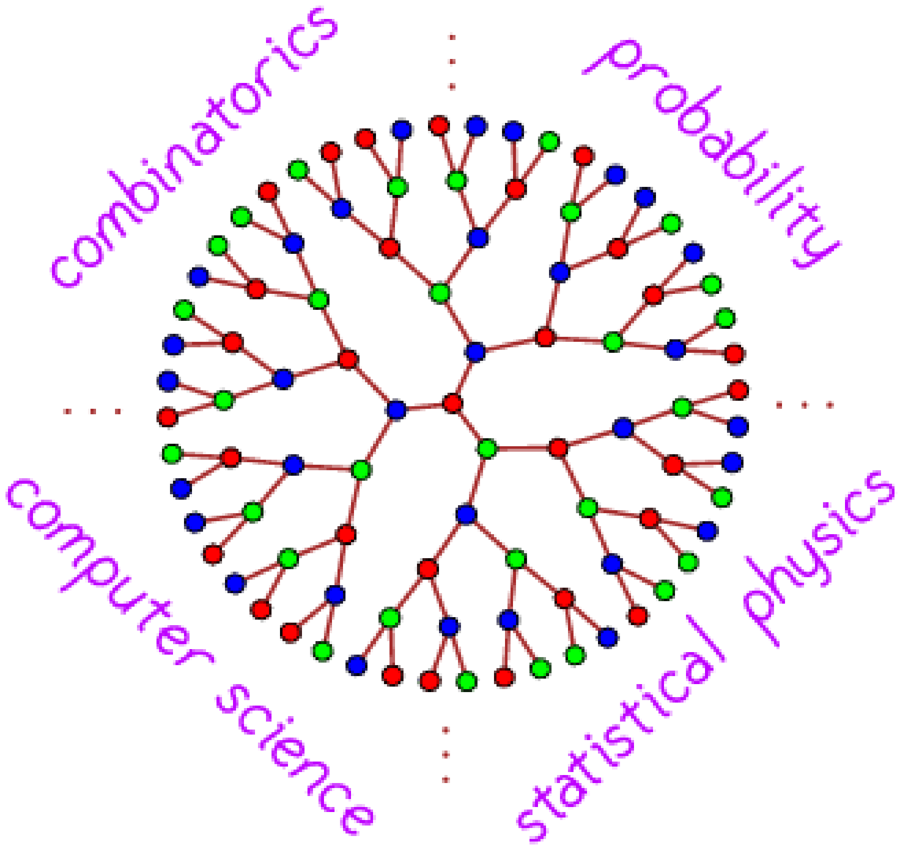


Where Statistical Physics Meets Combina-torics

Peter Winkler, Dartmouth

Objectives

What they (statistical physicists) want:

to understand the behaviour of large random systems.

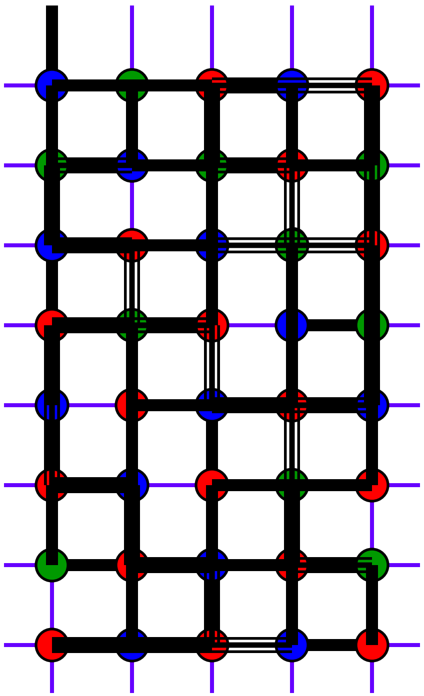
What some of us (combinatorialists) want:

to understand large random combinatorial objects.

