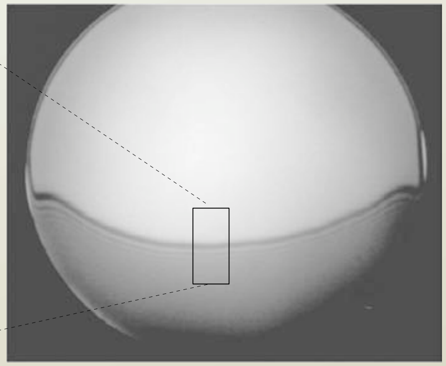
Extreme elastohydrodynamics: of films, flags, fishes and finches ....

L. Mahadevan
Engineering and Applied Sciences
Organismic and Evolutionary Biology
Harvard University



(Reiutord; 2006)

(H-Y Kim; 2006)

Extreme geometries + extreme rates

- how films heal, peel and fly
- how flags flutter
  and how fishes swim
- how tubes oscillate and birds sing

theory:
- M. Argentina
- S. Mandre

experiments:
- H-Y. Kim
- A. Mukherjee

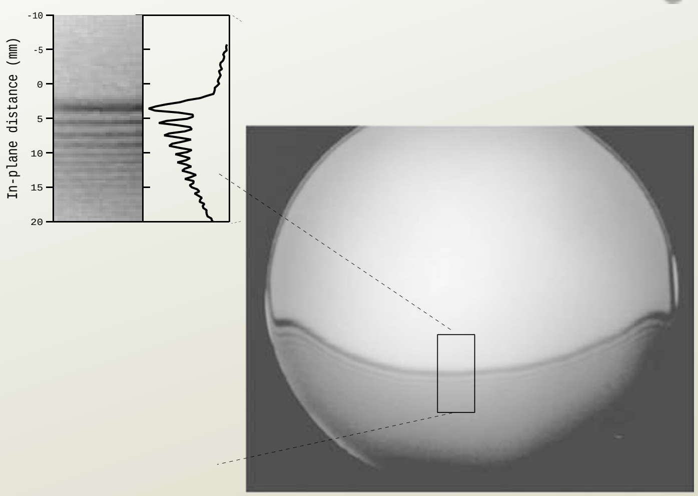
+ Lauder, Parker lab (Harvard)

Healing films

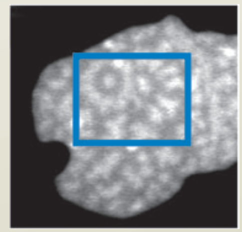
(Mandre, LM; 2010)

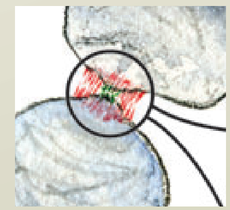
wafer bonding ...

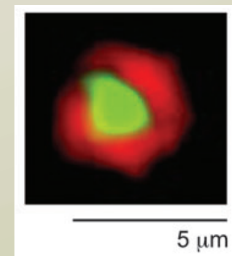
cellular recognition - bilayer adhesion ...

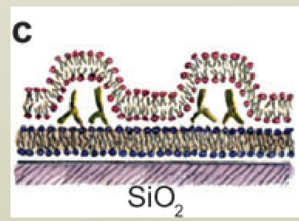


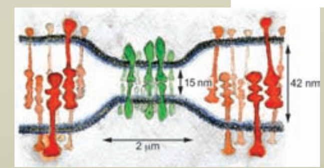
(Reiutord; 2006)





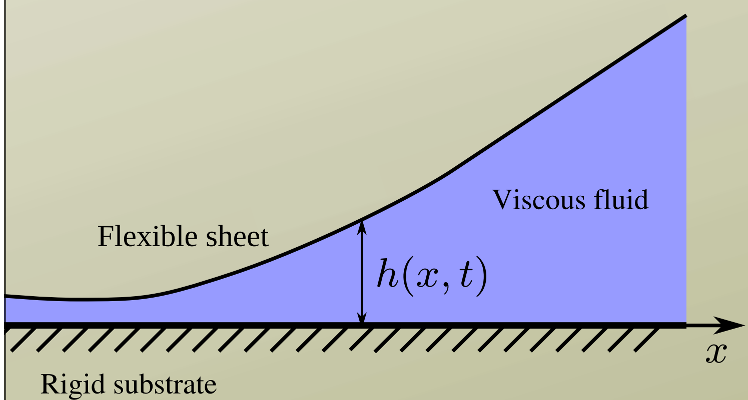






(Groves; 2004)

Q. Speed, shape of wafer/ bilayer contact line ? fluid flow is critical !



$$p(x,t) = Bh_{xxxx} + \frac{A}{\epsilon}\Phi'\left(\frac{h}{\epsilon}\right)$$

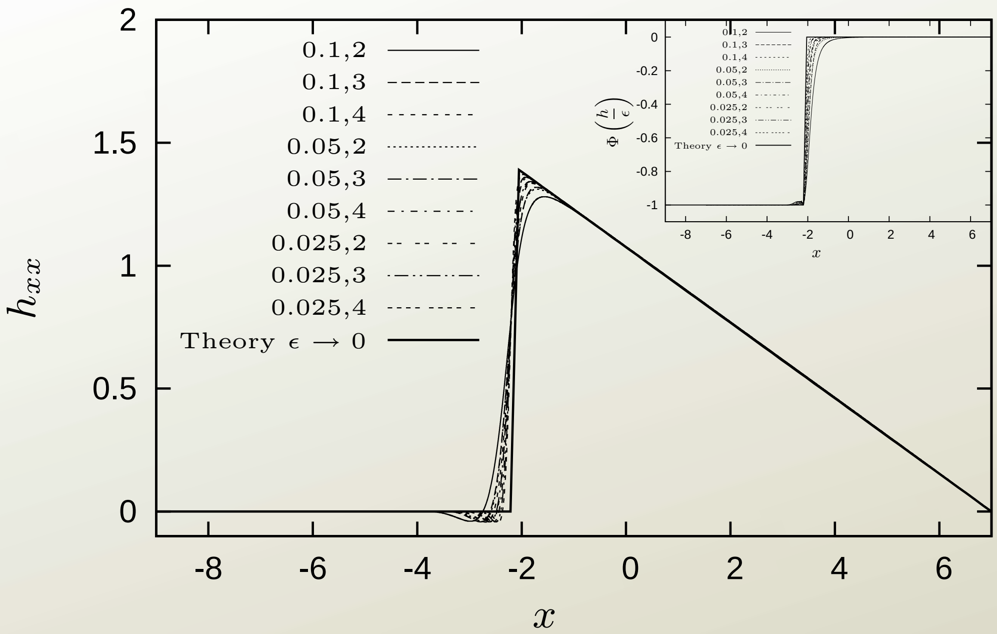
vertical force balance

$$\Phi(s) = 4\left(\frac{1}{s^{2m}} - \frac{1}{s^m}\right)$$

adhesion potential

$$12\mu h_t = \left(h^3 p_x\right)_x$$

hydrodynamics (+ continuity)



Statics ?

$$p = 0$$

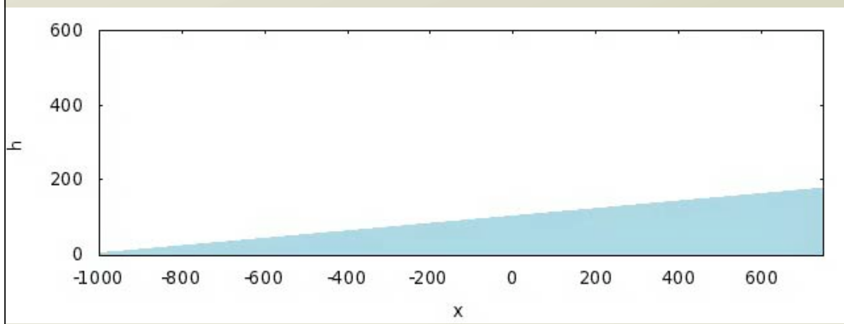
far from the contact line: $h = h_0; h_{xx} = 0$

at the contact line: $h = h_x = 0$

$$h_{xx} = \sqrt{2A/B}$$

Obreimov (1930) - measurement of adhesion !

