Usage of Fuzzy a Priori Information for Modeling Systems with Variated Parameters

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Modelling of systems with parameters varied in time is an urgent problem as such systems are widely spread in engineering and economics. There are various solutions of this problem which are viewed in [1], for example. Their characteristic property is that they are designed for the big samples and calculations in real time: parameters of system are estimated at the moment of time $t = T$ on T to supervision, estimations of parameters for $t < T$ are not corrected. Such approach is acceptable for management of engineering systems and is not very productive for solving problems of modelling economic systems with the purpose of studying their structure. Non-stationarity is a characteristic property of many economic systems as well as rather short time series of variables, which allow to construct model. The abovesaid is especially right for the annual data.

The model of non-stationary system construction method based on use of short time series of variables and fuzzy a priori information about the system is offered in the present research. We shall assume, that the whole segment of observation $[1, T]$ is devided on intervals with constant parameters of the system. Lengths of these intervals are arbitrary and their minimal value equals to 1 (in this case it is considered, that parameters of the system variate at each moment of time). Giving these reasons, the switching model of regression follows:

$$y_t = x_t'(i)\alpha_i^0 + \varepsilon_t, i = \overline{1, N}, t \in I_i = [t_i, T_i]\,, \qquad (1)$$

where $y_t \in \Re^n$ - is dependent variable, $x_t'\ (i) \in \Re^n$ - regressor, α_i^0- unknown parameter of regression, i - index of a point of switching, ε_t - noise (hereinafter "$'$" means transposing).

Parameters of regression are constant and equal α_i^0 on interval I_i with number of observations m_i . Let $t_1 = 1$, $t_i = T_{i-1}$, $i \geq 2$. We shall consider further, that points of switching t_i are already known, and the value of m_i may be less than n. The estimation problem, examined below, differs from described ones in literature.

Let parameters of regression on adjacent intervals I_i and I_{i+1} be near enough, this is possible to formulate as fuzzy restriction

$$g_i(\alpha) = \alpha_i - \alpha_{i+1} = \beta_i, \tag{2}$$

where $\alpha = (\alpha_1, ..., \alpha_N)'$, β_i - vector, its components - are fuzzy given numbers, which membership functions are concentrated in the areal of 0 .

Let's describe membership function β_i to fuzzy subset B_i of space $\Re^n$ as an expression

$$\varphi_{B_i}(\beta_i) = \exp(-\|\beta_i\|^2 /2). \tag{3}$$

The description of membership function as an exponential function (3), obviously, is not the only one. Its advantage is simplicity and possibility to change the sign of β_i.

Let A_i be a set of estimations of parameter of regression $\alpha^0 = ((\alpha_1^0)', ..., (\alpha_N^0)')'$, $\alpha^0 \in \Re^{nN}$, that satisfies restriction (2). Because β_i is assigned fuzzy, A_i is a fuzzy set. Let its membership function be $\varphi_{A_i}(\alpha)$. Thus,

$$A_i = \{\alpha : \varphi_{A_i}(\alpha), \alpha \in \Re^n\}. \tag{4}$$

The set A_i is prototype of B_i for the function $\beta_i = g_i(\alpha)$. Thus, we have

$$\varphi_{A_i}(\alpha) = \varphi_{B_i}(\beta_i). \tag{5}$$

Let us have $N-1$ restrictions of type (2). The set of admissible estimates of parameters of regression is crossing of fuzzy sets A_i , i=$\overline{1, N-1}$, $A = A_1 \cap A_2 \cap ... \cap A_{N-1}$. Membership function of fuzzy set A according to (3), (5) looks like $\varphi_A(\alpha) = \varphi_{A_1}(\alpha)...\varphi_{A_{N-1}}(\alpha) = \exp(-\frac{1}{2}\sum\limits_{i=1}^{N-1}\|\alpha_i - \alpha_{i+1}\|^2), N \geq 2.$

