Abstract

The Weighted Tree Augmentation Problem (WTAP) is a fundamental well-studied problem in the field of network design. Given an undirected tree $G = (V, E)$, an additional set of edges $L \subseteq V \times V$ disjoint from E called *links* and a cost vector $c \in \mathbb{R}^L_{\geq 0}$, WTAP asks to find a minimum-cost set $F \subseteq L$ with the property that $(V, E \cup F)$ is 2-edge connected. The special case where $c_\ell = 1$ for all $\ell \in L$ is called the Tree Augmentation Problem (TAP). For the class of bounded cost vectors, we present a first improved approximation algorithm for WTAP since more than three decades. Concretely, for any $M \in \mathbb{R}_{\geq 1}$ and $\epsilon > 0$, we present an LP based $(\delta + \epsilon)$-approximation for WTAP restricted to cost vectors c in $[1, M]^L$ for $\delta \approx 1.96417$. More generally, our result is a $(\delta+\epsilon)$-approximation algorithm with running time $n^{r^{O(1)}}$, where $r = c_{\max}/c_{\min}$ is the ratio between the largest and the smallest cost of any link. For the special case of TAP we improve this factor to $\frac{5}{3} + \epsilon$. Our results rely on several new ideas, including a new LP relaxation of WTAP and a two-phase rounding algorithm. In the first phase, the algorithm uses the fractional LP solution to guide a simple decomposition method that breaks the tree into well-structured trees and equips each tree with a part of the fraction LP solution. In the second phase, the fractional solution in each part of the decomposition is rounded to an integral solution with two rounding procedures, and the best outcome is included in the solution. One rounding procedure exploits the constraints in the new LP, while the other one exploits a connection to the Edge Cover Problem. We show that both procedures can not have a bad approximation guarantee simultaneously to obtain the claimed approximation factor.