Dynamics ?



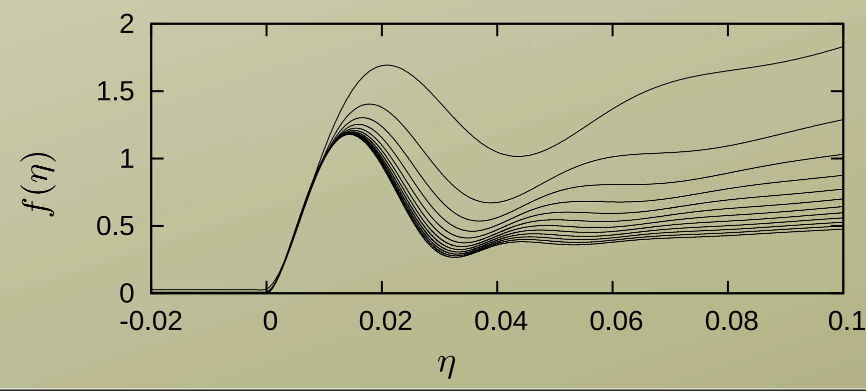
Similarity solution ?

$$h(x,t) = t^{\beta} f(\eta), \quad p(x,t) = t^{\kappa} g(\eta), \quad \eta = \frac{x - ct}{t^{\gamma}}.$$

$$\beta = 5/4; \kappa = -7/4; \gamma = 3/4$$

universal shape ... (traveling frame)

$$h \sim (x - ct)^{5/3}$$



Speed ?

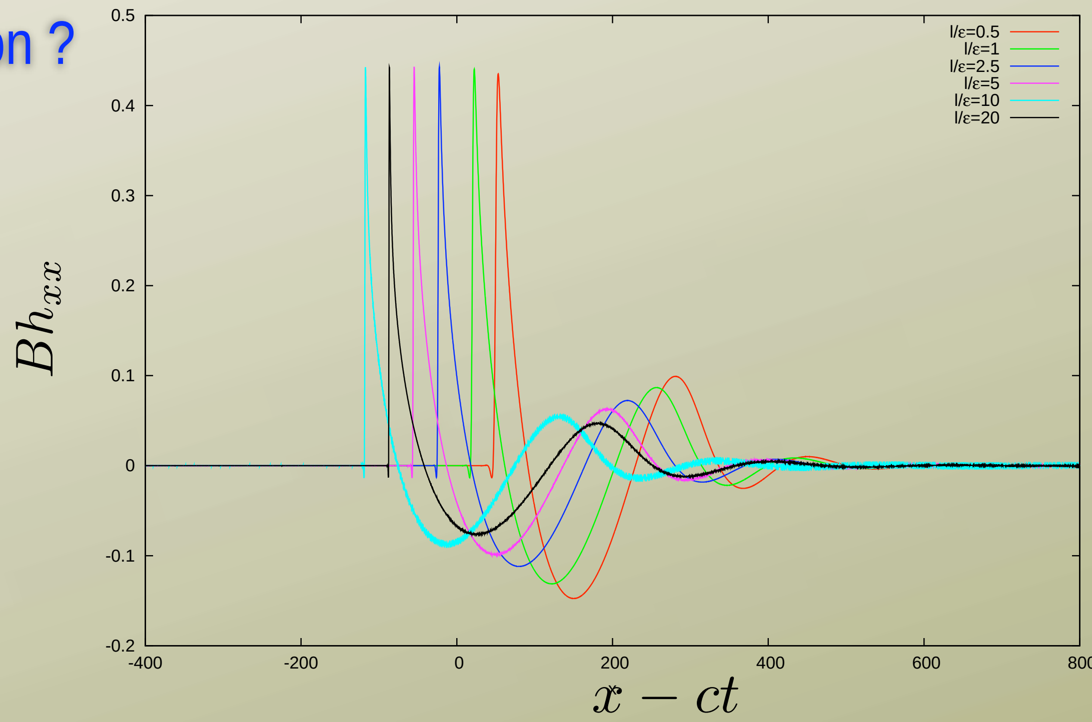
$$\underbrace{\frac{d}{dt}\int_0^L [Bh_{xx}^2}_{\text{bending}} + \overbrace{2A\Phi(\frac{h}{\epsilon})}^{\text{adhesive}}]dx = -\underbrace{\int_0^L h^3\frac{p_x^2}{6\mu}dx}_{\text{viscous}}$$

dimensionless power balance

Scaling ...

contact line condition ?

$$h_{xx} \sim 0.4\sqrt{A/B}$$

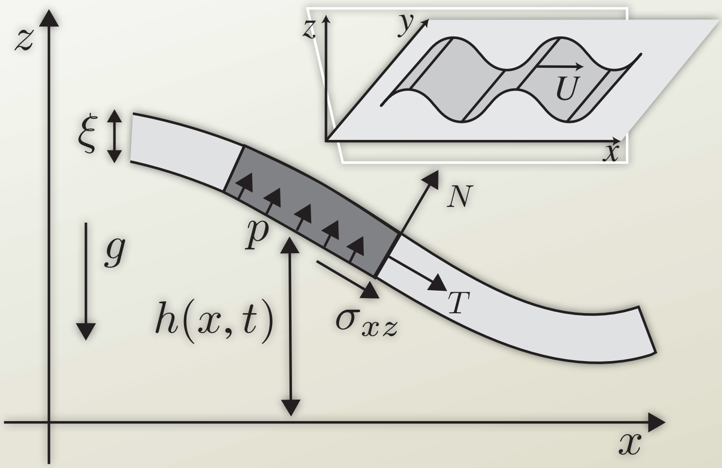


similar to static problem !

transverse stability ? dynamics and patterning at the immunological synapse ?

Wall bounded flying films - gait ?

Argentina, Skotheim, LM; PRL 2007



continuity $\quad \partial_t h + U\partial_x h - \partial_x(h^3\partial_x p) = 0$ (viscosity)

horiz. mom. $\quad \cancel{W\partial_t U} = -\int_0^1 \left(\frac{U}{h} + 3p\partial_x h\right)dx,$