We shall take the function $F_1(\alpha) = \frac{1}{2}\sum\limits_{i=1}^{N}\sum\limits_{t\in I_i}(y_t - x_t'(i)\alpha_i)^2$ as criterion for accuracy of estimation of parameters of regression. Lets choose such estimation of parameter α , that minimizes $F_1(\alpha)$ and at the same time its membership degree to admissible set A is maximized. Thus, we come down to two criterial estimation problem

$$F_1(\alpha) \to \min, \varphi_A(\alpha) \to \max. \tag{6}$$

According to lemma 2 [2], considering positivity of $\varphi_A(\alpha)$, it is possible to replace the second criterion in (6) for $ln\varphi_A(\alpha)$. Thus, (6) is equivalent to a problem

$$F_1(\alpha) = \frac{1}{2}\sum_{i=1}^{N}\sum_{t\in I_i}(y_t - x_t'(i)\alpha_i)^2 \to \min, F_2(\alpha) = \frac{1}{2}\sum_{i=1}^{N-1}\|\alpha_i - \alpha_{i+1}\|^2 \to \min, N \geq 2. \tag{7}$$

We shall consider Pareto-optimal solutions (further P-estimations of parameters of regression) as the solution for (7). We shall introduce the following matrixes:

$X(i) = [x_{tk}(i)]$, $t = \overline{t_i, T_i}$, $k = \overline{1,n}$ (dimension $T \times n$); $X = diag(X(i))$, $i = \overline{1,N}$ (dimension $T \times N$), where $T = \sum\limits_{i=1}^{N} m_i$, $x_{tk}(i)$ - k component $x_t(i)$. We shall generate matrix $M = \begin{bmatrix} X \\ \sqrt{r}C \end{bmatrix}$. Here $r > 0$, J_k - is unitary matrix $k \times k$, $C = L \otimes J_n$, where a matrix

$$L = \begin{bmatrix} 1 & -1 & 0 & ... & 0 & 0 & \\ 0 & 1 & -1 & ... & 0 & 0 & 0 \\ ... & ... & ... & ... & ... & ... & \\ 0 & 0 & 0 & ... & 1 & -1 & \end{bmatrix}$$ has dimension $(N-1) \times N$.

We have $M = [M_{j\times i}]$, $i = \overline{1,N}$, $j = \overline{1,L}$, where $M_{j\times i}$ - is a vector, with dimensions $T + n(N-1)$, and $M_{j\times i} = \begin{bmatrix} V_{j\times i} \\ U_{j\times i} \end{bmatrix}$. Here $V_{j\times i} = \begin{bmatrix} O_{\rho_1} \\ x_j(i) \\ O_{\rho_2} \end{bmatrix}$, where $x_j(i) = \begin{bmatrix} x_{t_i j}(i) \\ x_{T_i j}(i) \end{bmatrix}$, $\rho_1 = \sum\limits_{k=1}^{i-1} m_k$, $\rho_2 = \sum\limits_{k=i+1}^{N} m_k, O_k$, - is a zero k-dimensional vector. Dimensions of $V_{j\times i}$ equal T . Components with index $j + n(i-1)$and $j + n(i-2)$ are equal to r and other components are equal to zero for vector $U_{j\times i}$ with dimensions $n(N-1)$.

Lets introduce the following assumption relatively to regressor

Assumption 1. Columns of the matrix $\widetilde{X} = \begin{bmatrix} X(1) \\ \vdots \\ X(N) \end{bmatrix}$ with dimensions $T \times n$ are linearly independent.

Lemma 1. *Let the assumption 1 hold and $T \geq n$.Then matrix M has a full rank.*

Proof. Necessary and sufficient condition for linear independence of vectors $M_{j\times i}$ is holding condition $\sum\limits_{j=1}^{n}\sum\limits_{i=1}^{N} p_{ji} M_{j\times i} = O_{T+n(N-1)}$ for all $p_{ji} = 0$. We get two systems of equations from the given equality

$$\sum_{j=1}^{n} p_{jk} x_j(k) = O_{m_k}, k = \overline{1,N}, \tag{8}$$

$$\sum_{j=1}^{n} p_{j1} u_{j1}(k) + \sum_{j=1}^{n} p_{j2} u_{j2}(k) + \cdots + \sum_{j=1}^{n} p_{jN} u_{jN}(k) = 0, k = \overline{1, q_u}, \tag{9}$$

where $u_{ji}(k)$ - is k component of vector $U_{j\times i}$. The number of equations in the first system is $q_v = T$, in the second is $q_u = n(N-1)$. Here q_v and q_u are dimensions of vectors $V_{j\times i}$ and $U_{j\times i}$ accordingly.

