IP1

Amphiphilic Morphology: Lipids, Proteins, and Entropy

The self organization of phospholipids into membranes is fundamental to the origin of life, allowing for protection of internal structure while necessitating machinery to open pores. Evolutionary pressure has designed a myriad of controls in the guise of surface proteins that adjust the entropy of the lipid-solvent interactions, raising and lower energy barriers to membrane fusion, budding, endocytosis, and inducing curvature vectors that encode for fenestration and helical structures. All of these actors orchestrate the delicate dynamics of the endoplasmic reticulum (ER) and associated Golgi apparatus, biology's original coherent structures. We embed the ER and its dynamics within an energy minimization problem whose coherent dynamics play out on a huge center-stable stage. We given an overview of the bifurcation structure, including a mechanism for the onset of morphological complexity observed in synthetic amphiphilic polymers, and describe the template ER problem: the evolution and budding of lipid droplets.

Keith Promislow

$\mathbf{IP2}$

Pilot-wave Hydrodynamics: From Chaotic Dynamics to Quantum-like Statistics

A decade ago, Yves Couder in Paris discovered that droplets walking on a vibrating fluid bath exhibit several features previously thought to be exclusive to the microscopic, quantum realm. These walking droplets propel themselves by virtue of a resonant interaction with their own wavefield, and so represent the first macroscopic realization of a pilot-wave system of the form proposed for microscopic quantum dynamics by Louis de Broglie in the 1920s. New experimental and theoretical results allow us to rationalize the emergence of quantum-like behavior in this hydrodynamic pilot-wave system in a number of settings, and explore its potential and limitations as a quantum analog.

John W. Bush

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IP3

Partial Differential Equations as Models for Social Complex Systems

Gentrification, urban crime, and rioting activity are all important social issues that have become increasingly relevant as technological advances and a growing population have increased the flow of people and information. Much effort has gone into understanding these topics from a variety of scientific methodologies, including through mathematical modeling via partial differential equations (PDEs), which have the power to shed light on the dynamics of these complex social systems by identifying spatio-temporal patterns that these systems demonstrate. In this talk I will present various PDE systems that have been introduced to model urban crime, rioting activity, and gentrification, including new models that have been recently proposed that have allowed for a deeper intuition and understanding of these social phenomena. The analysis of the these systems has required advances in numerical and theoretical techniques, which are interesting in their own right as generic techniques that can be employed in the analysis other PDE systems. Finally, I will highlight how the mathematical analysis of these systems can be combined with data from real-world events to provide a comprehensive picture of the phenomena, and further validate the ability of mathematical results to successfully predict and shed understanding on observed data.

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IP4

On the Way to the Limit: Oscillatory Stiffness and Low Frequency Dynamics in Climate and Weather Prediction

One of the most well-known breakthroughs in scientific computing came just after WWII when a group of mathematicians and scientists came together to create the world's first numerical weather prediction on one of the world's first computers - ENIAC. One of the most important lessons learned from that experience was that there is an intimate relationship between the mathematical structure of the governing equations, their numerical approximation, and understanding their dynamics. Building on that history, I will discuss one of the mathematical issues that leads to computational limitations for many different types of physical phenomenon including climate and weather prediction oscillatory stiffness in the PDEs from time-scale separation that leads to low-frequency dynamics. I will discuss some of the first mathematical discoveries from geophysical fluid dynamics about how nonlinear phenomenon gives rise to low-frequency solutions and the relationship to fast singular limits studied in PDEs analysis and numerical analysis. I will discuss some of the key mathematical ideas behind new time-parallel numerical integrators, where we use frequency-averaging to approximate the low frequency dynamics.

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IP5

Multi-scale Problems of Material Design in Sustainable Energies

The higher demand for sources of sustainable energies poses tremendous challenges in production and storage capabilities as well as the restructuring of existing and the creation of smart networks for transport of distributed energy. Mathematics plays a key role in understanding the complex problems that arise in exploiting underlying multiscale structures and processes. In this talk I will focus on the material science aspect and discuss several research topics that illustrate the multi-scale nature of material design in photovoltaics and highlight the impact of thin-film nano-structures on light harvesting and charge transport at complex interfaces. Similar mathematical problems on the interplay of material design and battery function will also be considered.

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IP6

Nonlinear Geometric Optics and Applications to Stable Singularity Formation

Quasilinear hyperbolic systems are evolution-type PDEs that arise in many physical and geometric contexts. In this talk, I will describe some of my recent work on stable singularity formation in solutions to large classes of such systems. The methods have robust features and apply in particular to systems in multiple spatial dimensions with multiple speeds of propagation. The proofs rely on new geo-analytic constructions, notably the development of a theory of nonlinear geometric optics for transport operators that is dynamically adapted to the singularity. I will also highlight some important open problems and connect the results to the broader goal of obtaining a rigorous mathematical theory modeling the long-time behavior of solutions. Some of the works I will discuss are joint with G. Holzegel, J. Luk, and W. Wong.

