Importance of Linear algebra in Engineering Design Methodology

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Most mathematicians define *Linear Algebra* **a**s that branch of mathematics that deals with the study of vectors, vector spaces and linear equations. Modern mathematics also relies upon linear transformations and systems of vector matrix. Analytic geometry utilizes the techniques learned during a study of linear algebra, for analytically computing complex geometrical shapes. In addition to science, engineering and mathematics, linear algebra has extensive applications in the natural as well as the social sciences. Linear algebra today has been extended to consider *n*-dimnsional space. Although it is very difficult to visualize vectors in *n*-space, such n-dimensional vectors are extremely useful in representing data. One can easily summarize and manipulate data efficiently in this framework, when data are *ordered* as a list of *n* components.

Since the students who take pre-calculus have very little knowledge about the subject of matrices, it has become very important to treat the subject matter in depth. In 1989, the NCTM recognized the need for greater emphasis on linear algebra and stated that "matrices and their applications" should receive "increased attention" in high school curriculum. It should be recognized that linear algebra is as important as calculus to scientists and engineers.

In *linear algebra* one studies sets of linear equations and their transformation properties. It is possible to consider the analysis of rotations in space, selected curve fitting techniques, differential equation solutions, as well as many other problems in science and engineering using techniques of linear algebra. Two tools are extensively used in linear algebra. They are : *The Matrix* and *The Determinant.* Solution to a vector matrix model equation is regarded as one of the most important of 'central problems' of linear algebra.

Study of vectors in two dimensional as well as three dimensional space is extremely important for design engineers. A course was specially designed to provide the engineering and engineering technology students with all the necessary mathematical tools that are essential and necessary for a four-year program. The course begins with the coverage of a wide variety of introductory topics such as complex numbers, partial fractions, determinants, Taylor and Maclaurin Theorems, etc. Vectors, vector spaces, scalar products and vector products are considered extremely important. About weeklong discussion is incorporated in the course to ensure that the students obtain a very strong foundation that pertains to vector operations. Matrices and Matrix operations are covered extensively. Eigenvalues and Eigenvectors, Diagonalization of Matrices are considered essential foundation for subsequent engineering courses and as such several homework exercises are necessarily assigned in this area.

Eigenvectors are extremely important while creating engineering models whether it be a satellite or a jet engine. Study of the dynamics of a football trajectory needs the knowledge of eigenvectors. Eigenvalues can be used to explain several aspects of musical performances. It is very well known that frequencies are vital in music performance. Tuning of an instrument actually means that their frequencies are matched. If some music is pleasing to the human ear, it is because of the frequency. Musicians may not study eigenvalues, but a study of eigenvalues explains why certain sounds are pleasant to the human ear while certain others sounds are not. When groups of people sing in harmony, the frequency matters a lot.

Engineers utilize Ordinary differential equations extensively and as such, solutions to separable, exact and homogeneous equations are discussed in great detail. Bernoulli and Riccati Equations are stressed in particular because of their importance in subjects such as Fluid Mechanics, Thermodynamics, Heat Transfer, etc. Discussion of second and higher order differential equations follows, however third and higher order equations are not discussed at length.

Laplace Transform has been identified as *the mathematical signature of an engineer.* This technique, in addition to the *z - transform* is heavily utilized while designing analog as well as discrete / digital control system components. Extra emphasis is placed while discussing topics such as Transfer Function, Initial and Final Value Theorems. The importance of *natural frequency* is stressed while discussing transfer functions. Study of *Tacoma Narrows Bridge Disaster* is considered extremely important and some student groups have created a web site pertaining to this disastrous event. Also, the 1831 collapse of a bridge in Manchester, England is discussed that was the result of conflicts between frequencies. Soldiers marching in step had caused the bridge to *oscillate* at its own *natural frequency.* This resulted in the building up of *large oscillations* , which ultimately resulted in the collapse of the bridge. Soldiers are therefore required to break cadence while crossing a bridge.

The Fourier series is another important mathematical tool engineers use often. About three weeks are spent discussing the Fourier Integral, the Fourier Transform, The Fourier Sine and Cosine Transforms. In addition several assignments reinforce the mathematical knowledge essential for engineers. A short list is given below.

- Linear Difference Equations and the Fibonacci Sequence.
- Erratic Behavior of a Sequence Generated by a Difference Equation.
- The Row-Reduced Echelon Form of a Matrix and its Rank. A Theorem on the Rank of Matrix Product ABC.
- Consistency of Augmented Coefficient Matrices, Solution by Back Substitution and Cramer’s Rule.
- A One-Way Traffic Flow Problem. Forces in Bridge Struts.

- Verifying and using the Cayley-Hamilton Theorem. Diagonalization of a Matrix.
- Orthogonal Vectors Computed by the Gram-Schmidt Method. Reduction of a Quadratic Form to Standard Form.
- The Hubble Space Telescope and Quadratic Forms. Dynamical Systems and Logging Operations to supply a sawmill.
- First Order Linear Differential Equations : Direction Fields and Integral Curves. Direction Fields and Isoclines.
- The Limit Cycle of the van der Pol Equation. Period of Oscillation of Nonlinear Pendulum.
- Erratic Behavior of a Sequence Generated by a Difference Equation.
- Laplace Transforms : Solution to a Third Order Initial Value Problem.
- Solving an Equation with the Heaviside Step Function in the Nonhomogeneous Term. Solving an Equation with the Dirac Delta Function in the Nonhomogeneous Term.
- Eigenvalues / Eigenvectors : Chebyshev Approximation, Legendre Approximation and Bessel Function Approximation.
- Fourier Series : Plotting Partial Sums, Examining the Gibbs Phenomenon

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