viscosity

vertical mom. $\quad p = B\partial_{xxxx}h + 1 - \partial_{xx}f.$

bending gravity active torque

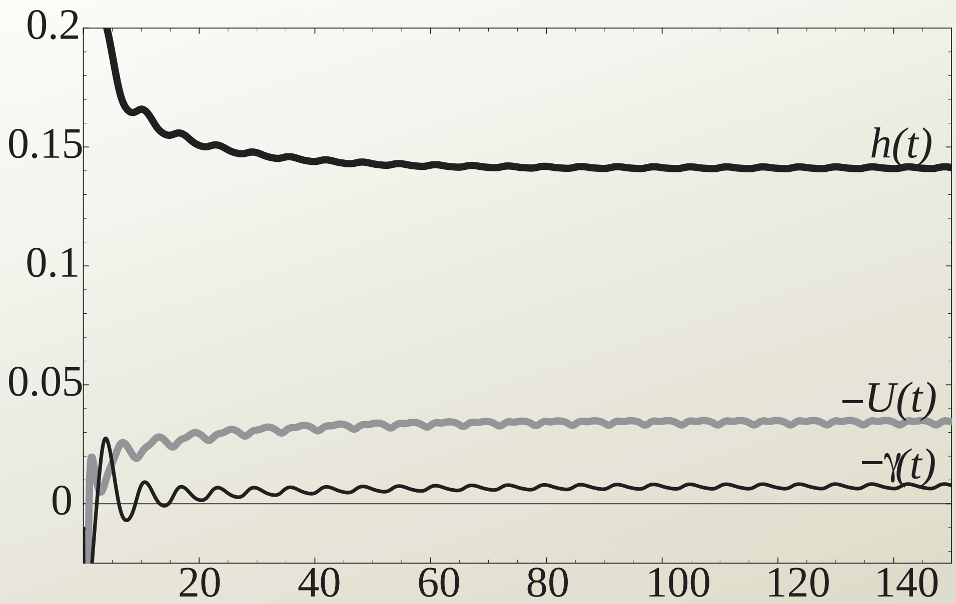
global balances: $\quad \int_0^1 p\,dx = 1 \qquad \int_0^1 p(x-1/2)dx = 0.$

Given $f(x,t)$ find $U(t), h(x,t), p(x,t)?$ Large phase space !

Semi-inverse method: $\quad h(t,x) = h_0(t) + \gamma(t)x + A\sin(\omega t - qx)$

Determine $h_0(t), \gamma(t)$

Deduce $f(x,t)$



Scaling laws:

vertical forces

$$p \sim \Delta\rho g\xi.$$

$$\mu U/h_0 \sim p\gamma$$

horizontal forces

$$\mu U\gamma \sim ph_0^3/L^2$$

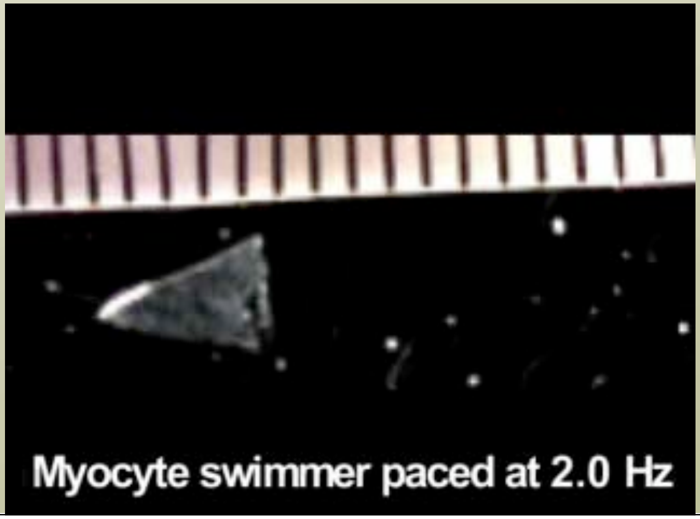
continuity

kinematics $\omega A/q \sim U\gamma L$ $\quad q \sim 1/L$

$$U \sim (A\omega)^{2/3}(\frac{\Delta\rho g\xi L}{\mu})^{1/3} \qquad h_0 \sim (A\omega)^{1/3}(\frac{\Delta\rho g\xi}{\mu})^{-1/3} \qquad \gamma \sim h_0/L$$

Comparisons ?

Facts ? Parker, Whitesides lab



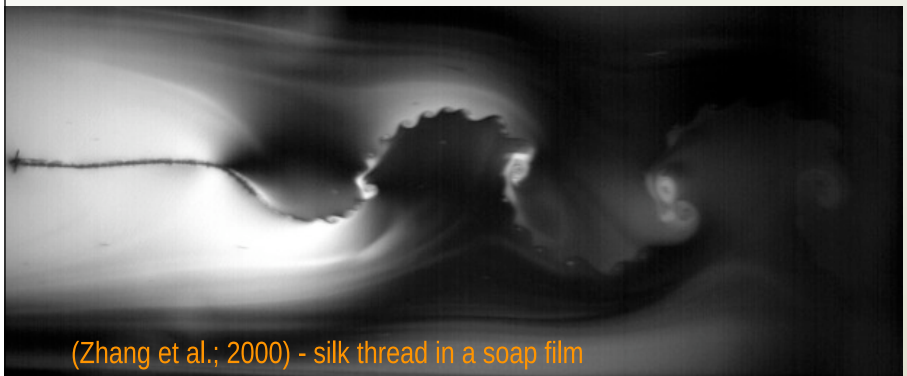
Fantasy ? Goscinny, Uderzo

Flutter of a flag

Argentina, LM; PNAS, 2005

flutter, flutter went the flag, first to the left, then to the right : H.G. Wells

Long history - Kelvin, Lamb, ... von Karman, Theoderson, Dowell, Peskin etc.



- mechanism / onset ? frequency ? wavelength ?

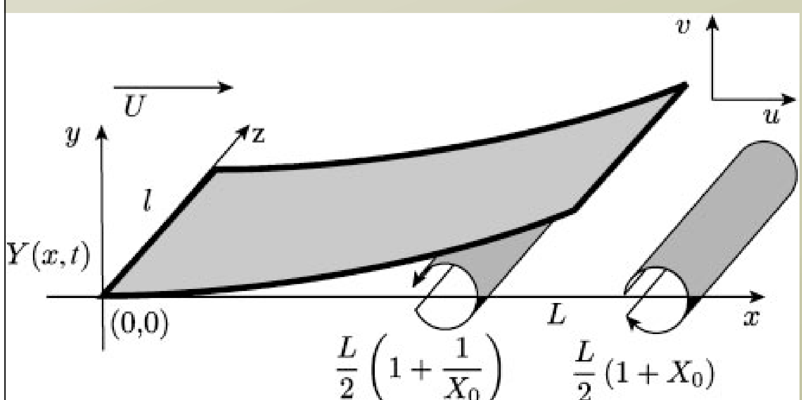
- fluid: $m_f \sim \rho_f L^2 w$ (inertia) $\quad p \sim \rho_f U^2$ (pressure)

- solid: $m_s \sim \rho_s r L w$ (inertia) $\quad B \sim E r^3 w$ (elasticity)

Parameters

$$\rho = \frac{\rho_f L}{\rho_s r} = \frac{m_f}{m_s} \qquad u_0 = \frac{UL}{r}\left(\frac{\rho_s}{E}\right)^{1/2} = \frac{\tau_s}{\tau_f} \qquad Re = \frac{UL}{\nu} \sim 10^4$$

vortex shedding ?



