

Abstract

In the minimum Multicut problem, the input is an edge-weighted supply graph $G = (V, E)$ and a demand graph $H = (V, F)$. Either G and H are directed (DMulC) or both are undirected (UMulC). The goal is to remove a minimum weight set of supply edges $E' \subseteq E$ such that in $G - E'$ there is no path from s to t for any demand edge $(s, t) \in F$. UMulC admits $O(\log k)$ -approximation where k is the number of edges in H while the best known approximation for DMulC is $\min\{k, \tilde{O}(|V|^{11/23})\}$. These approximations are obtained by proving corresponding results on the multicommodity flow-cut gap. In this paper we consider the role that the structure of the demand graph plays in determining the approximability of Multicut. We obtain several new positive and negative results. In undirected graphs our main result is a 2-approximation in $n^{O(t)}$ time when the demand graph excludes an induced matching of size t . This gives a constant factor approximation for a specific demand graph that motivated this work, and is based on a reduction to uniform metric labeling and not via the flow-cut gap. In contrast to the positive result for undirected graphs, we prove that in directed graphs such approximation algorithms can not exist. We prove that, assuming the Unique Games Conjecture (UGC), that for a large class of fixed demand graphs DMulC cannot be approximated to a factor better than the worst-case flow-cut gap. As a consequence we prove that for any fixed k , assuming UGC, DMulC with k demand pairs is hard to approximate to within a factor better than k . On the positive side, we obtain a k approximation when the demand graph excludes certain graphs as an induced subgraph. This generalizes the known 2 approximation for directed Multiway Cut to a larger class of demand graphs.