Jared Speck

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IP7

The Rainbow of Spatio-temporal Dynamics in Nonlinear Optics: The Story of Multi-color Light Filaments, Vortices and Other Patterns and the Mathematics Behind It

Since the first observation of a nonlinear process in light matter interaction in 1961, better lasers and designs of photonic structures have opened new ways to explore nonlinear phenomena with many important technological applications. In this talk we will focus our attention to spatiotemporal dynamics and coherent modes described by nonlinear Schrödinger-like equations. While the presentation will center on models and experiments of light filament propagation in air, we will also discuss recent optical experiments in quadratic media, multi-mode fibers and fiber arrays; most in need of a fresh theoretical formulation.

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$\mathbf{IP8}$

Propagating Waves in Nonlocal Neural Media

Traveling waves are among the most commonly spatiotemporal dynamics observed in the recording of electrical activity in the brain. They range from stimulus evoked waves, propagation of pathological behavior such as epilepsy and migraine auras to more complex patterns such as spiral waves and other types of rotating waves. In this talk, I will first discuss the different classes of traveling waves, mainly divided between waves in excitable media and so-called phase waves. Interactions in a piece of neural tissue are not mediated by simple diffusion, but rather, nonlocal connections that are mediated by convolutions in space. I will discuss recent mathematical and computational work on the existence of wave fronts, wave pulses, and nonlocal phase models. I will relate the mathematical results to some experimental findings and discuss some open problems.

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$\mathbf{SP1}$

Martin D. Kruskal Prize Lecture - On the Mathematical Theory of Graphene and its Artificial Analogues

Graphene is a two-dimensional material made up of a single atomic layer of carbon atoms arranged in honeycomb pattern. Many of its remarkable electronic properties, e.g. quasi-particles (wave-packets) that propagate as massless relativistic particles and topologically protected edge states, are closely related to the spectral properties of the underlying single-electron Hamiltonian: -Laplacian + V(x), where V(x) is a potential defined on the plane with the symmetries of a hexagonal tiling of the plane. Taking inspiration from graphene, there has been a great deal of activity in the fundamental and applied physics communities related to the properties of waves (photonic, acoustic, elastic,) in media whose material properties have honeycomb symmetry. In this talk l will review progress on the mathematical theory. In particular, Ill discuss propagation of wave-packets in bulk graphene (the infinite twodimensional honeycomb structure) and of robust "edge states along line defects (both sharp terminations of the bulk and domain walls) in graphene-like structures. Finally, I'll discuss ongoing work on metastable edge states which slowly radiate their energy from the edge into the bulk. The latter is a question in "beyond all orders asymptotics", a class of problems to which M.D. Kruskal made seminal contributions.

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CP1

Optical Solitons in Nematic Liquid Crystals: Model with Saturation Effects and Without Smallness Condition on the Deviation Angle

We present a 2-D system that couples a Schrödinger evolution equation to a nonlinear elliptic equation that models the propagation of a laser beam in a nematic liquid crystal.

$$\begin{aligned} \partial_z u &= \frac{1}{2} i \nabla^2 u + i \gamma (\sin^2(\theta_0 + \theta) - \sin^2(\theta_0)) u \\ \nu \nabla^2 \psi &= \frac{1}{2} E_0^2 \sin(2\theta_0) - \frac{1}{2} (E_0^2 + |u|^2) \sin(2(\theta_0 + \theta)) \end{aligned}$$

This system arises in experimental devices designed by G. Assanto and collaborators. The nonlinear elliptic equation, the second one, describes the response of the director angle to the laser beam electric field. In a previous work we showed a nonlinear approximation to this system given by a simpler non linear system, under a smallness condition on θ . In this work we present an analysis of the nonlinear system by using of nonlinear techniques. We show the existence of the local and global solutions of this system and that the deviation θ of the director field, remains bounded in $[0, \pi/2 - \theta_0)$ (saturation effect). And in addition, we never require the smallness of θ . At the end, for sufficiently large L^2 -norm for incident laser u, we show the existence of energy minimizing optical solitons with radial, positive and monotone profiles.

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CP1

Global Lagrangian Solutions of the Camassa-Holm Equation

In this talk, we construct global weak conservative solutions of the Camassa-Holm(CH) equation on the periodic domain. Using the geometric approach on PDEs pioneered by V. Arnold, we first express the equation in Lagrangian flow variable η and then transform it using a change of variable $\rho = \sqrt{\eta_x}$. The new variable removes the wave breaking singularity of the CH equation, and we obtain both global weak conservative solutions and global spatial smoothness of the Lagrangian trajectories of the CH equation. This work is motivated by J. Lenells who proved similar results for the Hunter-Saxton equation using the geometric interpretation.