Elasticity (linearized)

$$mY_{tt} = -BY_{xxxx} + l\underline{\Delta P}$$

inertia elasticity pressure

boundary conditions

$$Y(t,0) = 0, \quad Y_x(t,0) = 0,$$
$$Y_{xx}(t,L) = 0, \quad Y_{xxx}(t,L) = 0.$$

Aerodynamics (Theodorsen 1935)

$$\Delta(\phi_{nc} + \phi_\gamma) = 0$$

non-circulatory circulatory

$$\nabla\phi\cdot\mathbf{n}|_{Y=0} = Y_t + UY_x,$$
$$\nabla\phi \to 0 \qquad r \to \infty.$$

Bernoulli equation (linearized):

$$\Delta P = -2\rho_f(\partial_t + U\partial_x)(\phi_\gamma + \phi_{nc})$$

Kutta condition:

$$\partial_x(\phi_\gamma + \phi_{nc})|_{x=L} \quad \text{is} \quad \text{finite}$$

Dimensionless system ?

added mass

elasticity

Theodorsen function - vortex shedding

$$(1 + \rho n\,(s))\, h_{\tau\tau} = -h_{ssss} - (\rho u_0^2 h_s + \rho u_0 h_\tau) C[\gamma] f(s)$$

inertia

space irreversible

time irreversible

For a heavy flag in a fast flow $\rho \to 0; \rho u_0 \to 0 \quad \rho u_0^2 \to K, C[\gamma] \to 1$

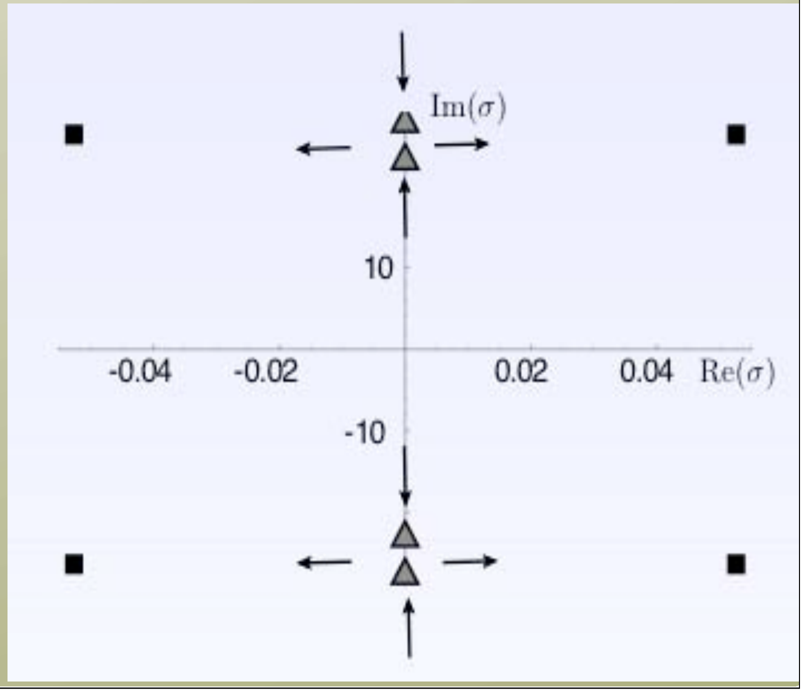
$$h_{\tau\tau} = -h_{ssss} - \rho u_0^2 h_s f(s)$$

time-reversible !

stability analysis ?

$$h(s,\tau) = \zeta(s) e^{\sigma\tau}$$

- 1:1 resonance



Mechanism ?

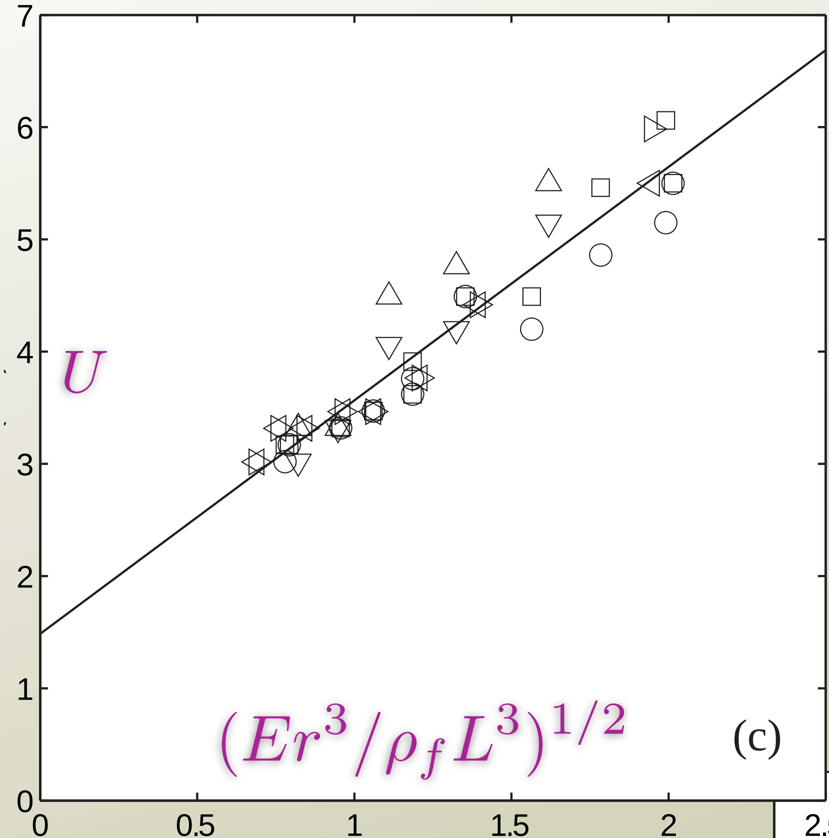
H-Y Kim, LM(2010)

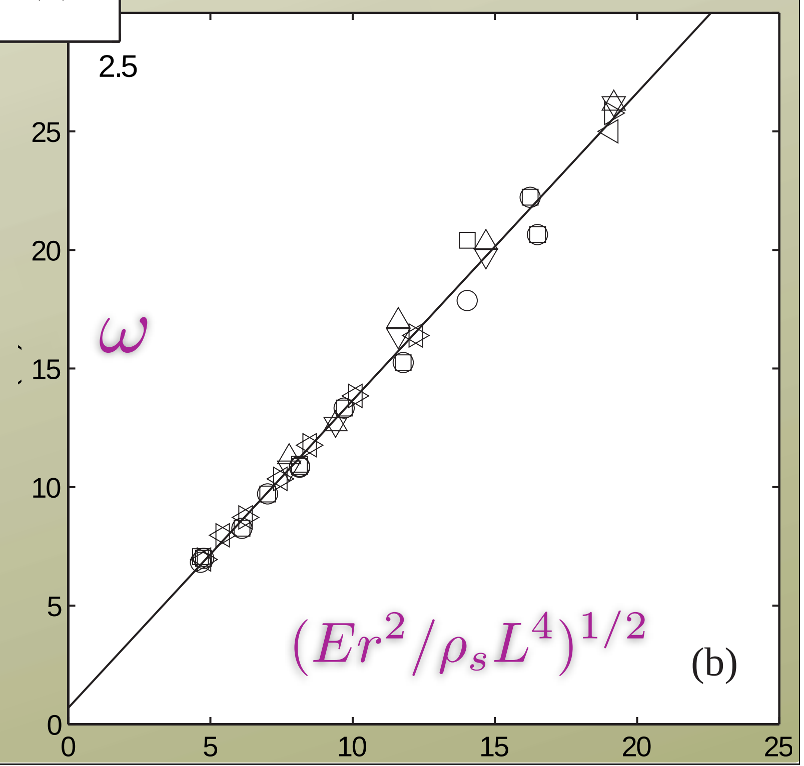
frequency “confusion”

$$\lambda \sim L$$

$$U \sim (\frac{E}{\rho_f})^{1/2}(\frac{r}{L})^{3/2}$$

$$\omega \sim (\frac{E}{\rho_s})^{1/2}(\frac{r}{L^2})$$





Other elastohydrodynamic instabilities ?

- low Re ? confined flows ?

Mandre, LM; P. R. Soc. (A) (2009)

1:1 resonance !

- weak effect of fluid
- clustered frequencies
- hydrodynamics is slaved

Unconfined swimming - gait ?

