# PULSE PROPAGATION IN GRANULAR SYSTEMS

## SIAM Conference on Nonlinear Waves and Coherent Structures

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**Vitali Nesterenko (UCSD)**
**Niraj Kumar (UCSD)**

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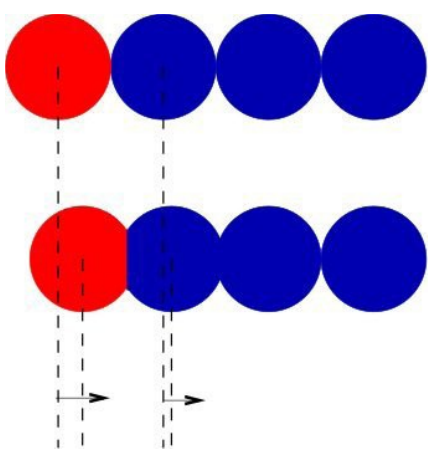
**PetroleumResearch Fund**
**UC Mexus – Conacyt**
**Various international agencies**



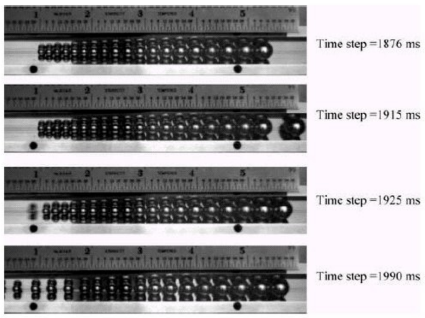


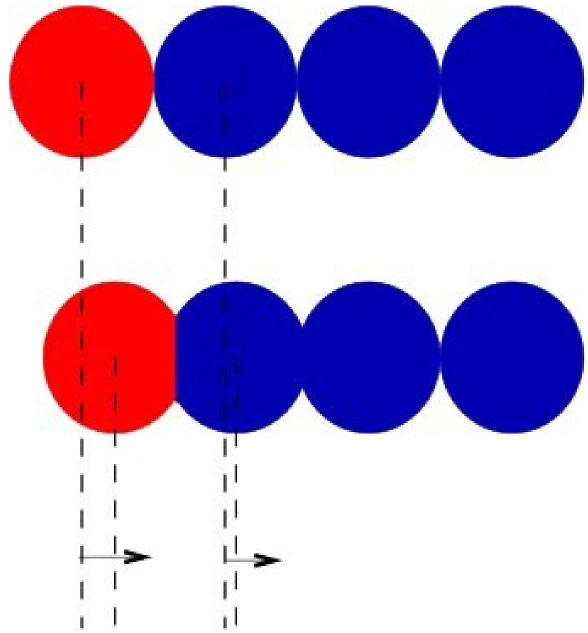

Fig. 5. Velocity of each ball in the tapered chain after the impact

Nakagawa et al., Gran. Matter **4**, 167 (2003)

# PULSE PROPAGATION ALONG GRANULAR CHAINS

**Granular chains serve as convenient model systems to study energy propagation experimentally, numerically, and theoretically**

**An uncompressed chain of granules that just touch, initially at rest except for first granule with velocity $v_0$**



**Sonic vacuum**

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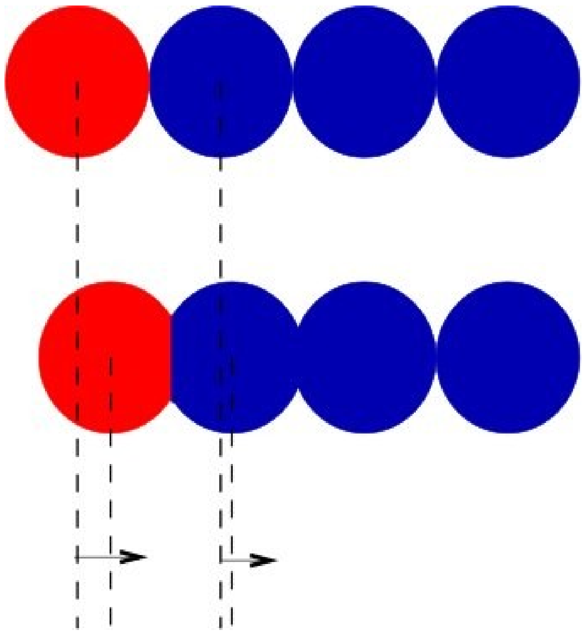

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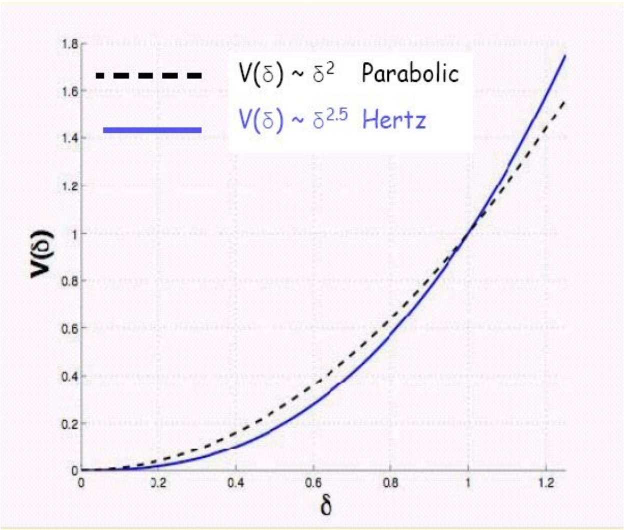


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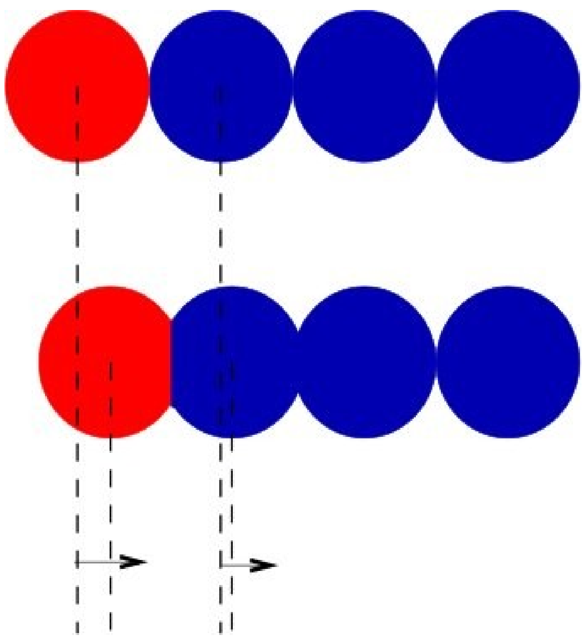

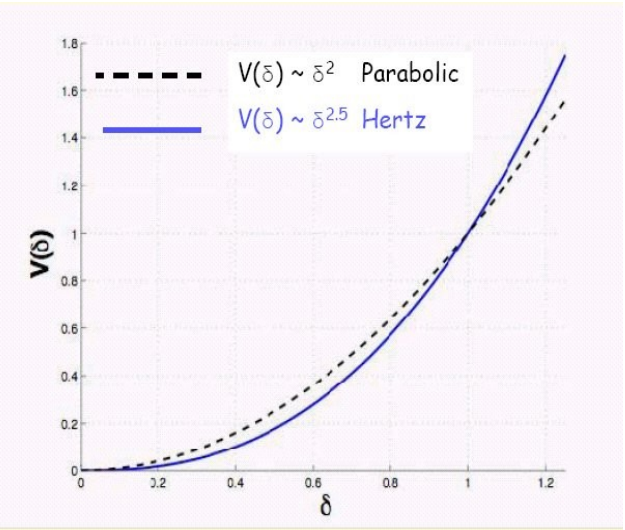


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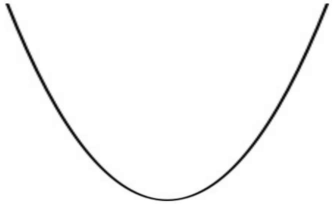
**The usual potential**

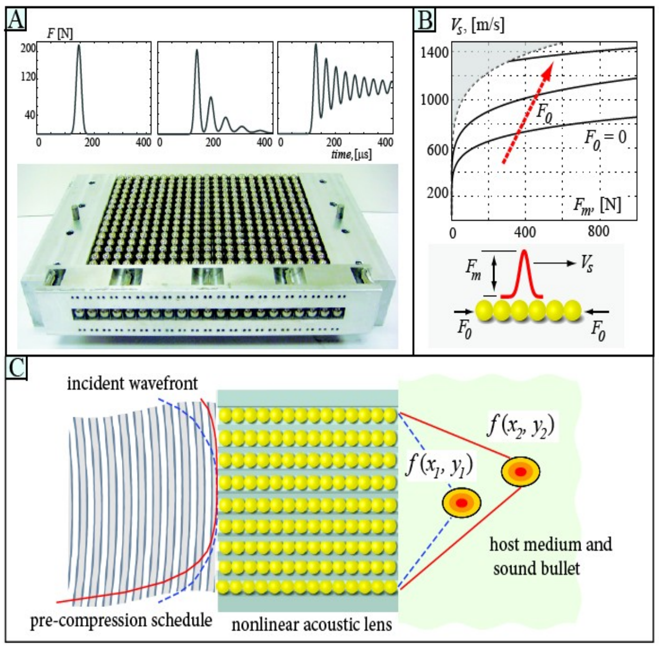


**Sonic vacuum**

# PULSE PROPAGATION ALONG GRANULAR CHAINS

## 1. How does a pulse propagate in a granular medium?

Distribution of energy
Dissipation of energy
Fragmentation




From Spadoni and Daraio, PNAS **107**, 7230 (2010)

Prototype of a nonlinear acoustic lens used to focus acoustic energy into a “sound bullet”

Some applications benefit from spreading of energy and fragmentation (e.g. shock absorption), others from focusing of energy (e.g. detection of buried objects, sound bullets).