Lets look at the first equation in (9), which coefficients equal to $u_{11}(1) = r$, $u_{12}(1) = -r$, $u_{j3}(1) = u_{j4}(1) = \ldots = u_{jN}(1) = 0$. It follows that $p_{11} = p_{12}$. Similarly, we get from the rest of equations in (9)

$$p_{j1} = p_{j2} = \ldots = p_{jN} = p_j, j = \overline{1,n}. \tag{10}$$

We get the system of equations $\widetilde{X}p = O_T$ from this expression and (8) , where $p = (p_1, \dots, p_n)'$. According to the condition of lemma its solution is $p = O_N$. From this expression and from (10) we get $p_{ji} = 0$, $j = \overline{1,n}$, $i = \overline{1,N}$. □

For definition of estimations of parameters of regression with switchings we shall convolute two criteria in one

$$F(\alpha) = F_1(\alpha) + rF_2(\alpha) \to \min, \alpha \in \Omega, r \geq 0. \qquad (11)$$

Theorem 2. *If conditions for lemma 1 and $\alpha \in \Omega$, where Ω is a convex set, are satisfied, then P-estimation of parameters of regression (1), which are appropriate for criteria in (7) is the solution for the problem in (11)*

Proof. After transformations we get

$$F(\alpha) = \frac{1}{2}y'y + \frac{1}{2}\alpha'\alpha - \alpha'X'y \qquad (12)$$

from (7), where $y = (y_1, \dots y_T)'$, $R = X'X + rC'C = M'M$.

But according to a lemma 1 M has a full rank. Therefore quadratic form $\alpha'M'M\alpha$ is defined positively and, hence, (11) has a single solution. The statement of the theorem follows from this conclusion and from [2], the theorem 8. □

From abovesaid, P-estimations are functions of r, and we shall mark them $\alpha(r)$. Function $\alpha(r)$ represents a compromise curve (CC) in the space of parameters . Obviously, not all points of CC are acceptable because of very high value of the first criterion. Therefore for the further analysis we shall select points of CC with $r \in Z = [r_0, r_1]$. The properties of criteria as the function of parameter r will be necessary to choose r_0 and r_1 . Denote $f_i(r) = F_i(\alpha(r))$, $i = 1, 2$.

Lemma 3. *If the assumption 1 is hold, then $f_2(r)$ decreases strictly monotonously if $r \geq 0$.*

Proof. Considering strict convexity of the function $F(\alpha)$ according to lemma 1 for $r_2 > r_1 > 0$ we have

$f_1(r_1) + r_1 f_2(r_1) < f_1(r_2) + r_1 f_2(r_2)$, $f_1(r_2) + r_2 f_2(r_2) < f_1(r_1) + r_2 f_2(r_1)$.

We shall receive $(r_1 - r_2)(f_2(r_2) - f_2(r_1)) < 0$ from this. The statement in lemma follows from the given inequality. □

Lemma 4. *Let function $f(\alpha)$ be strictly convex and let function $g(\alpha)$ be convex. Then function $q(y) = \min_{\alpha \in \Omega} \{f(\alpha) : g(\alpha) < y\}$ is strictly convex.*

Proof of lemma repeats the proof of the theorem 5.3 [3] with little differences.

Lemma 5. *If the assumption 1 holds, then $f_1(r)$ grows strictly monotonously if $r \geq 0$.*

Proof. According to [4], p.168 the solution for (11) $\alpha(r)$ is also the solution for the problem

$F_1(\alpha) \to \min,\ F_2(\alpha) \le F_2(\alpha(r)) = f_2(r) = f_2,\ \alpha \in \Omega.$

Lets assume $q(f_2) = \min_{\alpha\in\Omega}\{F_1(\alpha) : F_2(\alpha) \le f_2\}$. According to lemma 4, $q(f_2)$ is strictly convex function. It decreases monotonously [3], the theorem 5.2. Thus, $q(f_2)$ decreases strictly monotonously for $f_2 \ge 0$. Then, considering $f_1(r) = q(f_2) = q(f_2(r))$ and a lemma 3, we shall receive $f_1(r_2) = q(f_2(r_2)) = f_1(r_1)$ for $r_2 > r_1$. □