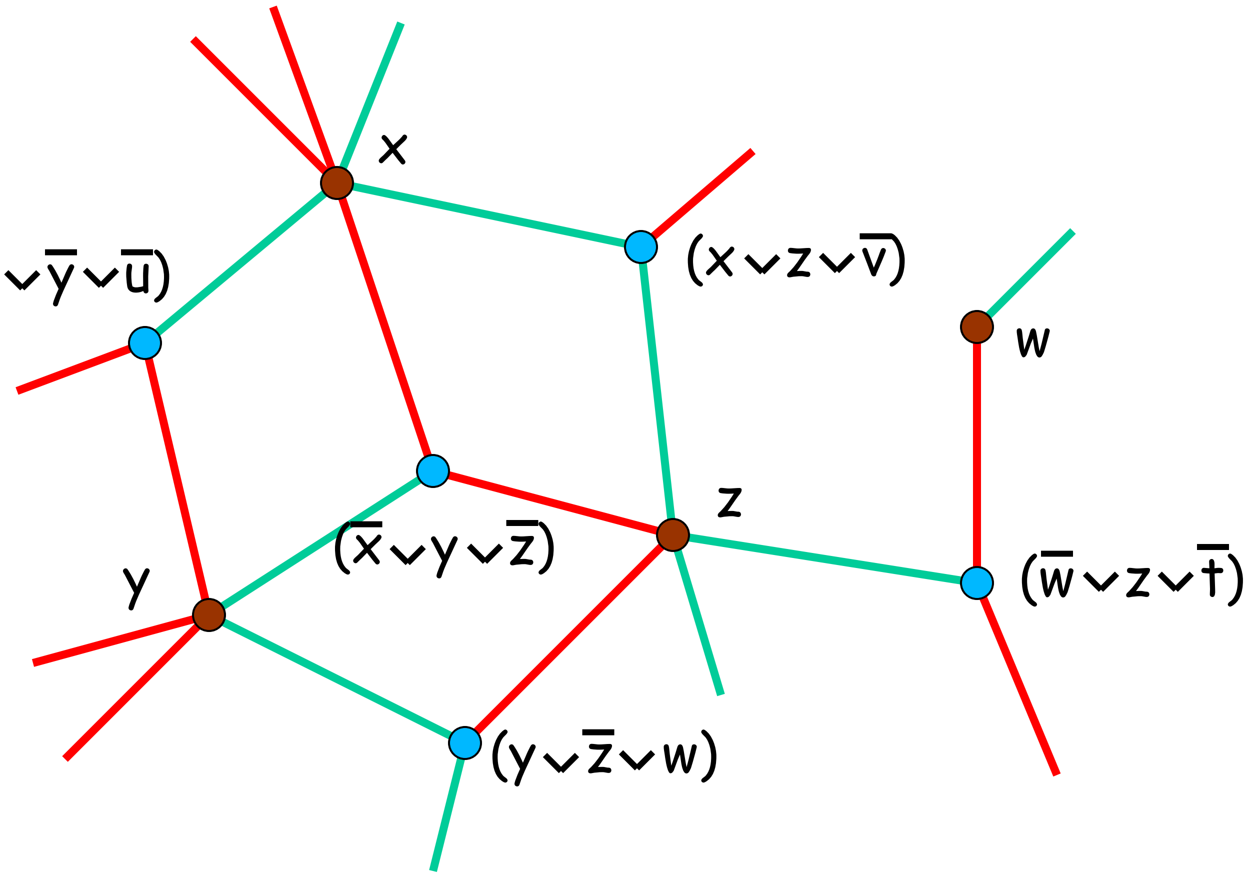
The area of greatest common interest is
systems with hard constraints, such as . . .

Statistical physics ←→ Combinatorics

hard-core model	random independent sets
monomer-dimer	random matchings
Potts model	random colorings
percolation	random subgraphs
linear polymers	self-avoiding random walks
branched polymers	random lattice trees