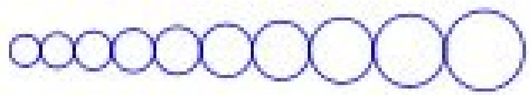
# PULSE PROPAGATION ALONG GRANULAR CHAINS

1. **How does a pulse propagate in a granular medium?**

2. **How does the physical design of the medium affect these behaviors?**
(Figs: Sen et al., Herbold et al.)

**Monodisperse chains**

**Tapered chains**



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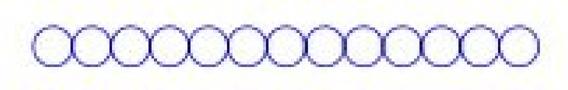
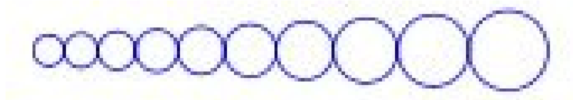
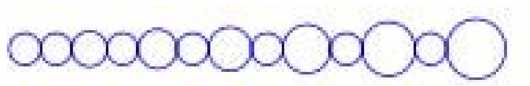
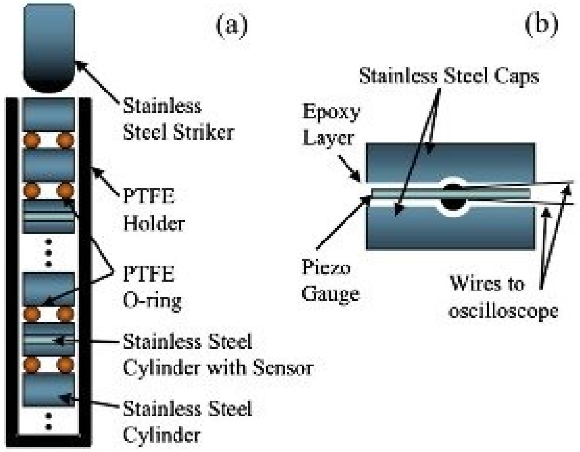


**Tapered chains**



**Decorated chains, other profiles**





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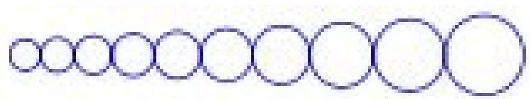

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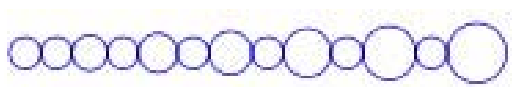
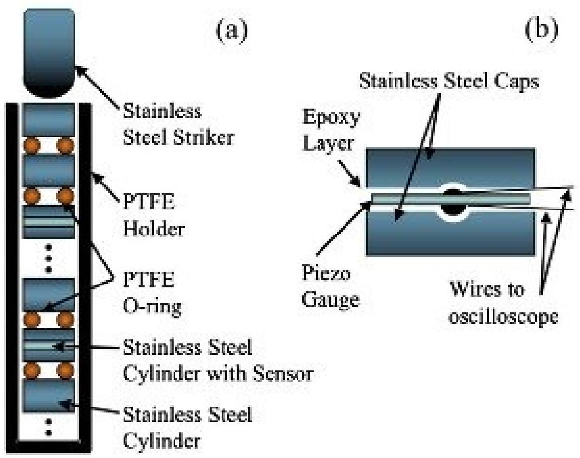
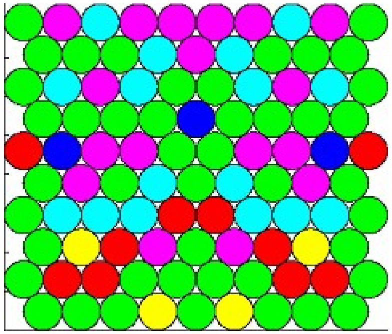


**Mixed material chains**

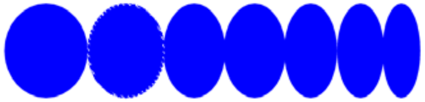


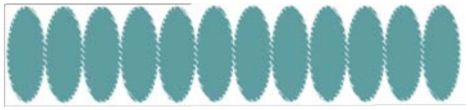
**Two-dimensional hexagonal granular bed** (Daraio)

## 3. Very long list of interesting questions, including:

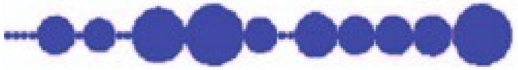
**Different initial conditions: precompression, no longer sonic vacuum, sound waves can propagate. Profound effects on pulse propagation.**



**Shapes of granules, e.g. cylinders or ovals instead of spheres. Profound effects on pulse propagation.**



**Mass and size distribution profile of the granules in the chain, including random distributions. Profound effects on pulse propagation.**



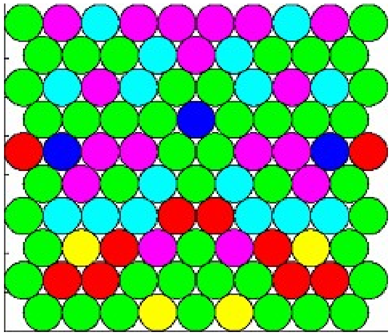
**(Fraternali et al.)**

## 3. Very long list of interesting questions, continued:

**Manner of excitation, e.g. a continuous excitation profile on the first granule instead of an impulse.**

**Higher dimensions: packing geometry, rotational motion, everything entirely different.**

One dimension



Two dimensions

**FRICTION has major consequences on energy propagation**

# GOALS – and how to achieve them

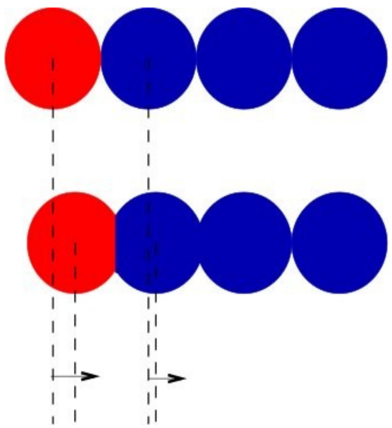
**Experiments: there are many well-controlled experiments in a variety of one-dimensional chains and a few in two-dimensional granular beds.**

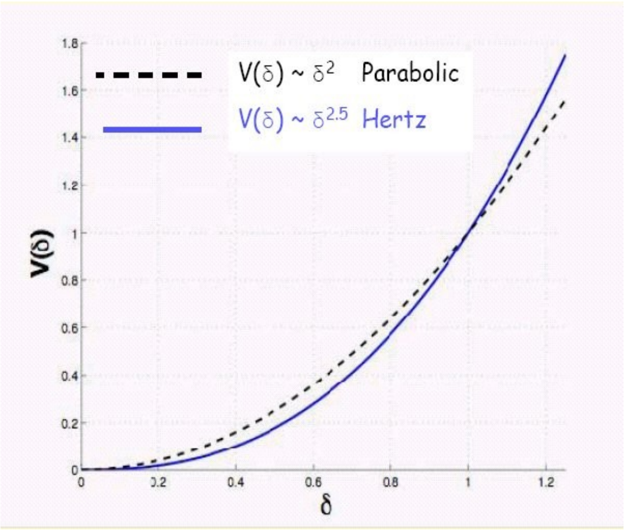
**Numerical simulations: While there are lots of them, they are severely limited because there are so many parameters. Simulations are typically done in small systems because of computational constraints.**

**ANALYTIC THEORY: OUR GOAL is to greatly extend what is now available in order to overcome experimental and numerical limitations.**

## ANALYTIC APPROACHES:
## F=ma

**An uncompressed chain of granules that just touch, initially at rest except for first granule with velocity $v_0$**





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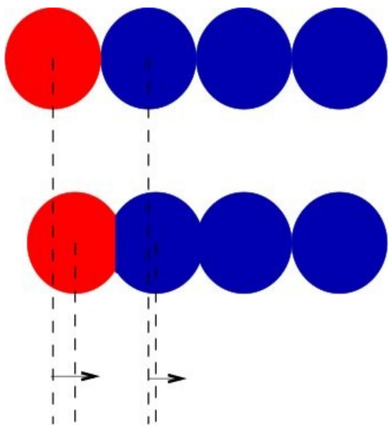