Single estimation of α^0 is necessary to get model (1). This estimation corresponds to some parameter $r^* \in Z$. For definition of r^* lets use the Bellman-Zade principle, according to which the viewed problem can be treated as a problem with two fuzzy objectives of choice, because the first criterion increases and the second criterion decreases with the increase of r and vice versa. Some fuzzy subset is a fuzzy i objective, $i = 1, 2$, in set Z. We shall define it as $\widetilde{Z_i}$. Membership function of $\widetilde{Z}_i$ is $\varphi_i(r) = (f_{i2} - f_i(r))/(f_{i2} - f_{i1})$, where , $f_{i1} = \min_{r\in Z} f_i(r)$, $f_{i2} = \max_{r\in Z} f_i(r)$.

According to lemmas 3, 5 $\varphi_1(r)$ decreases from 1 to 0, and $\varphi_2(r)$ increases from 0 to 1. The set $\widetilde{Z}_1 \cap \widetilde{Z}_2$ is a fuzzy solution for problem of achievement of a fuzzy objective . Its membership function is $\mu(r) = \min(\varphi_1(r), \varphi_2(r))$, since the condition $\widetilde{Z}_1 \subseteq \widetilde{Z}_2$ is not hold. A single solution for a problem will be $r = r^*$, which maximizes $\mu(r)$: it has the maximal membership degree to the crossing of sets $\widetilde{Z}_1$and $\widetilde{Z}_2$.

Theorem 6. *If the assumption 1 and $\alpha(r) \in \Omega$ (Ω - convex set) is hold, then $\mu(r)$ is unimodal function.*

This result follows directly from a lemma 1.

From the theorem 6 follows, that the single solution for the problem will be $r = r^*$, which maximizes $\mu(r)$. It can be found by the method of golden section with known r_0 and r_1 . From lemmas 3, 5 follows, that it is possible to set $r_0 = 0$ and to find the value of $r_1$from the condition

$$f_{i2}/\overline{y}(T - nN) = \delta \le F_1(\alpha(0))/\overline{y}(T - nN), \tag{13}$$

where $\overline{y}$ is the average value of a dependent variable on segment $[1, T]$, δ is a maximum admissible relative error of model (set value).

From abovesaid, the problem (11) needs to be solved repeatedly for defining the estimation of parameter of regression $\alpha(r^*)$. We shall consider the algorithm for its solution, which allowes to reduce the duration of calculations.

From (11), (12) we have $\alpha(r) = R^{-1}Z$, where $Z = X'y$. It is easy to see, that R is the block matrix and its inversion consists from the consecutive matrix inversions

$$R_i = \begin{bmatrix} R_{i-1} & P_{i-1} \\ P'_{i-1} & D_{i-1} \end{bmatrix}, i = \overline{2, N}, \tag{14}$$

where $P_i = \begin{bmatrix} O_{n(i-1)n,n} \\ -rJ_n \end{bmatrix}$, $i = \overline{2, N-1}$; $D_i = X'(i+1)X(i+1) + q_{i+1}J_n$, $i =$

$\overline{1, N-1}$ (O_{kl} is a zero matrix $k \times l$). Here, $D_0 = R_1 = X'(1)X(1) + q_1 J_n$; $P_1 = -rJ_n$; $R_N = R$; $q_i = rk_i$, $k_1 = k_N = 1$, $k_i = 2$, $i = \overline{1, N-1}$. Having applied the formula of inverse block matrixes to (14) we shall receive

$$R_i^{-1} = \begin{bmatrix} \widetilde{R}_{i-1} & \widetilde{P}_{i-1} \\ \widetilde{P}'_{i-1} & \widetilde{D}_{i-1} \end{bmatrix}, i = \overline{2, N}, \tag{15}$$

where

$$\widetilde{R}_{i-1} = R_{i-1}^{-1} + r^2 \begin{bmatrix} M_{i-1}\widetilde{P}_{i-2} & M_{i-1}\widetilde{D}_{i-2} \\ N_{i-1}\widetilde{P}_{i-2} & N_{i-1}\widetilde{D}_{i-2} \end{bmatrix}, i = \overline{2, N}; \tag{16}$$