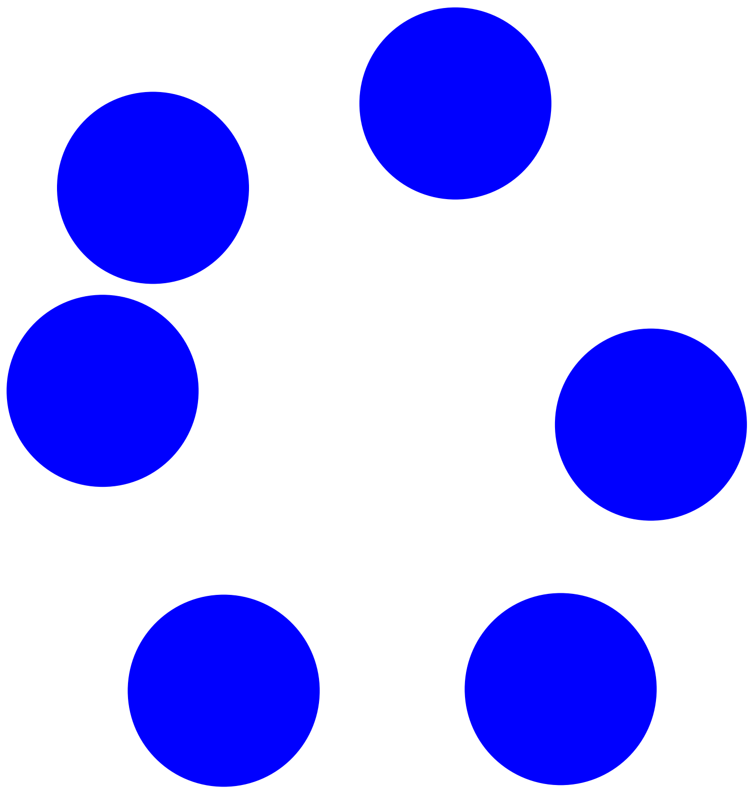


CS theory's favorite hard-constraint model

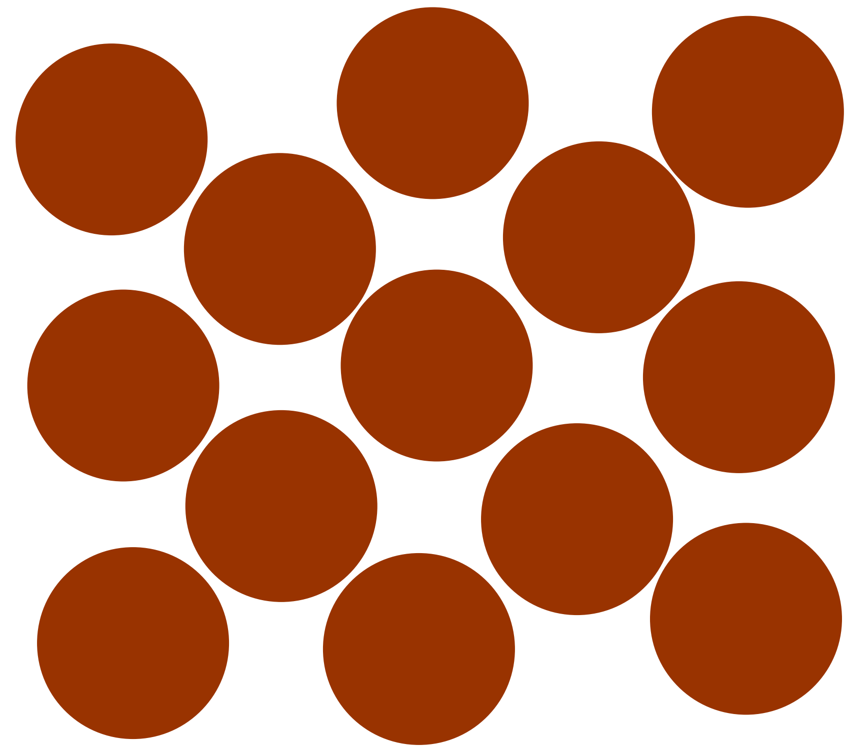


Physics techniques (e.g., "cavity method") have helped to make major progress in understanding satisfiability. [Mezard, Parisi and Virasoro '85]

Hard-core (non-overlapping) disks in the plane



low density



high density

Problem 1: Is there a "solid state"?

The hard-core model is supposed to exhibit fluid behaviour at low density, solid at high.

Indeed, at low density, it exhibits disordered behaviour, short-range correlation, and rapid mixing [Kannan, Mahoney and Montenegro '03].

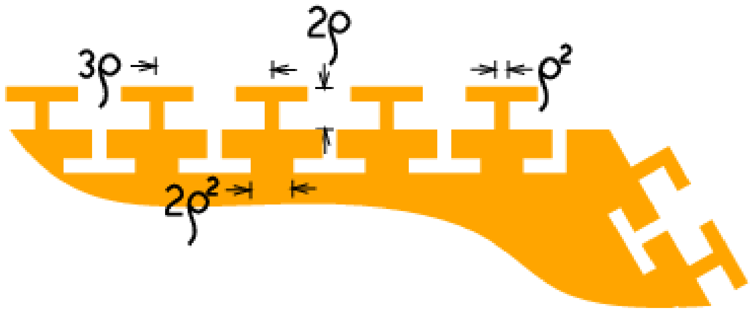
But no one has managed to prove that at high density it does the opposite!

Maybe if you change the shape!?