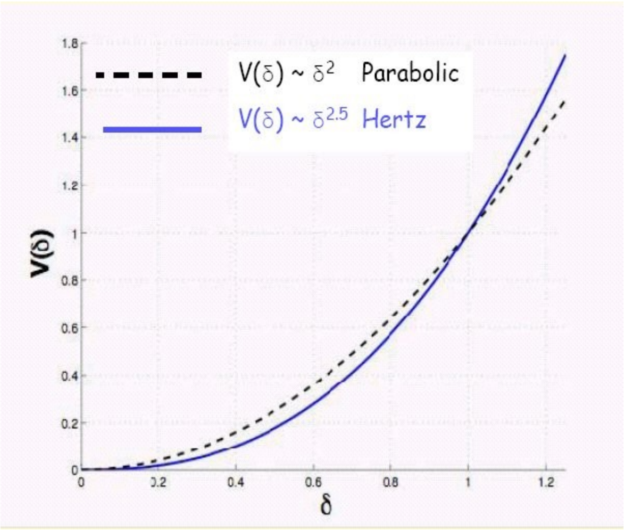


**Equations of motion:**

$$m_k \ddot{x}_k = -r_{k-1}(x_k - x_{k+1})^{n-1} + r_k(x_{k-1} - x_k)^{n-1}$$

**Only when k and k+1 overlap. Otherwise =0**

**Only when k-1 and k overlap. Otherwise =0**

**Plus: initial pulse**

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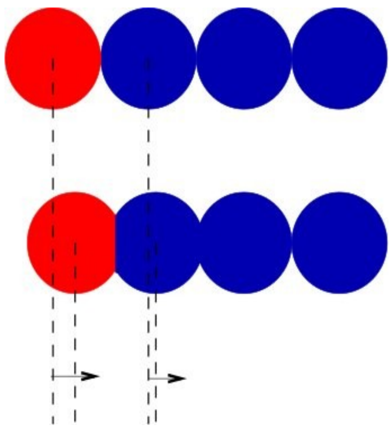

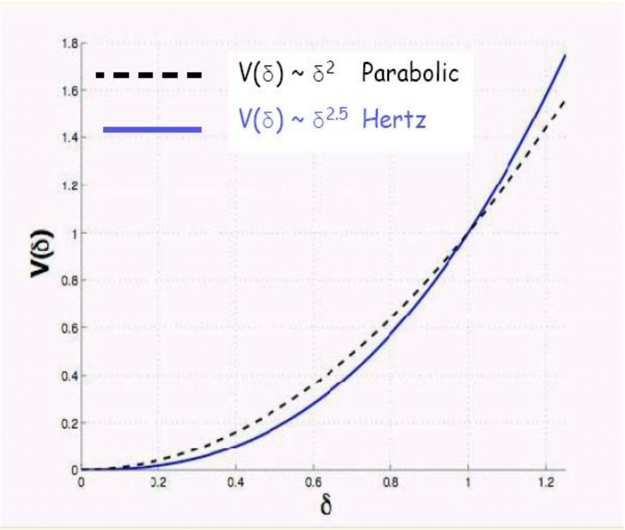


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**How to solve??**

**How does pulse propagate?**

## Two very different analytic approaches

### 1. Continuum approximation (Nesterenko, 80's)

**Equations of motion for chain of monodisperse cylinders (*n*=2)** **but only when there is overlap**

$$\ddot{x}_k = -(x_k - x_{k+1}) + (x_{k-1} - x_k)$$

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**Obtain a traveling pulse solution**

$$x(k,t) = b(t) f\left(\frac{k-t}{\lambda(t)}\right)$$

**Amplitude *b*, shape *f*, width λ**

## PULSE ALONG A CHAIN OF MONODISPERSE CYLINDERS:

$$x(k,t) = b(t) f\left(\frac{k-t}{\lambda(t)}\right)$$

**The pulse retains 95% of the initial energy**

**Shape:**

$$f(z) = 2.431 \int_z^{\infty} Ai^2(2^{1/3}(2^{1/3}y))dy$$

**Width and amplitude:**

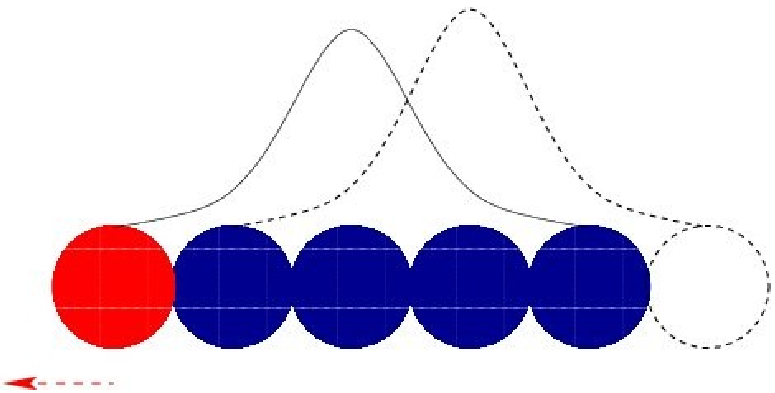
$$\lambda \sim t^{1/3} \qquad\qquad b \sim t^{1/6}$$

**This describes a pulse that gets bigger and fatter with time**

UPSHOT: Pulse travels forward with increasing width and amplitude

Momentum conservation requires that particles be ejected backwards

## CHAIN FRAGMENTATION



$$\dot{x}_k = -0.158k^{-5/6}$$

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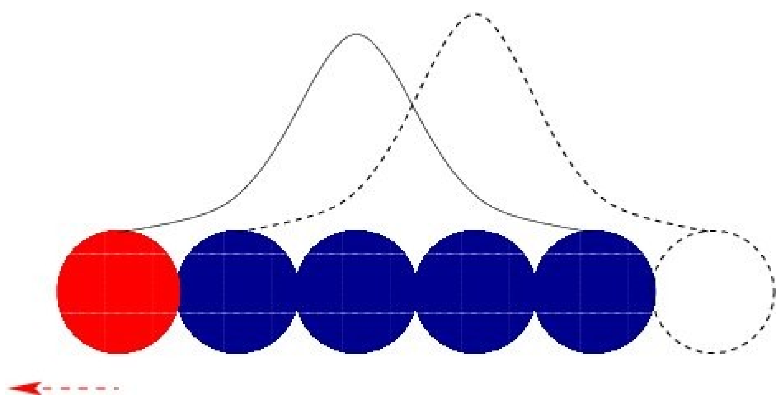

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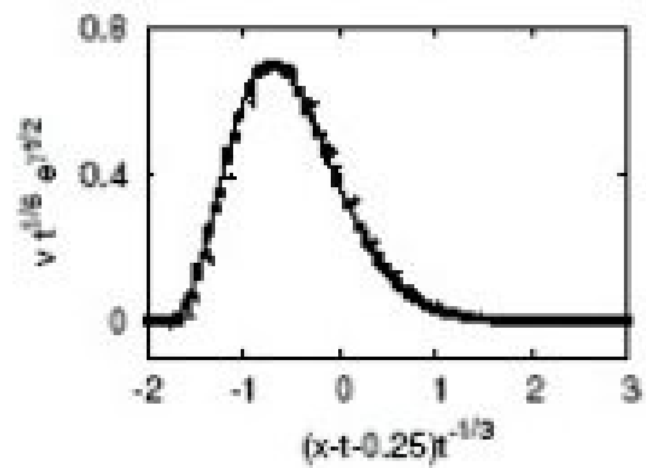

## Equations of motion for chain of monodisperse spheres (*n*=5/2)

$$\ddot{x}_k = -(x_k - x_{k+1})^{3/2} + (x_{k-1} - x_k)^{3/2}$$

## Continuum approximation + discreteness correction

$$\frac{\partial^2 x}{\partial t^2} = \frac{\partial}{\partial k}\left[-\left(-\frac{\partial x}{\partial k}\right)^{3/2} + \frac{1}{16}\left(-\frac{\partial x}{\partial k}\right)^{1/2}\frac{\partial^3 x}{\partial k^3}\right] - \frac{1}{24}\frac{\partial^3}{\partial k^3}\left[\left(-\frac{\partial x}{\partial k}\right)^{3/2}\right]$$

## Obtain a traveling pulse solution (Nesterenko)

$$\left(-\frac{\partial x}{\partial k}\right) = -\left(\frac{5c_0^2}{4}\right)^2 sin^4\sqrt{\frac{2}{5}}(k - c_0 t)$$

## PULSE ALONG A CHAIN OF MONODISPERSE SPHERICAL GRANULES:

$$\left(-\frac{\partial x}{\partial k}\right) = -\left(\frac{5c_0^2}{4}\right)^2 sin^4\sqrt{\frac{2}{5}}(k - c_0 t)$$