$$M_{i-1} = \widetilde{P}_{i-2}\widetilde{D}_{i-1}, N_{i-1} = \widetilde{D}_{i-2}\widetilde{D}_{i-1}, i = \overline{2, N}; \tag{17}$$

$$\widetilde{P}_{i-1} = r \begin{bmatrix} \widetilde{P}_{i-2} \\ \widetilde{D}_{i-2} \end{bmatrix} \widetilde{D}_{i-1}, i = \overline{2, N}; \widetilde{P}_0 = \widetilde{D}_0^{-1} = D_0^{-1} = \widetilde{R}_1; \tag{18}$$

$\widetilde{D}_{i-1} = (D_{i-1} - r^2 \widetilde{D}_{i-2})^{-1}, i = \overline{2, N}$.

Lets set: $A_i^0 = ((\alpha_1^0)', \dots (\alpha_i^0)'$; $A_{i|\theta}(r)$ and $\alpha_{i|\theta}(r)$ are estimations of A_i^0 and α_i^0 from supervisions at θ intervals ($i \le \theta \le N$) accordingly provided that estimation α_{i+1}^0 is equal to 0 if $i < N$. Obviously, $A_{N|N}(r) = \alpha(r)$. Then

$$A_{i|i}(r) = \begin{bmatrix} A_{i|i-1}(r) \\ \alpha_{i|i}(r) \end{bmatrix} = R_i^{-1} Z_i = \begin{bmatrix} \widetilde{R}_{i-1} & \widetilde{P}_{i-1} \\ \widetilde{P}'_{i-1} & \widetilde{D}_{i-1} \end{bmatrix} \begin{bmatrix} Z_{i-1} \\ z_i \end{bmatrix}, \tag{19}$$

where $Z_i = (z'_1, \dots, z'_i)' = (Z'_{i-1}, z'_i)'$, $z_i \in \Re^n$, $Z_i \in \Re^{in}$, $i = \overline{2, N}$.

Theorem 7. *If the conditions of lemma 1 hold, then estimations of parameters of regression are defined by the following expressions:*

$$\alpha_{i|i}(r) = \widetilde{D}_{i-1}(r\alpha_{i-1|i-1}(r) + z_i), i = \overline{2, N}, \alpha_{1|1}(r) = R_1^{-1} Z_1, \tag{20}$$

$$A_{i-1|i}(r) = \alpha_{i-1|i-1}(r) + \widetilde{P}_{i-1}(r\alpha_{i-1|i-1}(r) + z_i), i = \overline{2, N}. \tag{21}$$

Proof. After transformation we get from (15) - (17), (19)

$$A_{i-1|i}(r) = R_{i-1}^{-1} Z_{i-1} + r \begin{bmatrix} M_{i-1}(r\widetilde{P}'_{i-2} Z_{i-2} + r\widetilde{D}_{i-2} z_{i-1} + z_i) \\ N_{i-1}(r\widetilde{P}'_{i-2} Z_{i-2} + r\widetilde{D}_{i-2} z_{i-1} + z_i) \end{bmatrix}.$$

We get $\alpha_{i-1|i-1} = \widetilde{P}'_{i-2} Z_{i-2} + \widetilde{D}_{i-2} z_{i-1}$. Inserting this expression in formulas for $A_{i-1|i}(r)$ and $\alpha_{i|i}(r)$ in (19) we shall finally receive (20), (21).

Modelling the distributed lag is one of the supplements of the model (1). The distributed lags are applied to modelling the level of discount rate, a rate of exchange, process of reproduction of a fixed capital etc.

The problem of estimating the structure of a lag, despite of the fact that many people worked on it for a long time, cannot be considered to be solved. One of the not very developed questions is modelling the distributed lag with variable structure. Lets consider its solution with reference to modelling the investment process. The non-stationary model looks like

$$y_t = \sum_{j=0}^{L} \alpha_j^0(t) K_{t-j} + \varepsilon_t, t \in I_i = [t_i, T_i], i = \overline{1,N}, t = L+1, L+2, \ldots, T, \quad (22)$$

where y_t - are the capital assets put into operation in t year; K_{t-j} are fixed capital investments in $t-j$ year, $\alpha_j^0(t)$ is true value of j coefficient of lag in a t year (unknown value); L - is the maximal lag.

