B that allows for saving all leaves of the tree. Prior to this work, the best known approximation ratios were an $O(1)$-approximation for the Firefighter problem and an $O(\log^* n)$-approximation for RMFC, both being LP-based and essentially matching the integrality gaps of two natural LP relaxations. We improve on both approximations by presenting a PTAS for the Firefighter problem and an O(1)-approximation for RMFC, both qualitatively matching the known hardness results. Our results are obtained through a combination of the known LPs with several new techniques, which allow for efficiently enumerating over super-constant size sets of constraints to strengthen the natural LPs.