**With** $c_0 = 0.836$

**The pulse retains 99.7% of the initial energy – NO BACKSCATTERING, no fragmentation**

**Velocity depends on amplitude**

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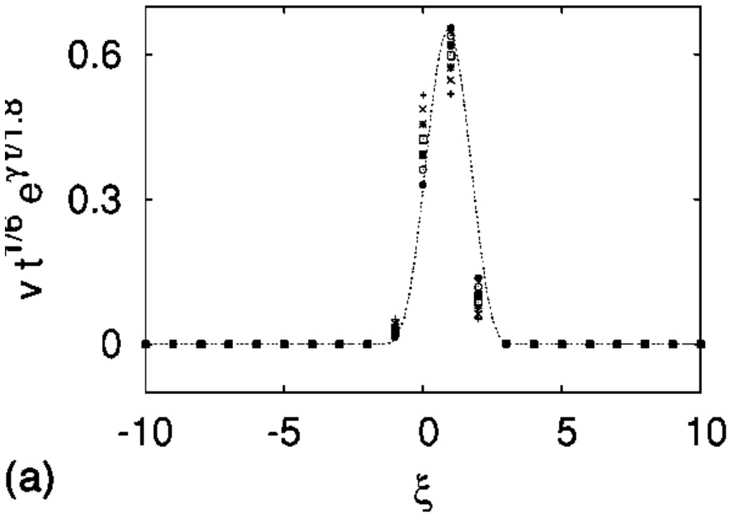


(a)

## Taking stock so far

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*In a chain of *cylindrical granules*:

Pulse grows in amplitude and width as it propagates
Pulse speed is constant, independent of amplitude
Chain fragments
Pulse carries 95% of initial energy

*In a chain of *spherical granules*:

Pulse remains constant in shape and speed as it propagates
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Some additional pluses:

Continuum approximation can handle weakly nonlinear and even linear cases,
Successfully predicts not only solitary waves but also periodic and shock waves.

Some serious minuses:

It can not easily handle large variations in granule radius or size
Difficult to extend to higher dimensions

WHAT TO DO? A surprising solution

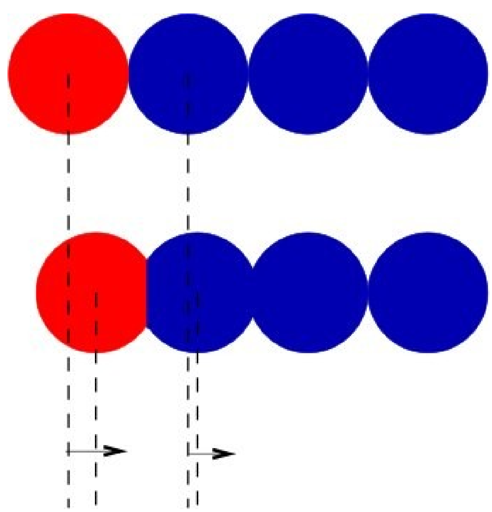
## 2. "Opposite" to Continuum Theory: BINARY COLLISION APPROXIMATION

**Simplest version: transfer of energy along the chain occurs via a succession of two-particle collisions.**

**PartIcle k=1 collides with initially stationary particle k=2, which then acquires a velocity and collides with stationary particle k=3, and so on.**

**Velocities after each collision follow from conservation of energy and momentum.**

**QUESTION: Does this work?**



**FULL EQUATION OF MOTION when granules overlap:**

$$m_k \ddot{x}_k = -r_k (x_k - x_{k+1})^{3/2} + r_{k-1} (x_{k-1} - x_k)^{3/2}$$

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## BINARY COLLISION APPROXIMATION – SIMPLEST CASE

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**CAN SOLVE RECURSIVELY**

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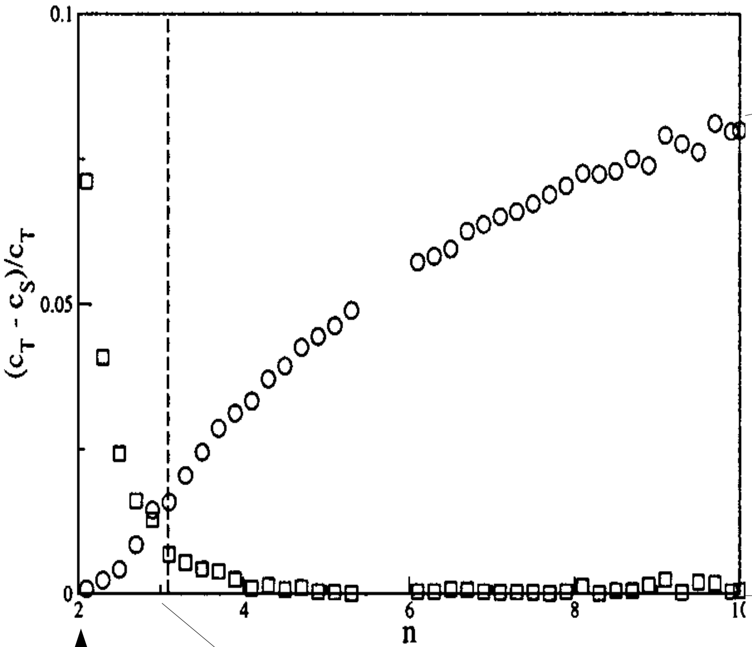
**CAN SOLVE RECURSIVELY**

**PLUSES: Can handle most granular configurations "two by two" (see caveat later)**

**MINUSES: Can only handle narrow pulses, pulse fronts**

# RELATIVE ERROR IN PULSE VELOCITY

## Continuum approximation vs Binary Collision approximation




Continuum

Binary Collision

spheres

cylinders

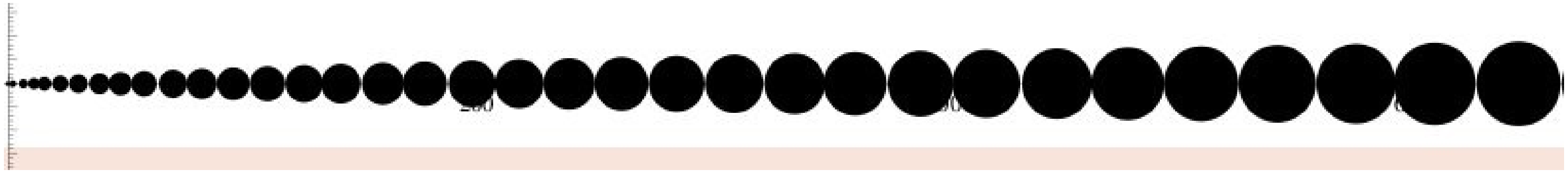
**Relative error in pulse velocity as a function of power of the potential for monodisperse chains**

**For cylinders the binary collision approximation is not appropriate.**

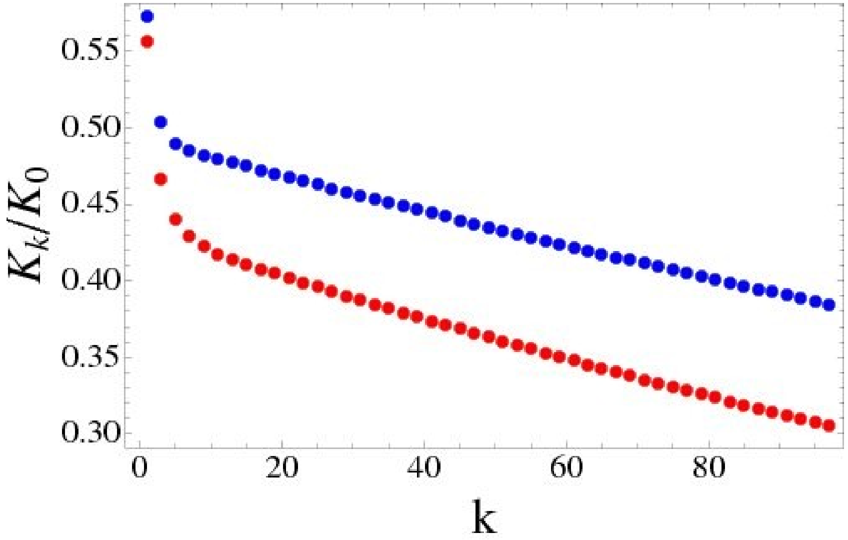
**For spherical granules, the error with either is smaller than 2%!!**

## TAPERED CHAINS, DECORATED CHAINS

### BACKWARD tapered chain



Good shock absorption properties: tapering in **either** direction attenuates energy




Kinetic energy along the chain in units of kinetic energy of the first grain.