Argentina, LM; 2010

Long history - Gray, Taylor, Lighthill, Wu ... - prescribed kinematics !

Transverse mom. balance (local)

$$h_{\tau\tau} = -[a(s)h_{ss}]_{ss} \quad + \Delta P \quad + \rho u_0^{3/2}\frac{4}{3\sqrt{Re}}[b(s)h_s]_s \quad + F_{ss}$$

inertia elasticity pressure boundary layer drag internal torque

pressure $\Delta P = -(\rho u_0^2 h_s + \rho u_0 h_\tau)C[\gamma]f(s) - \rho n(s)h_{\tau\tau}$

$$F(s,\tau) = g(s)\cos\omega^*\tau$$

muscular torque

Longitudinal mom. balance (global)

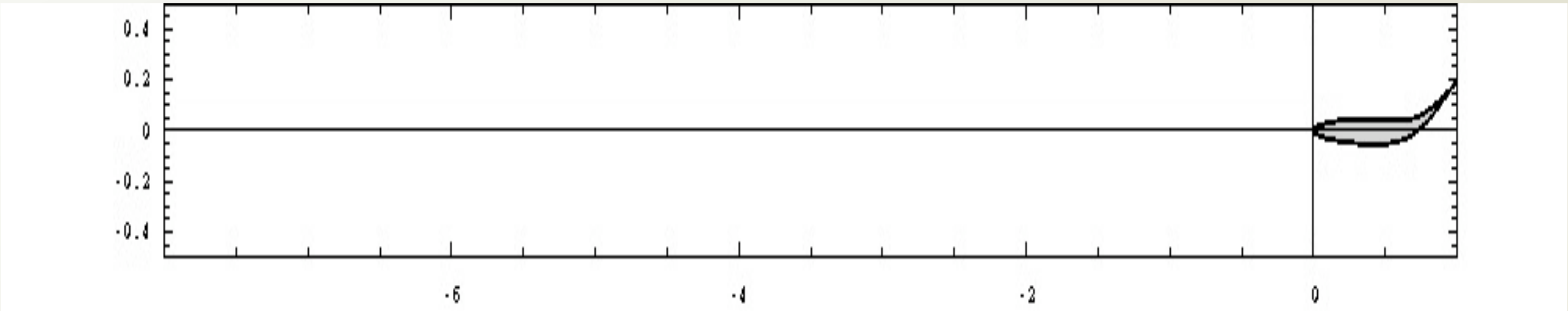
$$\int_0^1 \Delta P h_s ds - \frac{4}{3\sqrt{Re}}\rho u_0^{3/2} = \cancel{u_{0\tau}}$$

pressure X slope boundary layer drag

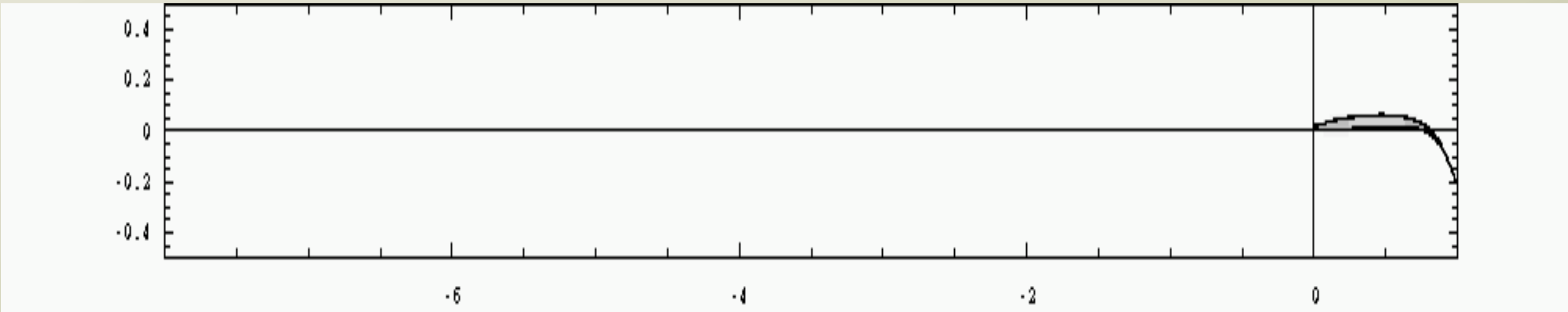
$u_0, h(s)$ Nonlinear eigenvalue problem

Carangiform mode:

small added mass



large added mass

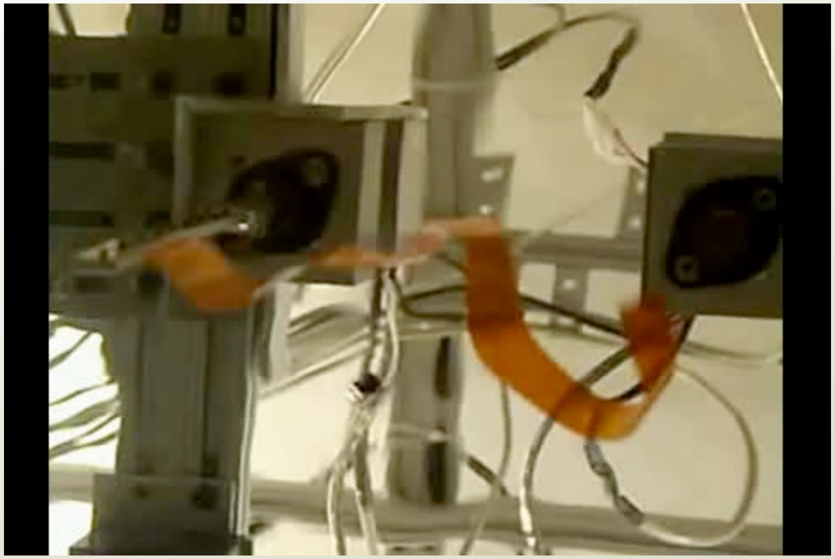


Resonant interaction ? ... Flapping fish ?

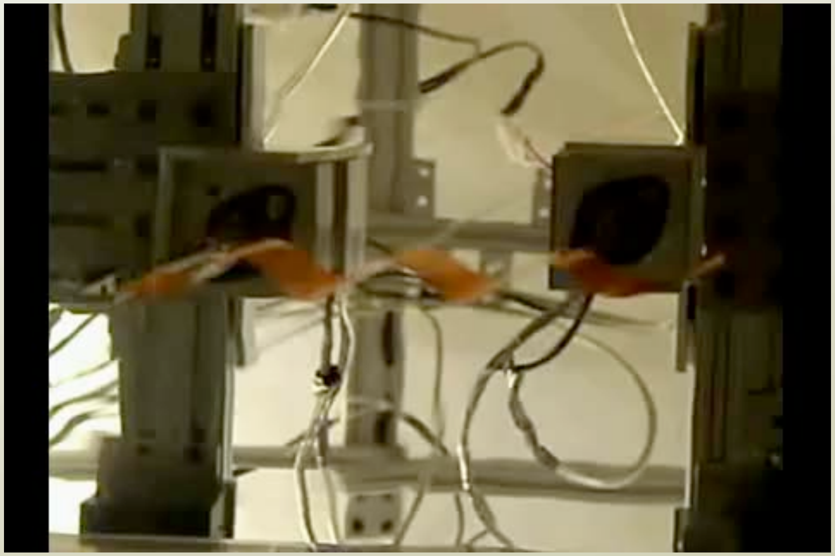
Paralyzed Trout in a wake (Liao et al., 2003)

Swimming flag ?

