Using Model Reduction techniques for simulating the heat transfer in electronic systems.

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Abstract:
There is an increasing need for quicker heat transfer simulations of electronic systems during its design process. Typically, FEM simulations require a lot of computational time and memory. Our work focuses on creating reduced order models of the thermal models of such systems. More specifically, Krylov subspace and Proper Orthogonal Decomposition methods are being used to create such reduced order models. Eventually, we are aiming to create a library of reduced order models of different components of electronic systems so that the heat transfer simulation of the entire system can be obtained when we interconnect several such reduced order models. This paper proposes a method for applying model reduction techniques to heat transfer problems in electronic systems and we state the initial results of our study and the main problems that have to be dealt with in our future work.

Introduction:
There is a great need for quick and accurate heat transfer simulations in the micro-packaging community. This is especially true in today's world where packages are getting smaller and problems of inefficient heat dissipation are beginning to affect the performance of micro-devices. Hence, searching a larger part of the micro-packaging design space has become very important. The bottlenecks which prevents one from exploring larger parts of the design space are the increased computational costs and storage. In this regard, apart from further improvements in machine speed and numerical algorithm accuracy, one would definitely like to reduce the size of the heat transfer models as much as possible without unduly compromising on accuracy. Currently, the electronic thermal design community uses very large numerical simulations (involving thousands of states) to model multi-mode heat transfer and fluid flow in packaged electronics [1]. This results in enormous computational costs. Previous efforts included compact modeling [2], which aimed at simulating steady state heat conduction within geometrically complex structures subject to convective and/or radiative surface conditions. We propose to use model reduction techniques like krylov subspace methods [3] and proper orthogonal decomposition [4] in order to reduce the size of the models.
The structure of this paper is as follows. We explain our solution technique, i.e. the way we discretized the original heat transfer model and the various model reduction techniques that we considered (along with their brief descriptions) and the reasons why we chose the krylov subspace based model reduction technique for solving the heat conduction problem and the proper orthogonal decomposition method for solving the convection and radiation heat transfer problem. Finally, we state our initial results and future directions for our work.

Proposed Method of Solution:

Initially we have limited the problem is limited to a simple 2-D heat conduction problem (though in reality one has to include convection and radiation effects), namely, given a printed circuit board which has materials with different heat conduction coefficients on it and suppose we model the Joule heating of different materials with simple heat sources, how do you efficiently reduce the size of the model?

A brief list of issues that are used in comparing various model reduction techniques are as follows (in roughly decreasing order of priority): stability (whether the resultant low order model is stable, given the stability of the original system), availability of ***apriori*** error bounds, ease of computing the transformation that gets the reduced order models from the original system and frequency weighting (whether it is possible to focus the low order model to accurately predict the system behavior in a desired range of input and output frequencies, rather than for all frequencies).

Some of the most common approaches to model reduction are :

a) Balanced Model Reduction (Balanced Truncation) [5] .
b) Krylov Subspace based model reduction [3] .
c) Proper Orthogonal Reduction [4].

Balanced truncation and Krylov subspace based model reduction are very well suited to large linear or almost linear models (Proper Orthogonal Decomposition is most suited to problems with nonlinear physics). Hence both these methods are obvious choices for solving the heat conduction problem in solid components of electronic chips.

We propose to create a reduced order model of the dynamics of heat conduction on the chip. In order to apply krylov subspace or balanced truncation techniques, one needs to first create a LTI model of the PDE that describes the actual heat conduction problem. This can be done with the help of FEMLAB (which computes the solution of the PDE at a given time and linearizes around that solution to create the LTI model). Though one needs to solve the PDE once, to get the LTI system – which seemingly beats the whole purpose of model reduction – the point to be noted is that once we get the reduced order model of the chip, we can use it in the future design process (say, by predicting system behavior when reduced order models of several such chips are connected together etc.). We can even create a library of reduced order models of different components of an electronic system, and use it for example, to interconnect various components in this library in order to optimize the design for the best heat dissipation. [6] addresses this issue- namely, how one should model reduce subsystems so that, when interconnected, the system level model reduction errors will be a minimum. This paper proves that a reduced coupled subsystem must be accurate at the natural frequencies of the entire system. Hence the following stability result can be concluded [7] : the entire system stability will be preserved if we keep the sub-system model reduction error, weighted by the entire system frequency, small. Hence, we propose [7] that the sub-system model reduction question be weighted by an estimate of the entire system frequency response.