**RED: forward tapered, grains get smaller**

**BLUE: backward tapered, grains get larger**

## Recursive BINARY COLLISION solutions

**Pulse velocity**

$$v_k = \dot{x}_k = \prod_{j=1}^{k-1} \frac{2}{1 + \frac{m_{j+1}}{m_j}}$$

***We say that the pulse "arrives at granule k" when the velocity of granule k surpasses that of granule k-1. The residence time on granule k is the time that granule k takes to transfer the pulse from k-1 to k+1.***

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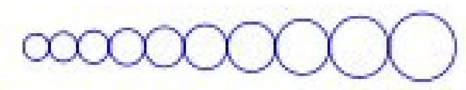
**Residence time**

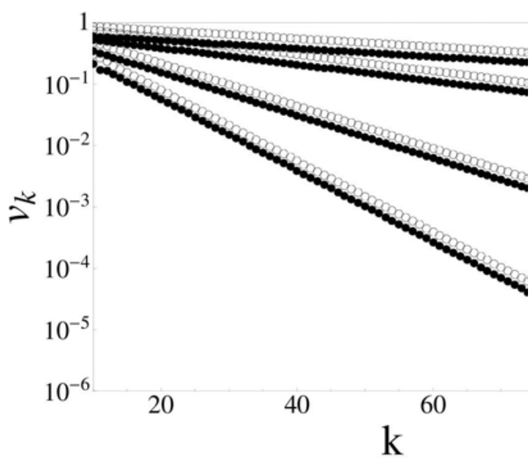
$$T_k = \sqrt{\pi}\left(\frac{5\mu_k}{4r_k}\right)^{2/5} v_k^{-1/5}\,\frac{\Gamma(7/5)}{\Gamma(9/10)}$$

**Now apply results to different tapering protocols**

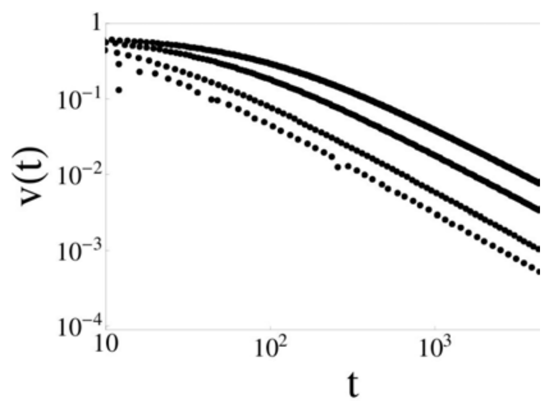
## An example: backward geometrically tapered chain



Decay of pulse amplitude along chain




Decay of pulse amplitude with time




Top to bottom: q=0.01, 0.02, 0.05, 0.08

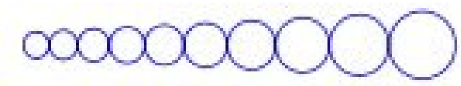
$$v_k = A(q)exp\left[-k \ln A(q)\right]$$

**Decay is exponential**

$$v(t) \sim t^{-f(q)}$$

**Decay is algebraic, power law**

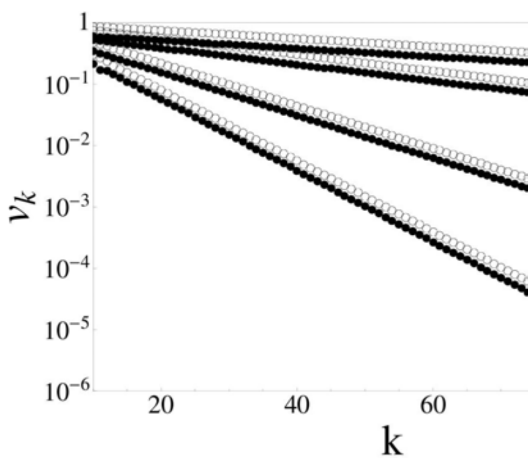
## An example: backward geometrically tapered chain



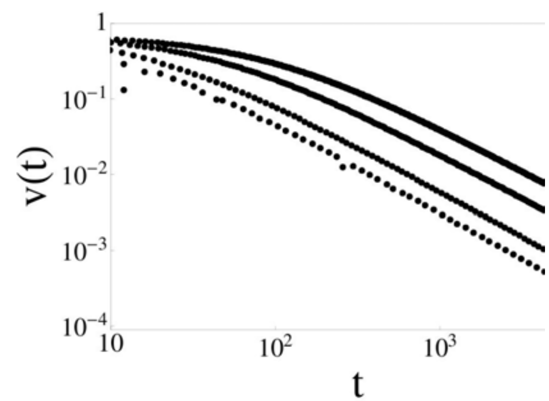
$$\frac{R_{k+1}}{R_k} = (1-q), \qquad A(q) = \frac{1}{2}\left(1 + \frac{1}{(1-q)^3}\right)$$

$$\eta(q) = \ln\left(\frac{[A(q)]^{1/5}}{1-q}\right), \qquad f(q) = \frac{1}{\eta(q)} \ln A(q)$$

Decay of pulse amplitude along chain




Decay of pulse amplitude with time

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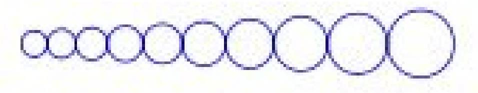
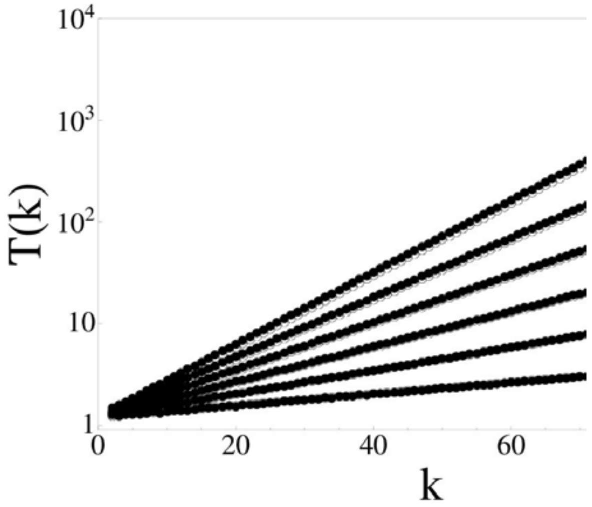
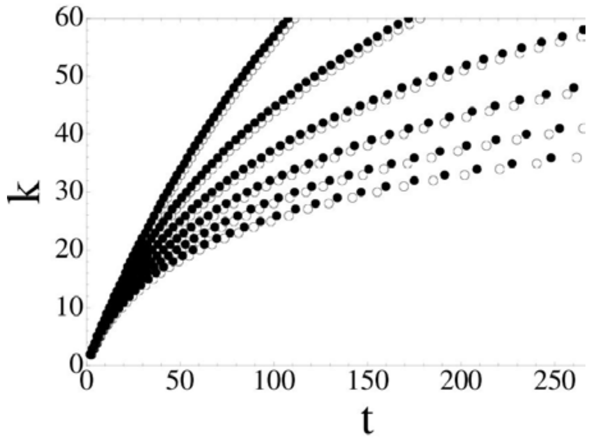
## BACKWARD GEOMETRICALLY TAPERED CHAIN






Pulse residence time

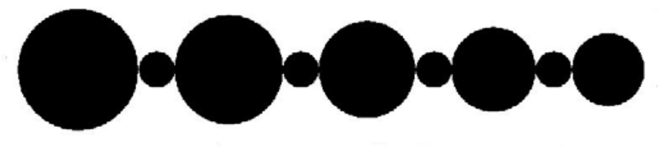



Pulse position vs time

q = 0.01 to 0.06 in steps of 0.01, top to bottom

# DECORATED CHAINS

Simple decorated chain

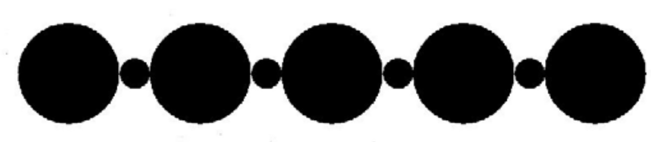


Tapered decorated chain

Requires an extended version of the Binary Collision approximation

REASON: Small granules rattle back and forth

# DECORATED CHAINS



Simple decorated chain

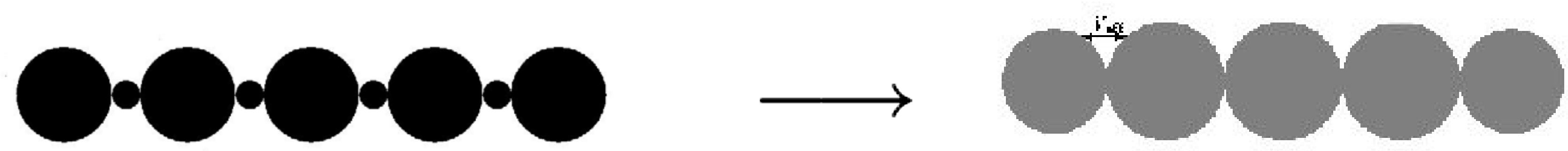
Tapered decorated chain

Requires an extended version of the Binary Collision approximation

REASON: Small granules rattle back and forth

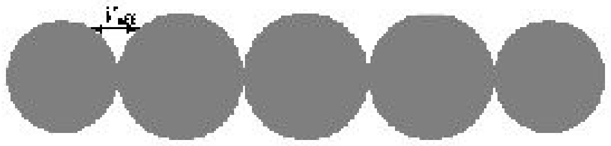
STRATEGY: Replace decorated chain with an effective undecorated chain