During one time period the coefficients of lag are considered constant:

$$\alpha_j^0(t_i) = \alpha_j^0(t_i+1) = \ldots = \alpha_j^0(T_i) = \alpha_{ij}^0, i = \overline{1,N}, j = \overline{0,L}. \quad (23)$$

Lets assume $\alpha_i^0 = (\alpha_{i0}^0, \ldots, \alpha_{iL}^0)'$, $x_t(i) = (K_{t_i}, K_{t_i-1}, \ldots, K_{t_i-L})^T$. Then the model of the distributed lag (22) will be reduced to model (1).

The coefficients of a lag vary slowly enough because of sluggishness of economic processes on macrolevel, which is possible to write down as a fuzzy restriction (2).

The coefficient of a lag shows, what part of fixed capital investments in the year $t-j$ transforms into capital assets in a the year t, therefore

$$0 \le \alpha_{ij} \le 1, i = \overline{1,N}, j = \overline{0,L}. \quad (24)$$

The top index "0" beside coefficients of a lag is omitted hereinafter. It means, that the given value varies.

Fixed capital investments for any year are fully or partly capitalized which generates restriction

$$M_t = \sum_{j=0}^{L} \alpha_j(t+j) \le 1, t = L+1, \ldots, T-L, \quad (25)$$

and according to (23) $\alpha_j(t) = \alpha_{ij},\ \ t \in I_i,\ i = \overline{1,N}$.

The given expressions allow to estimate coefficients of a lag, having solved problem (11), where Ω is set by expressions (24), (25).

The described method was applied for modelling process of transformation of fixed capital investments inot capital assets of Ukraine in 1960-2000. Cardinal changes occured in economy of Ukraine during the viewed period of time: transition from a planned economy to market. Apparently, the factors of a lag during this period were not constant. In order to consider those changes, the analyzed period of time was divided into two intervals: the first is 1960-1991, the second is 1992-2000. The results of modelling for the second interval are stated below. Essential changes in price and a tax policy occured during the second interval. Therefore the number of segments chosen with the constant coefficients of lag equals 9, each of 1-year length: in (1) $N = 9$, $t_i = T_i$, $i = \overline{1,N}$. The value of L in (22) is inexpedient to choose more than 2 since at $L = 2$ redundancy of the data is insignificant and equals $(T-2)/n = 2.33$, where $T = 9$, $n = L+1 = 3$ is a number of estimated coefficients of a lag for each year.

The initial data for calculations were taken from "The Statistical Yearbook of Ukraine, 1996-2000". In (13) put $\delta = 0.1$. The mean square error of the residual equaled 7.1 % from the average value of cost of the capital assets put into operation for the viewed period of time. Estimations of coefficients of lag $\widetilde{\alpha}_j$, $j = \overline{0, L}$ are submitted in the table.

The table

Interval of time	I_1	I_2	I_3	I_4	I_5	I_6
Number of year	1	2	3	4	5	6
Year	1992	1993	1994	1995	1996	1997
$\widetilde{\alpha}_0$	0.468	0.456	0.479	0.503	0.532	0.574
$\widetilde{\alpha}_1$	0.105	0.152	0.201	0.25	0.31	0.321
$\widetilde{\alpha}_2$	0	0.037	0.074	0.112	0.15	0.187

Interval of time	I_7	I_8	I_9
Number of year	7	8	9
Year	1998	1999	2000
$\widetilde{\alpha}_0$	0.593	0.615	0.621
$\widetilde{\alpha}_1$	0.309	0.306	0.311
$\widetilde{\alpha}_2$	0.147	0.117	0.101

The estimation $\widetilde{\alpha}_{i2}$ for the viewed period increased first, and then started to decrease as seen from the table. Its increase shows the increase of the big objects share in construction, and some subsequent decrease shows the increase of efficiency of construction due to reduction of its terms. The growth of efficiency of construction is confirmed with increase of coefficients $\widetilde{\alpha}_{i0}$, $i = \overline{1, 9.}$

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