A brief description of the balanced truncation and krylov subspace based model reduction techniques methods is given below.

Balanced Truncation [5]:
The main idea in balanced truncation is to transform the original stable LTI system into a representation of the same size, but with the property that those states of the transformed system that are uncontrollable are also unobservable and vice versa. In this form, the system is said to be 'balanced'. Once this is done, one simply discards those states of the transformed system that are the least controllable (and hence the least observable) which gives rise to the term 'truncation'. Thus, after truncation, one has a LTI model with (typically) far fewer states than the original model.

Krylov subspace techniques (KST) [3]:
The basic idea behind KSTs is to match the first few moments of the reduced order model and the original system. The moments are defined as the value and subsequent derivatives of the transfer function at $s = s$, where s is any particular frequency (the moments are the coefficients of the power series expansion of the transfer function about s). Thus if the original system has the transfer function G(s) and the reduced order model has the transfer function $G_{red}(s)$, then KSTs provide a transformation matrix that transforms the original system to the reduced system in such a way that the first (say) j moments of the original system G(s) match the first j moments of the reduced order model $G_{red}(s)$. Since, KST is local in nature, it is difficult to come up with apriori global error bounds. [3] suggests a method for global error bounds. Frequency weighting is another important issue and is related to the choice of interpolation points in the reduced order model and is discussed in [3].

If we denote the number of states in the original and reduced order model as n and r respectively, then for balanced truncation the computational cost involved in creating the reduced order model is O(n^3) and for krylov based techniques it is O(nr^2) [8]. The storage requirement in balanced truncation is O(n^2) and in krylov based techniques it is O(nr). Since the heat conduction problem that we propose to solve has a large number of states (approximately 2000), we have chosen to use the krylov subspace based approach to model reduction.

For solving the problem due to the convection and radiation heat transfer processes, we propose to use the Proper Orthogonal Decomposition (POD) method. This is because of the fact that the physics involved is nonlinear and POD has been demonstrated to perform very well in nonlinear problems [9].

A detailed explanation of POD can be found in [4]. Simply put, POD creates a small set of 'basis functions' which can be used as a reduced order model of the system. Suppose, one intended to create a reduced order model of the velocity field in a flow, then POD determines orthogonal basis functions such that the projection onto the first (say) n functions has the smallest L_2 error [4]. In practice, using the method of snapshots (A snapshot in this example is defined as a vector of velocity values at different points in the flow. It is constructed experimentally or using FEM simulations.) a statistically representative sample of the velocity field at various times and finds a low order subspace (the first n basis functions) that approximate the complete system (in this case velocity field) behavior is computed. Due to its methodology, POD is very well suited for nonlinear physics (turbulent flow [4]). Since, the basis functions are computed from the snapshots (which are themselves got from the actual flow), the POD reduced order model inherits the stability and the frequency behavior of the original system.

Summary of results and future work:

At present we are involved in getting the simple 2-D heat conduction problem to work for arbitrary heat inputs (with the krylov subspace based model reduction technique). We have tested the performance of the krylov based technique on arbitrary state space models. The L_2 errors between the original and reduced order models are got to within 5% with less than 3 inputs for systems with upto 150 states. However as we increase the number of states and the number of inputs, we face problems with stability of the model. We suspect that these are related to the placement of interpolation points of the model and the method of projection that we have chosen to project the actual dynamics onto the krylov subspaces. Another observation that we have is that the performance of the reduced order model gets worse as the rightmost eigenvalues of the original system get closer to zero. We are exploring alternative methods of projecting onto the krylov subspace instead of the standard Gram-Schmidt type methods of projection proposed by [3]. The heat conduction problem has very fast dynamics [10], and hence in the future we plan to have better strategies for choosing the interpolation points in order to achieve better frequency weighting. We hope to extend the results to the 3-D heat conduction problem and use POD to solve the convection and radiation problem. The strategies explained in [6] can then be used to interconnect the resulting library of reduced order models.

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