TRICKY: Choice of masses and interactions in the effective undecorated chain



**OR**

$\longrightarrow$



**Effective chain**

**STRATEGY** (a non-trivial exercise):

* **Approximate the motion of the small granule**

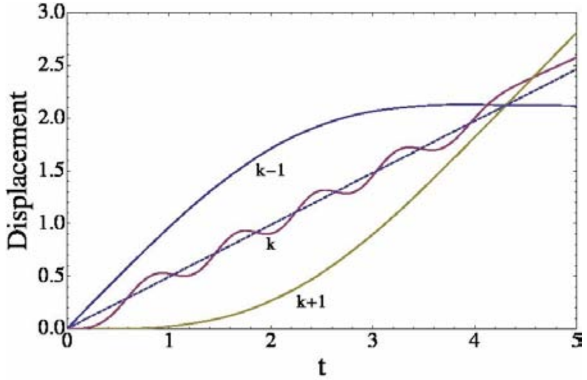
$$x_k(t) = \bar{x}_k(t) + Asin(\omega t + \phi)$$

* **Determine** $\bar{x}_k, \omega, A$

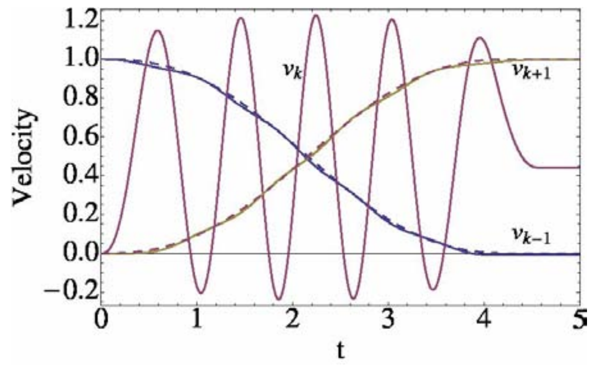
* **Eliminate equations for small granules**

* **End up with effective chain of modified end masses, modified interior masses, and modified interactions that all depend on radius and mass of small masses**

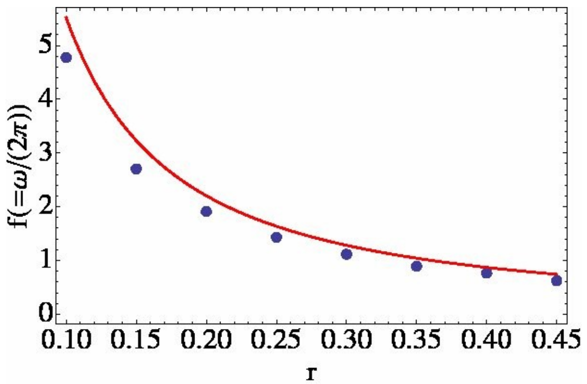
## THREE-GRANULE TEST CASE: BIG, SMALL, BIG




**Displacements of the three granules. Dashed: average displacement of small granule.**

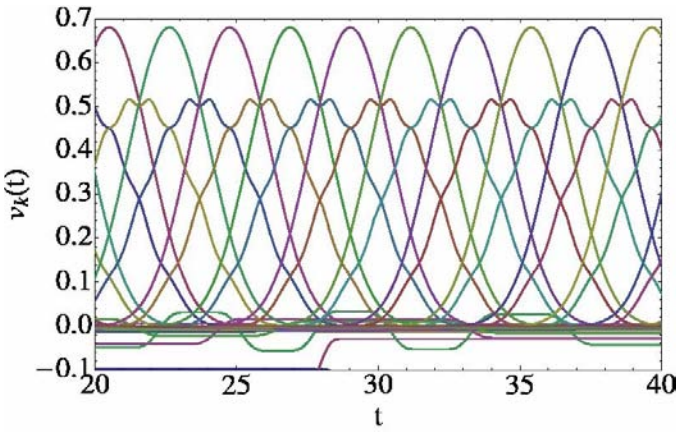



**Velocity profiles of the three granules.**
**Solid: exact.**
**Dashed: theory.**



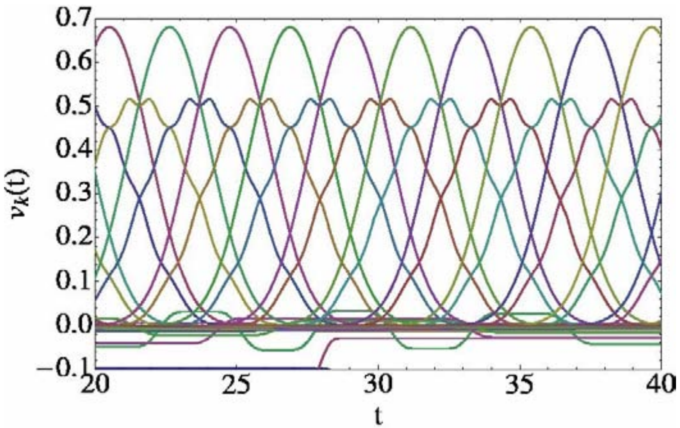

**Frequency of small granule.**
**Dots: exact.**
**Line: theory.**

# SIMPLE DECORATED CHAIN



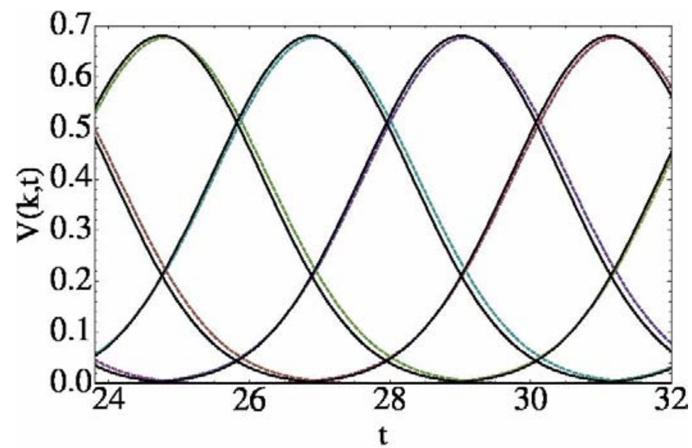

Velocity profiles of granules: the energy of the small granules is negligible compared to that of the large granules.

# SIMPLE DECORATED CHAIN

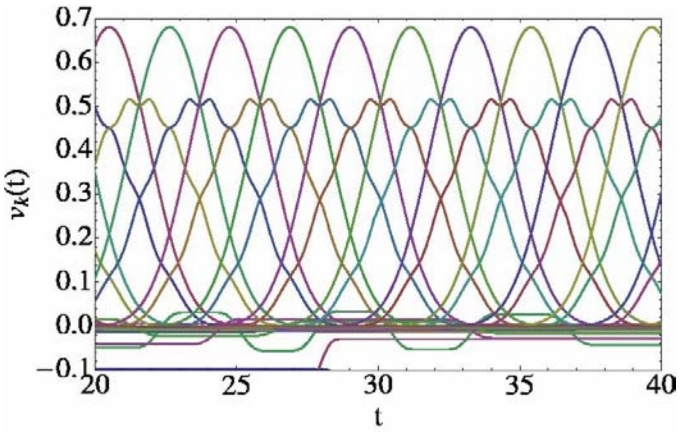



⟵ Velocity profiles of granules: the energy of the small granules is negligible compared to that of the large granules.

Velocity profiles of large granules.
Solid: exact. Dashed: theory ⟶

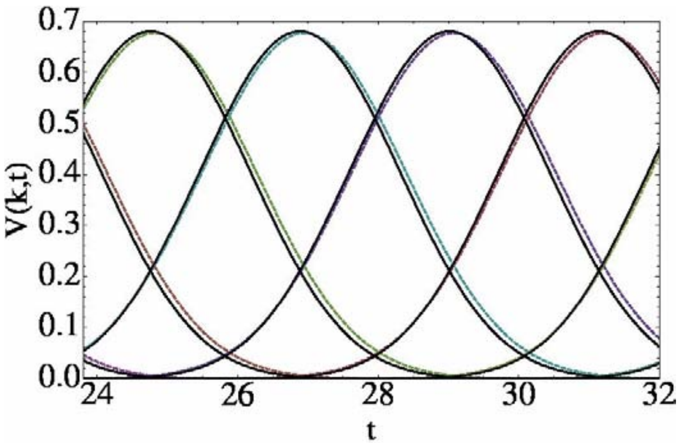


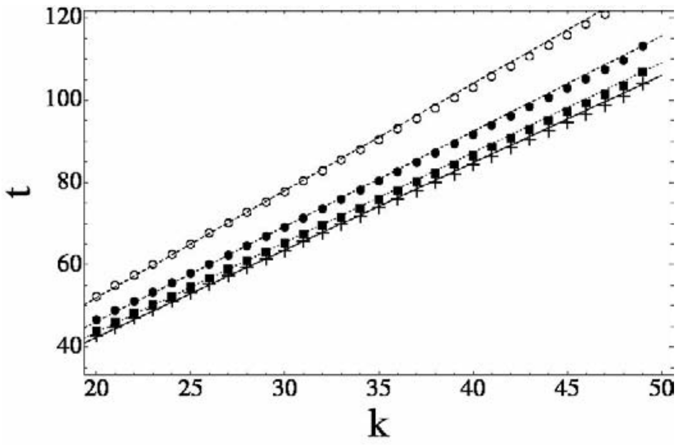
## SIMPLE DECORATED CHAIN




Velocity profiles of granules: the energy of the small granules is negligible compared to that of the large granules.