Cheating with the “zipper” tile

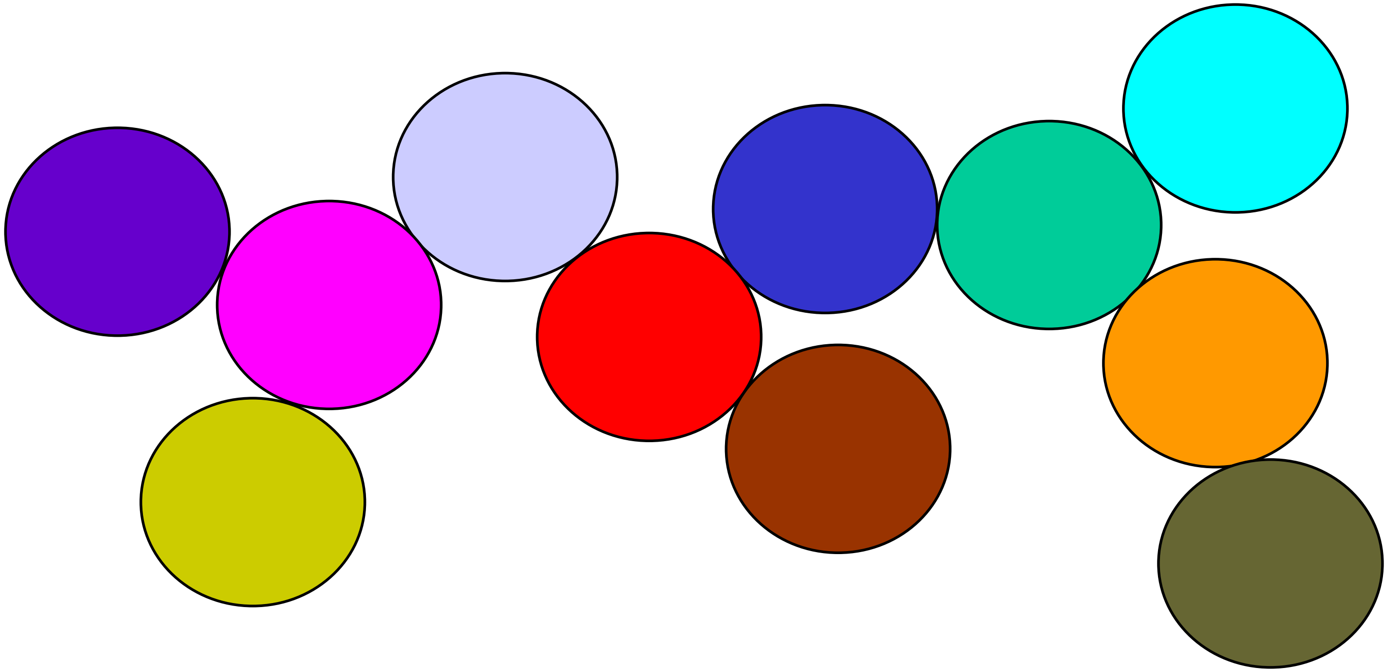
[Bowen, Lyons, Radin, and W. '07]

close-up of fringe:



Problem 2: How big is a random branched polymer?

A ***branched polymer*** is a connected set of labeled, non-overlapping unit balls in space.