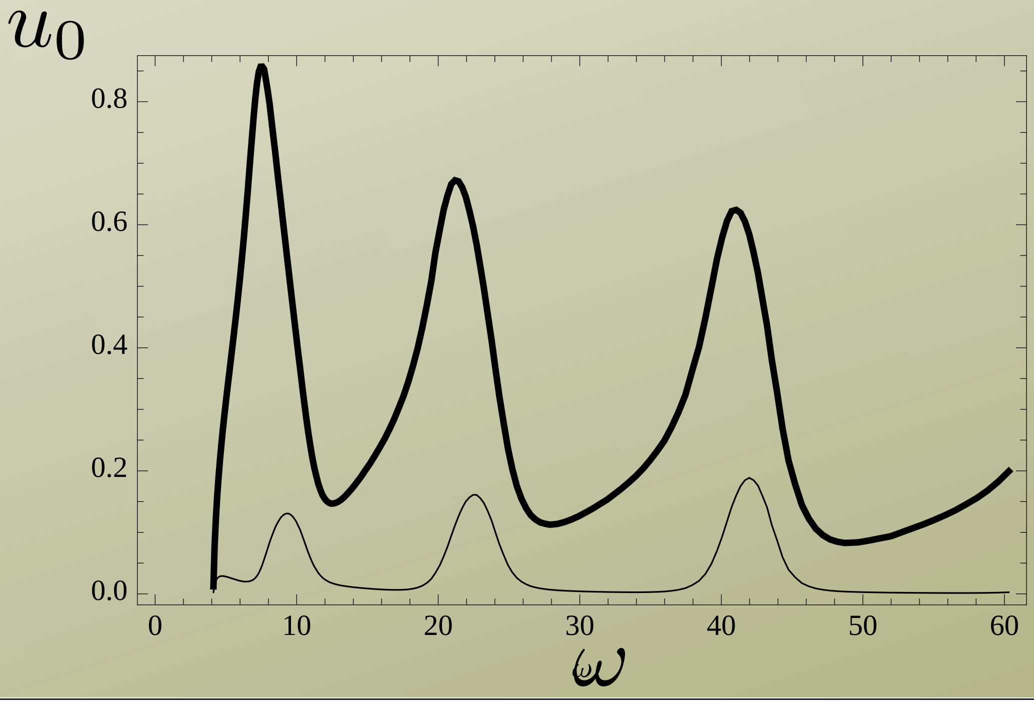
J. Lim (Lauder lab)



Oscillating pivot



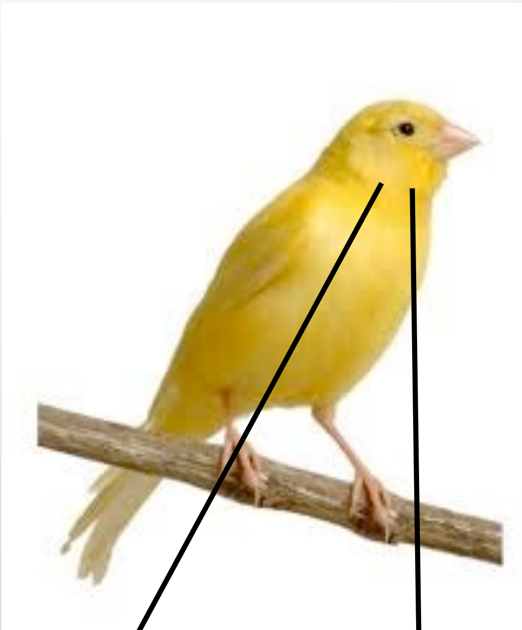
Oscillating pivot + flow (to cancel thrust !)



$$h(0,t) = \frac{A}{2}\sin\omega t$$

Resonance !

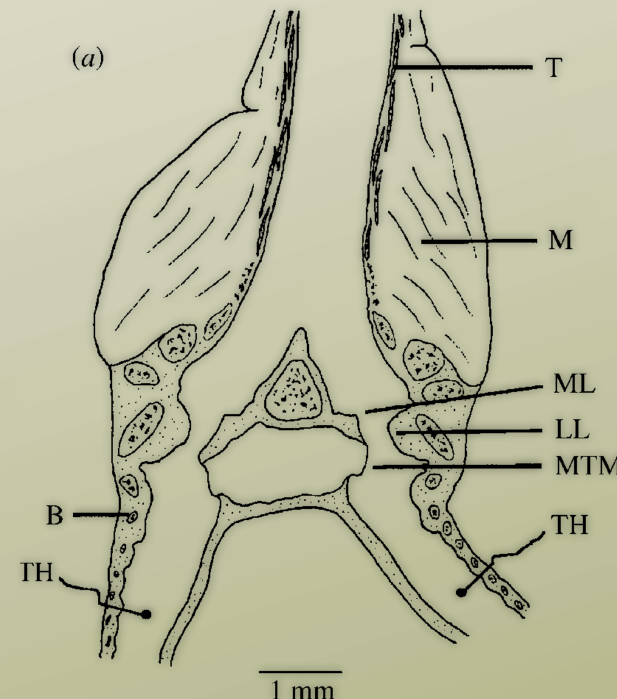
Birdsong ?



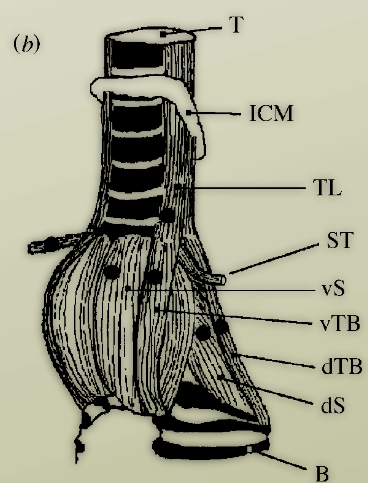
Q. - Ethology ? Behavior ?
- (Acoustic) ecology ?
- Neuroscience ? Memory, learning, adaptation ...

- Dynamics and control

of an extended neuromuscular system ?



