Abstract

A *conflict-free k-coloring* of a graph assigns one of k different colors to some of the vertices such that, for every vertex v, there is a color that is assigned to exactly one vertex among v and v's neighbors. Such colorings have applications in wireless networking, robotics, and geometry, and are well-studied in graph theory. Here we study the natural problem of the *conflict-free chromatic number* $\chi_{CF}(G)$ (the smallest k for which conflict-free k-colorings exist), with a focus on planar graphs. For general graphs, we prove the conflict-free variant of the famous Hadwiger Conjecture: If G does not contain K_{k+1} as a minor, then $\chi_{CF}(G) \leq k$. For planar graphs, we obtain a tight worst-case bound: three colors are sometimes necessary and always sufficient. In addition, we give a complete characterization of the algorithmic/computational complexity of conflict-free coloring. It is NP-complete to decide whether a planar graph has a conflict-free coloring with *one* color, while for outerplanar graphs, this can be decided in polynomial time. Furthermore, it is NP-complete to decide whether a planar graph has a conflict-free coloring with *two* colors, while for outerplanar graphs, two colors always suffice. For the *bicriteria* problem of minimizing the number of colored vertices subject to a given bound k on the number of colors, we give a full algorithmic characterization in terms of complexity and approximation for outerplanar and planar graphs.