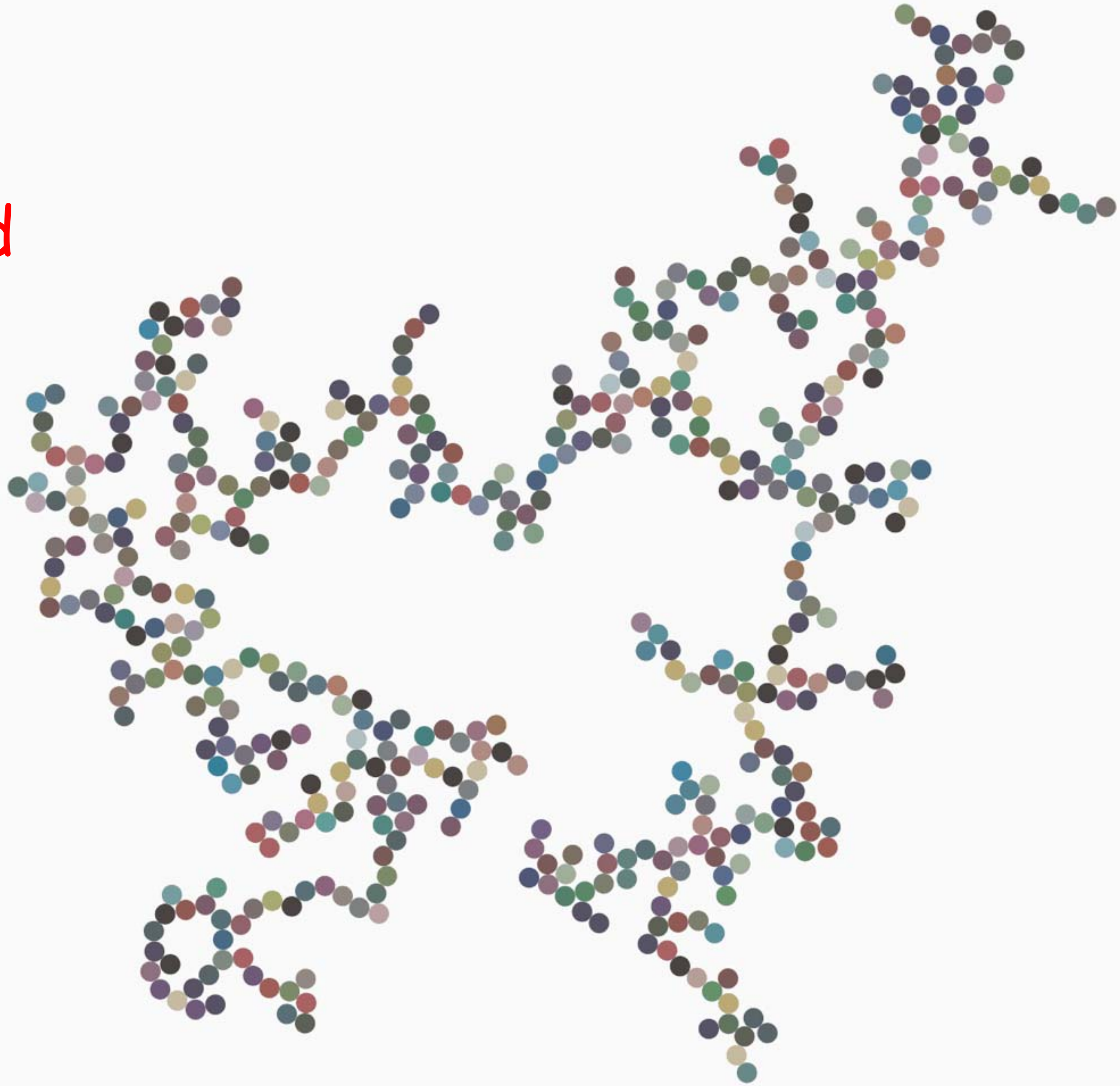


This one is order 11, dimension 2.

What is the diameter of a random branched polymer in the plane?

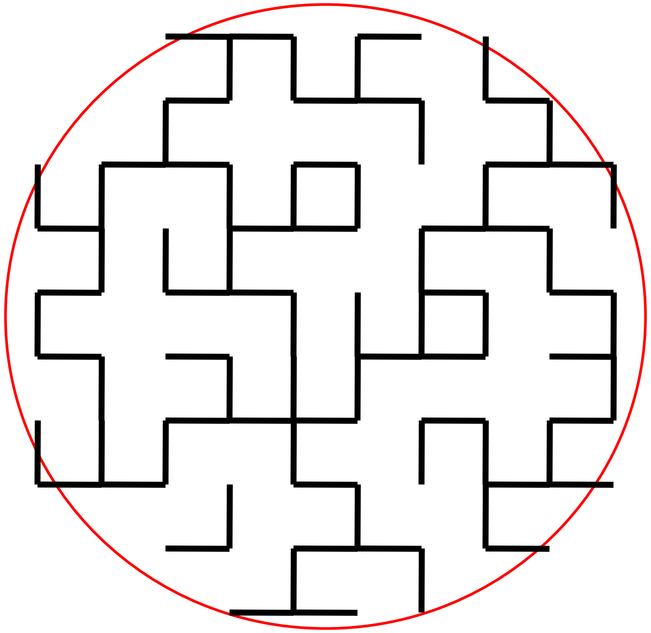
In *3*-space, the diameter is order $\sqrt{n}$.

(Kenyon & W., using work of Brydges & Imbrie)



Problem 3: What's the best percolating graph?

A plane graph with edge-probabilities percolates if it probably has no big faces.



If an edge of length x and reliability p costs $-x^2 / \log p$, what is the cheapest percolating graph?

Conclusion

Statistical mechanics is a great source of well-motivated questions for combinatorialists (and vice-versa). Don't be afraid!