Velocity profiles of large granules.
Solid: exact. Dashed: theory



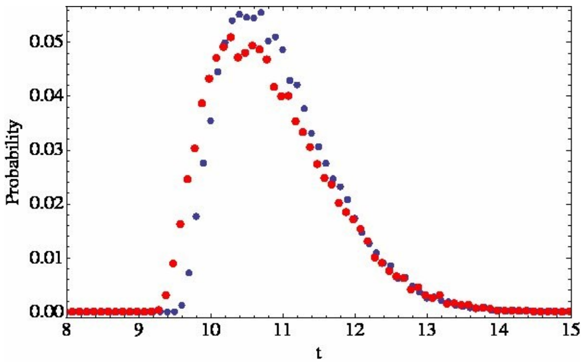

Time taken by the pulse to reach the kth granule for different small-granule radii.
Symbols: theory.

**Results equally good for tapered decorated chain**

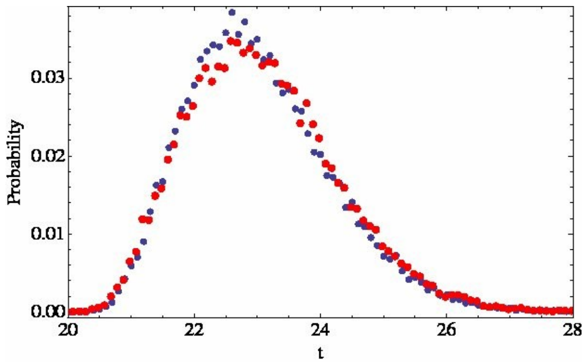
## MORE COMPLICATED PROFILES IN BABY STEPS

### A random radius/mass distribution of small granules in the decorated chain

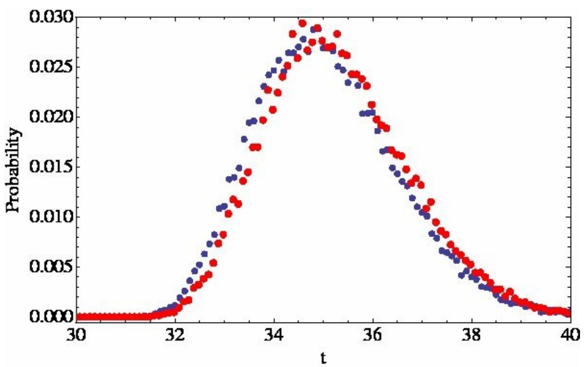
Distribution of time of arrival of the pulse to the penultimate large granule



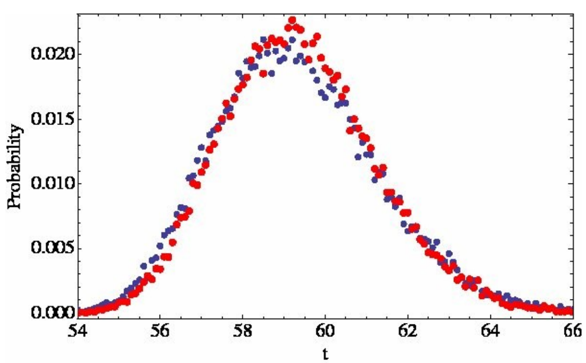

N = 11




N = 21




N = 31

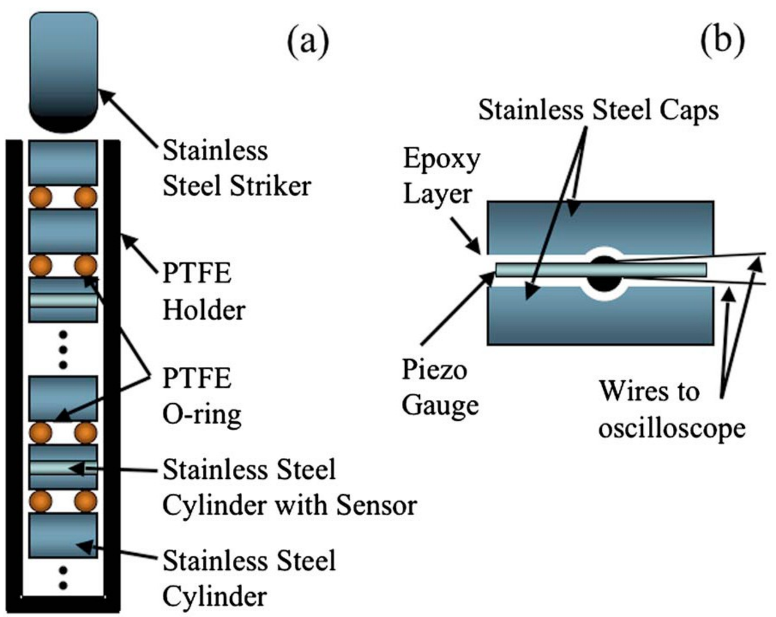



N = 51

**Ultimate goal: arbitrary profile OPTIMIZATION!**

## Most recent work based on the Binary Collision Approximation:

**Based on experiments of Herbold and Nesterenko on chains of o-rings (APL Vol. 90, 2007)**




Toroidal rings between rigid cylinders that act as nonlinear springs

**Two-term potential PLUS pre-compression (*f*)**

$$\ddot{x}_k = f - (x_k - x_{k+1})^{3/2} - b(x_k - x_{k+1})^6$$

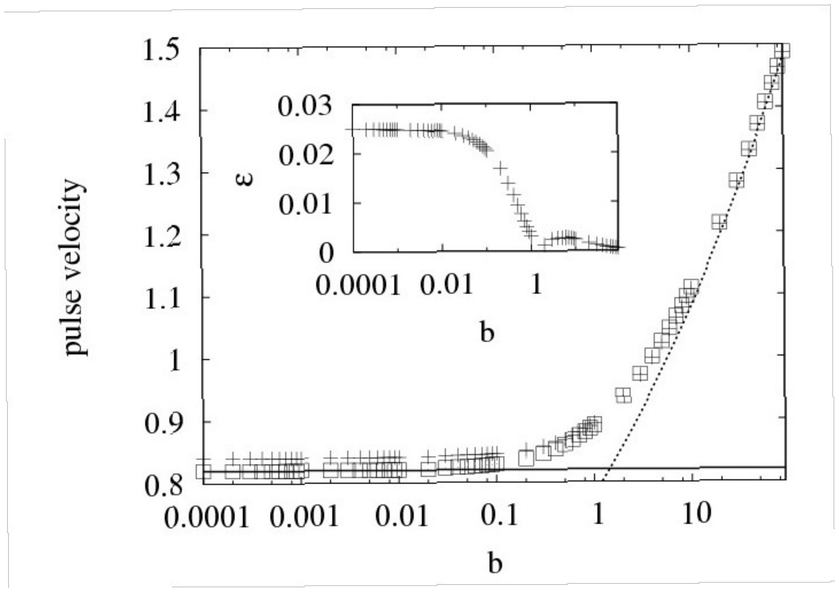
$$\ddot{x}_{k+1} = -f + (x_k - x_{k+1})^{3/2} + b(x_k - x_{k+1})^6$$

## RESULTS FOR CHAINS OF O-RINGS

**BCA works well as long as pre-compression is not too strong.**

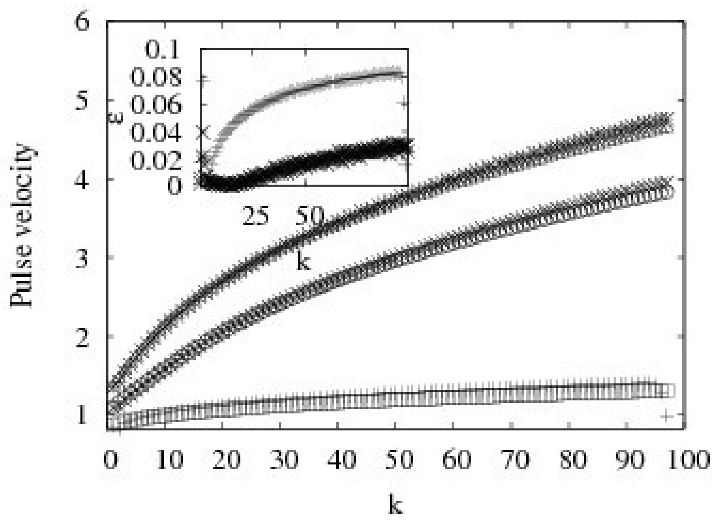
**In the case of gravity, it works well as long as chain is not too long.**

