NP-Completeness and Approximation Algorithms

David Johnson

AT&T Labs - Research

12 Open Problems from [G&J, 1979]

- Graph Isomorphism
  Still Open (but unlikely to be NPC)

- Fixed Subgraph Homeomorphism
  In P [Robertson-Seymour, 1986]

- Graph Genus
  NPC [Thomassen, 1990]

- Chordal Graph Completion
  NPC [Yannakakis, 1981]

- Chromatic Index
  NPC [Holyer, 1981]

- Spanning Tree Parity
  In P [Lovasz, 1980]

- Partial Order Dimension
  NPC [Yannakakis, 1982]

- 3-Processor Scheduling
  Still Open

- Linear Programming
  In P [Khachian, 1979]

- Total Unimodularity
  In P [Seymour, 1980]

- Composite Number
  In P [Agrawal-Kayal-Saxena, 2002]

- Minimum Length Triangulation
  NPC [Mulzer-Rote, 2008]

3-Processor Scheduling

- **Given:** Set T of unit length tasks, partial order < on T, and a positive integer deadline D.

Factoring

- **Given:** Positive integers $a < b \leq c$.

- **Question:** Is there a factor x of c such that $a < x < b$?
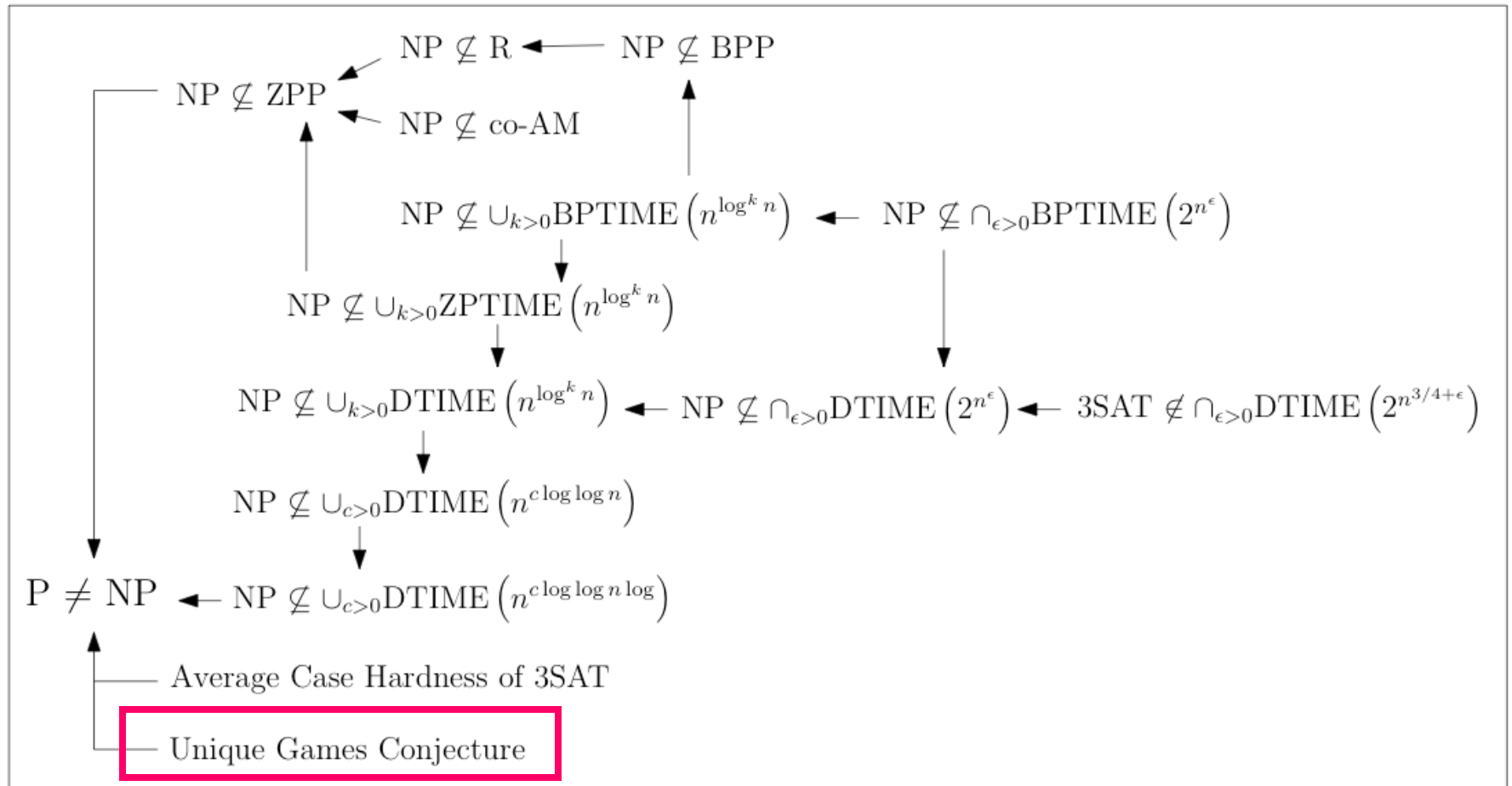
The $1,000,000 question*:

Does P = NP?

Progress since 1979

Flowering of Approximation Algorithm Research since 1990

Now-Classic Results

Conjectures Used in Inapproximability Results

Assuming the Unique Games Conjecture:

- The classic factor-of-2 algorithm for VERTEX COVER is essentially best possible.

- The Goemans-Williams "semidefinite programming + rounding" algorithm for MAX-CUT, with its worst-case ratio of $\min_{0<\Theta \leq \pi}((\pi(1-\cos(\Theta))/(2\Theta)) \sim 1.138$ is also best possible.

- Many other similar results

The Unique Games Conjecture

My Favorite Open Problems