SYRINX



- elastohydrodynamic flutter

+ controls + filters

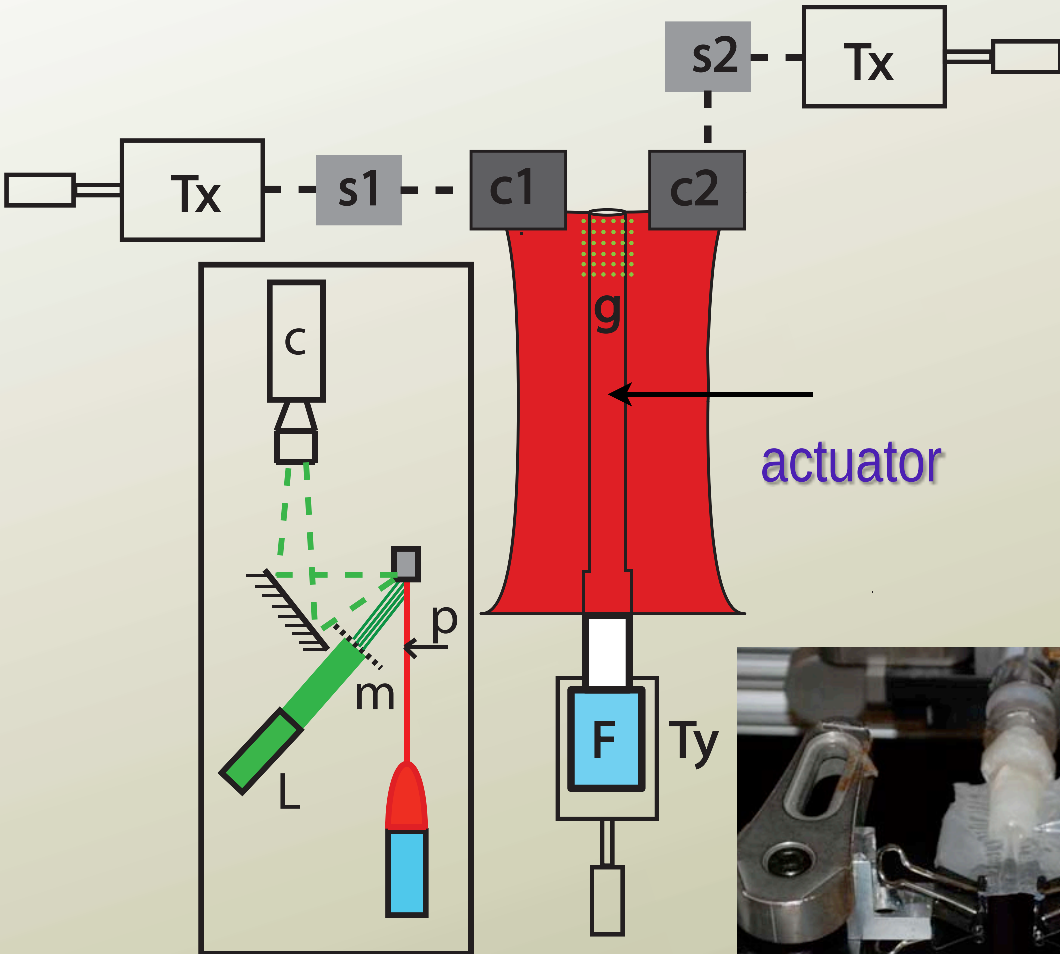
Short (physical/mathematical) history - N. Fletcher, Ishikawa-Flanagan (1970s) +
M. Fee, R. Suthers, Goller etc. - Physics of birdsong - G. Mindlin

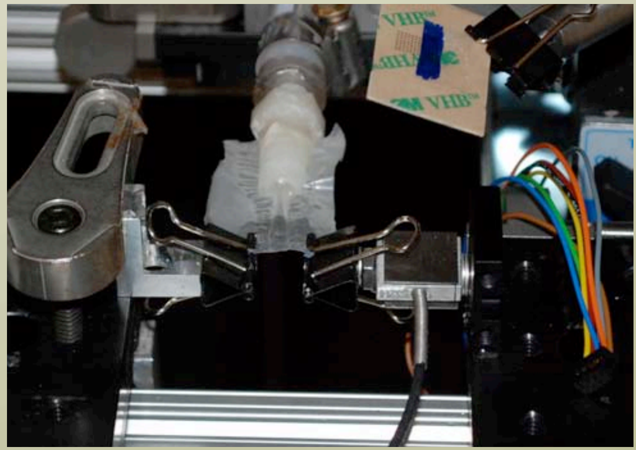
Parameters - pressure, flow rate, passive tension, active muscular force ....

A minimal system - physically ? mathematically ?

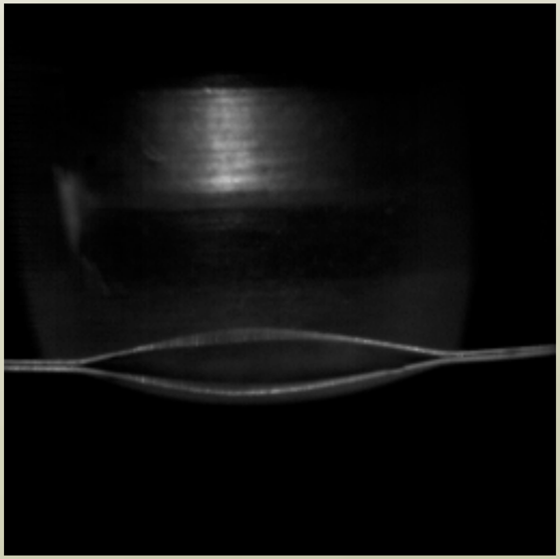
Mukherjee, LM; 2010

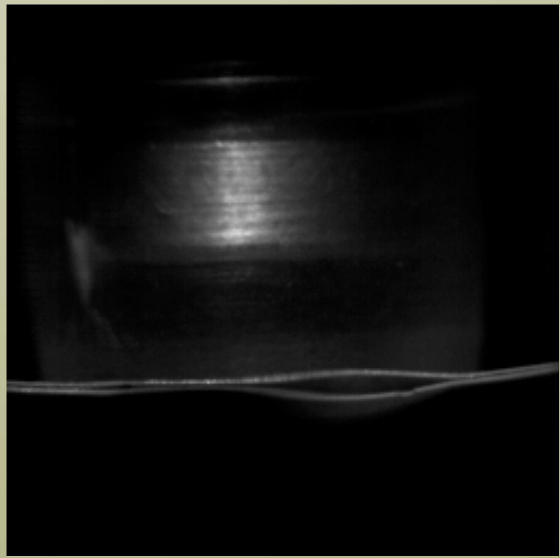
A stretched rubber tube + a single displacement actuator ... !