**It works well for current experimental scenarions (Herbold and Nesterenko)**




No pre-compression

Simulations and analytic results




Gravitational force

Simulations and analytic results

# AMONG OUR CURRENT INTERESTS: TWO-DIMENSIONAL GRANULAR BEDS

**So far: purely numerical**

**Hexagonal packing**

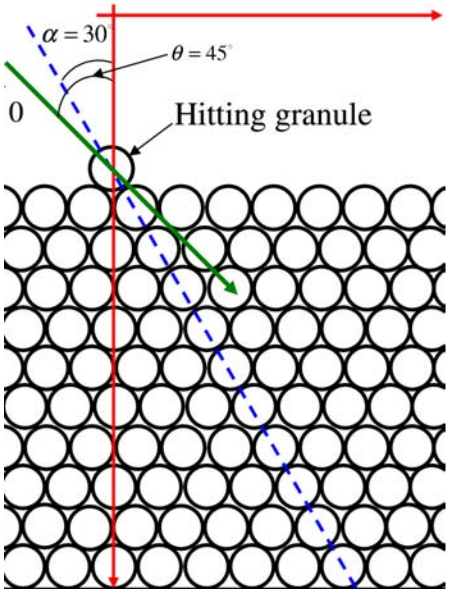
$$F_n = a_n r_{kk'} |h_{kk'}|^{3/2} + \gamma \dot{h}_{kk'}$$

**Hitting angle** $\theta$

$$F_t = -|h_{kk'}|^{1/2}(a_t \Delta s_t + \gamma_t v_t)$$

**Observation angle** $\alpha$

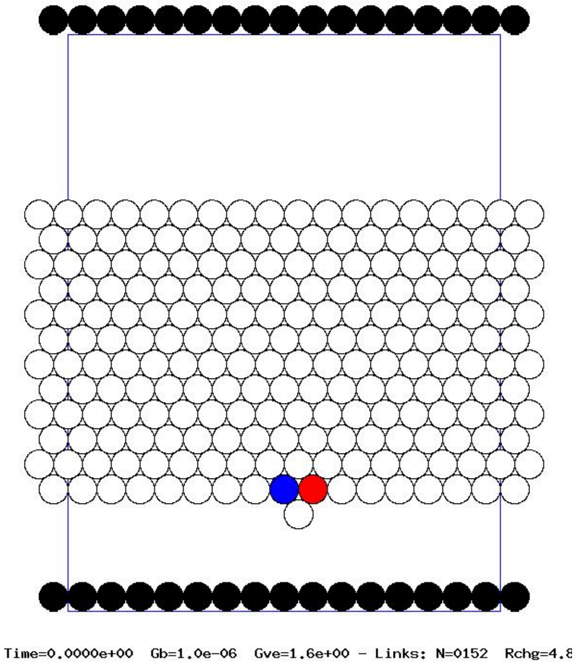
**Normal overlap** $h_{kk'} = y_k - y_{k'}$

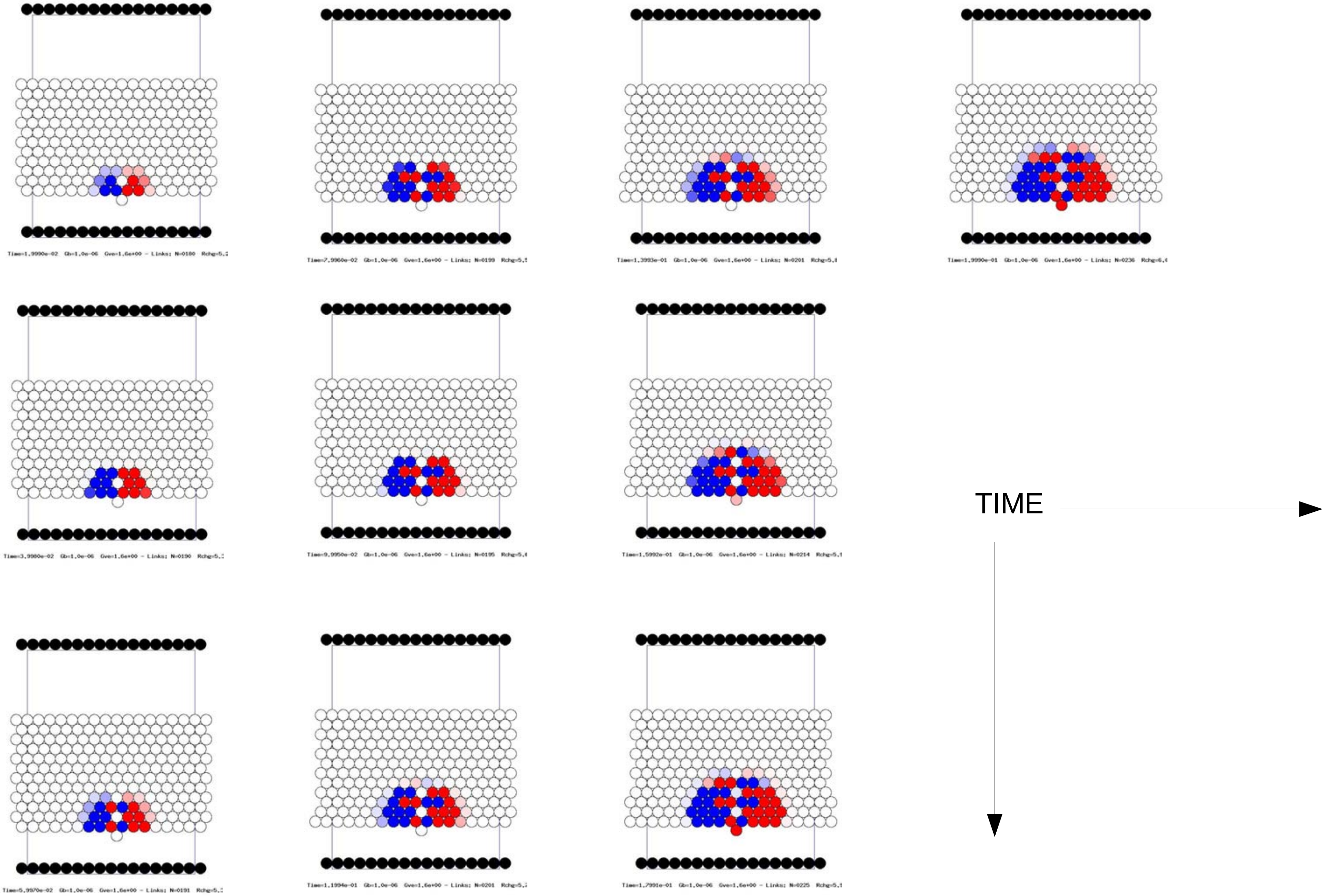



**RED: motion to the right**

**BLUE: motion to the left**

INTENSITY: magnitude of velocity




TIME
Time=1,9990e-02 Gb=1,0e-06 Gve=1,6e+00 - Links; N=0180 Rchg=5,
Time=7,9960e-02 Gb=1,0e-06 Gve=1,6e+00 - Links; N=0199 Rchg=5,
Time=1,3993e-01 Gb=1,0e-06 Gve=1,6e+00 - Links; N=0201 Rchg=5,
Time=1,9990e-01 Gb=1,0e-06 Gve=1,6e+00 - Links; N=0236 Rchg=6,
Time=3,9980e-02 Gb=1,0e-06 Gve=1,6e+00 - Links; N=0190 Rchg=5,
Time=9,9950e-02 Gb=1,0e-06 Gve=1,6e+00 - Links; N=0195 Rchg=5,
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Time=1,7991e-01 Gb=1,0e-06 Gve=1,6e+00 - Links; N=0225 Rchg=5,

# END NOTES

I hope I have convinced you that the study of pulse propagation in granular materials
Is mathematically and physically  interesting.  These systems are highly nonlinear and
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We have been particularly interested in developing theoretical tools for this purpose. Our newest results rely on a binary collision approximation that has been extremely successful in a number of chain geometries. We have developed generalizations of this approximation to deal with more complex geometries. This promises to provide a direct way to optimization.

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ONGOING WORK INCLUDES EXTENSION OF THE THEORY TO ARBITRARY PROFILES AND UNDERSTANDING ENERGY PROPAGATION IN HIGHER DIMENSIONAL GRANULAR BEDS.

## Our publications on the subject

## And the work continues......

1. Dynamics of two granules, PRE 68, 021303 (2003).

2. Pulse dynamics in a chain of granules with friction, PRE 68, 041304 (2003).

3. Self-similarity in random collision processes, PRE 68, 050103(R) (2003).

4. Pulse propagation in chains with nonlinear interactions, PRE 69, 016615 (2004).

5. Pulse velocity in a granular chain, PRE 69, 037601 (2004).

6. Velocity distribution in a viscous granular gas, PRE 71, 032301 (2005).

7. Observation of two-wave structure in strongly nonlinear dissipative granular chains, PRL 98, 164301 (2007).

8. Sort-pulse dynamics in strongly nonlinear dissipative granular chains, PRE 78, 051303 (2008).

9. Energy transport in a one-dimensional granular gas, PRE 79, 061307 (2009).

10. Pulse propagation in tapered granular chains: An analytic study, PRE 80, 031303 (2009).

11. Pulse propagation in decorated granular chains: An analytical approach, PRE 80, 051302 (2009).

12. Pulse propagation in randomly decorated chains, PRE 82, 011306 (2010).

plus preprints under review and in preparation