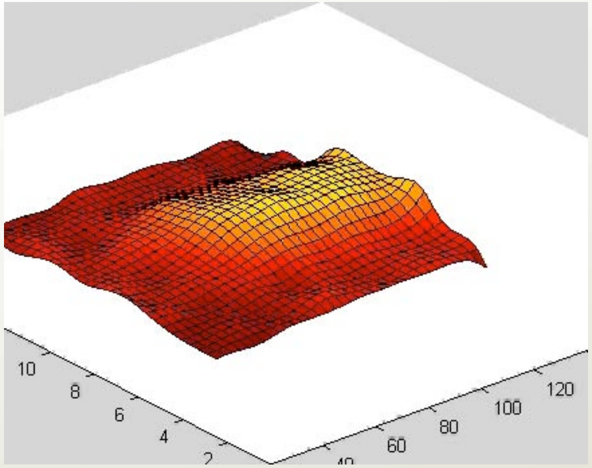


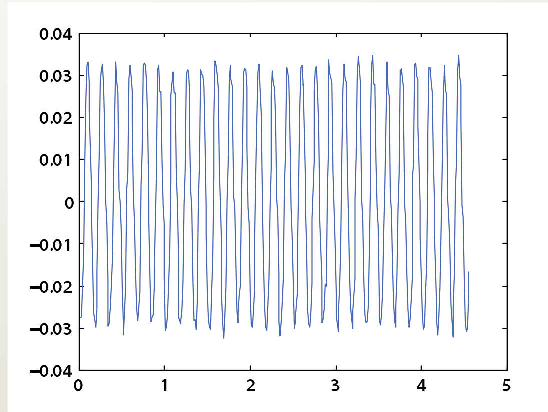
Actuated flutter modes



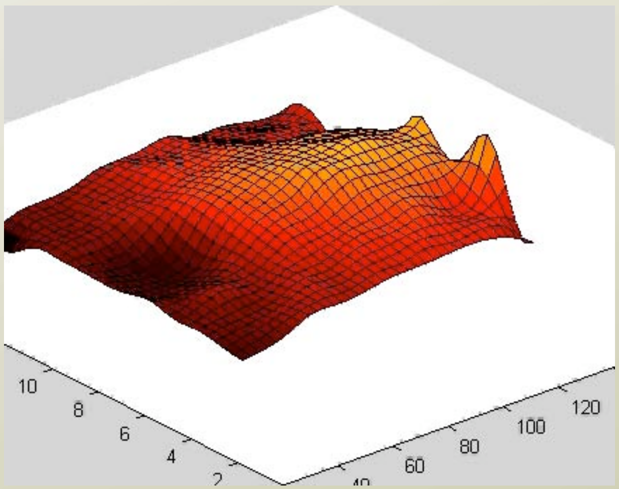


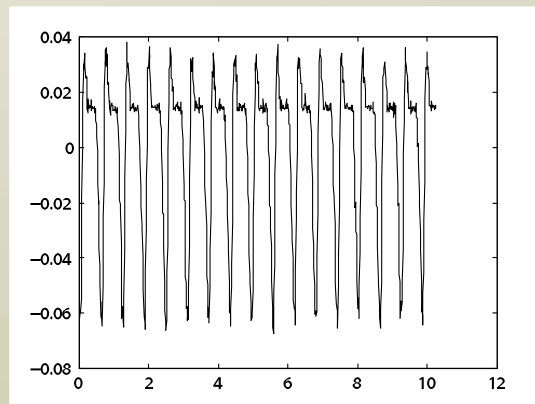
- high speed imaging
- audio recording
- control of tensions, flow rate, actuation





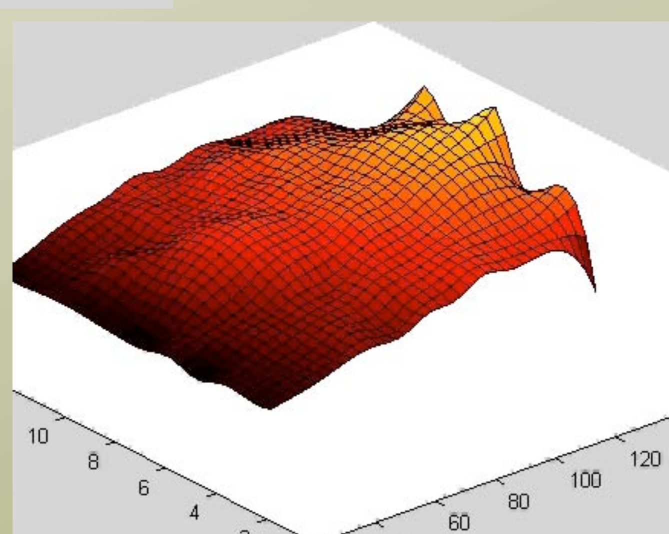
Pure tone

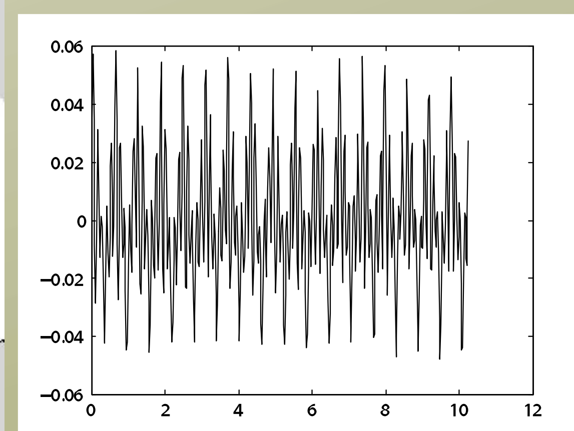




Solitary wave
(excitable)

Chatter - chaos ?





Bird song via optimal control of an oscillator ?

Mandre, LM; 2010

song $u(t)$ mimic $x(t)$ $\ddot{x} + r(t)\dot{x} + \omega^2(t)x = 0,$ (1)

(or variant ...)

$r(t), \omega(t)$ control (flow rate, piston displacement, tension)

slowly varying .... relative to base frequency (kHz !)

- muscular response ~ 10 ms or larger !

$$\underset{r(t),\omega(t)}{\text{minimize}} \int_0^T (x - u(t))^2 + W_1 \left(\frac{d\omega}{dt}\right)^2 + W_2 \left(\frac{dr}{dt}\right)^2 dt$$

subject to (1)

large !

Global optimization !

Sequence of local optimization problems:

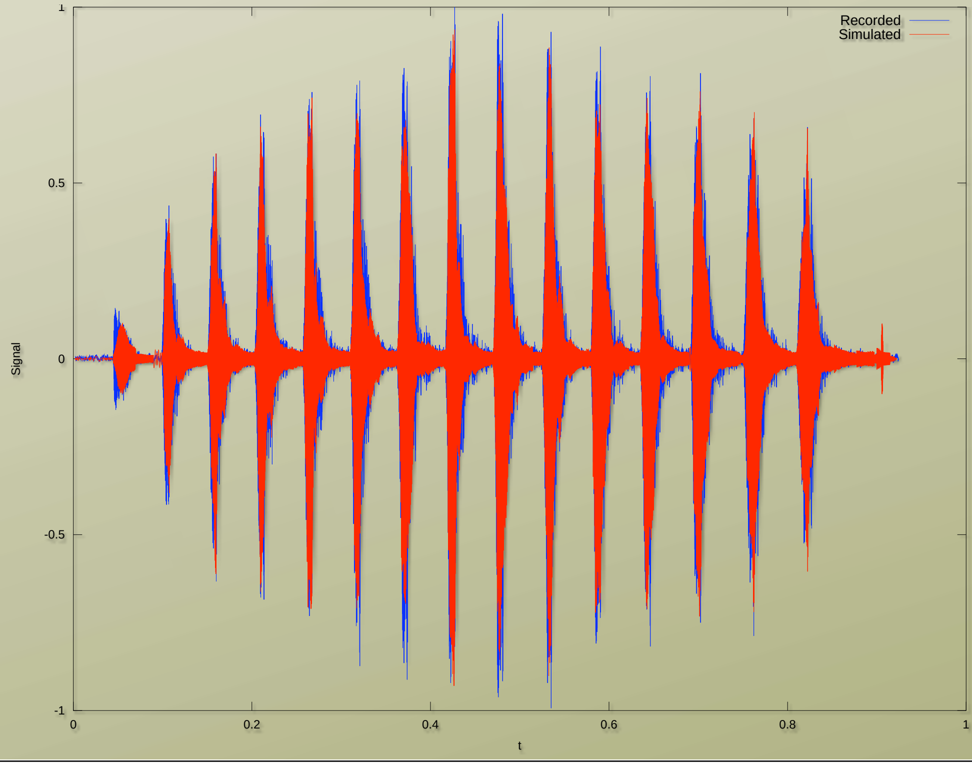
$$\underset{r_k,\,\omega_k}{\text{minimize}} \quad \int_{t_{k-1}}^{t_k} (x - u(t))^2 + \tilde{W}_1 \left(\omega_k - \omega_{k-1}\right)^2 + \tilde{W}_2 \left(r_k - r_{k-1}\right)^2 dt$$

$r(t) = r_k; \quad \omega(t) = \omega_k$ $\qquad$ $A_k; B_k$ continuity of $x(t), \dot{x}(t)$

Numerical optimization - Matlab ... (or variants)

E. *vireo*

pure tone song ...



The neglected borderlands between two branches of knowledge
is often that which best repays cultivation ...

.... the greatest benefits may be derived from a cross-fertilization of the sciences

- Lord Rayleigh, 1884.

elasticity + hydrodynamics + biology ....

- healing films - kinetics ++ ?

- flag flutter - 3d effects ? far from onset ? rippling instabilities ?

- fishes and flying films - energetics ? optimality ? biomimetics ?

- bird song - controlling the nonlinear dynamical system ?