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CP1

Steady Three-dimensional Ideal Flows with Nonvanishing Vorticity in Domains with Edges

We consider steady, inviscid, incompressible flows governed by the Euler equations. In two dimensions these can be studied using a stream function and three-dimensional irrotational flows can be understood through the introduction of a harmonic velocity potential. However neither of these approaches are possible if we seek a three-dimensional flow with nonvanishing vorticity. An earlier result by Alber [H. D. Alber, Existence of threedimensional, steady, inviscid, incompressible flows with nonvanishing vorticity, Math. Ann. 292, pp. 493-528, (1992)] shows the existence of such flows with H^3 regularity in domains with a smooth boundary. These flows are found by perturbing a given reference flow using boundary conditions on the vorticity. The aim of this paper is to extend Alber's result to a type of generalized cylinders, which are domains bounded by three smooth surfaces meeting at right angles along two edges (for example a finite cylinder with a general smooth cross section). A similar existence and regularity result for the generalized cylinders requires some additional compatibility conditions to be imposed on the given data. These conditions are given explicitly.

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$\mathbf{CP2}$

Coherent Vortex Detection from Particles Trajectories Analysis

We study the transport properties of mesoscale eddies (i.e. vortices between about 10 and 500 km in diameter) over a finite time duration. While these oceanic structures are well known to stir and mix surrounding water masses by their swirling motion, they can also carry and transport organic matter and marine in a coherent manner. Here we are particularly interested in those that remain coherent, despite the chaotic nature of the system. Existing techniques to analyze such Coherent Vortices rely on the classical geometric theory of invariants of the CauchyGreen strain tensor, where closed orbits of line fields associated with this later are defined as vortices. Others define coherent vortices from vorticity, by calculating its averaged deviation. In this work, we take a very different approach, based upon a rigorous analysis of particles trajectories. Our method identifies coherent vortices and their centers acting as observed attractors for light particles in an automatic manner. We illustrate our new method on ocean surface velocities derived from satellite altimetry.

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CP2

Modeling Ultrashort Electromagnetic Pulses with a Generalized Kadomtsev-Petviashvili Equation

In this talk I propose a properly scaled model for the nonlinear propagation of intense, ultrashort, mid-infrared electromagnetic pulses (10-100 femtoseconds) through an arbitrary dispersive medium. A physical derivation results in a generalized Kadomtsev-Petviashvili (gKP) equation. In contrast to envelope-based models such as the Nonlinear Schrödinger (NLS) equation, the gKP equation describes the dynamics of the field's actual carrier wave. It is important to resolve these dynamics when modeling ultrashort pulses. I proceed by sketching a proof of sufficient conditions on the initial pulse for a singularity to form in the field after a finite propagation distance. The model is numerically simulated in 2D using a spectral-solver with initial data and physical parameters highlighting our theoretical results.

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$\mathbf{CP2}$

Wave Kinetic Equation in a Nonstationnary and Inhomogeneous Medium with a Weak Quadratic Nonlinearity

A systematic derivation is presented for the wave kinetic equation describing the dynamics of an incoherent wave field propagating in a medium with a weak quadratic nonlinearity. The medium is allowed to be nonstationnary and inhomogeneous. Primarily based on the Weyl phasespace representation, our derivation makes use of the well-known ordering assumptions of geometrical optics and of a statistical closure based on the random phase approximation. The resulting wave kinetic equation describes the wave dynamics in the ray phase space. It captures linear effects, such as refraction, linear damping, and external sources, as well as nonlinear wave scattering. This general formalism could potentially serve as a stepping stone for future studies of weak wave turbulence interacting with mean fields in nonstationnary and inhomogeneous media.

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CP2

Rare Events in an Actively Mode-locked Laser Model with Multiple Paths

We consider a soliton-based, actively mode-locked laser model where errors arise with very low probability due to pulse position slips relative to the active driving. We consider the case where the pulse position undergoes damped oscillations in the absence of noise. In the presence of noise, this lead to multiple error paths that are differentiated by the number of oscillations experienced by the pulse position before a slip occurs. We study the probability associated with these error modes using dynamic importance sampling and demonstrate how to account for the multiple paths.

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CP2

A Computational Characterization of Dipolar **Bose-Einstein Condensate Ground States**

A nonlinear Schrödinger equation with dipolar interactions is studied computationally. The variations and possible properties of ground state solutions are identified. Using multiple numerical methods, it is verified that ground states appear for a variety of parameter-regimes. The relationship between the amplitudes, widths, powers and energies of the ground states is discussed. These results shed new light on how to computationally attain various ground states of the dipolar NLS equation.

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CP3

Application of Koopman Operator Theory to Highway Traffic Dynamics

such as human interaction and weather lead to a very complicated high-dimensional nonlinear dynamical system. This makes it difficult to obtain an analytic or Artificial Intelligence (AI) model that robustly captures the time evolution of vehicular traffic dynamics. This has led to a greater need for developing data-driven methods that can identify dynamically important structures within traffic data. We demonstrate how the Koopman operator theory can offer a model and parameter-free, data-driven approach to accurately analyzing and forecasting traffic dynamics. The Koopman operator framework is a rapidly developing theory in dynamical systems that offers powerful methods for analyzing nonlinear systems. The effectiveness of this framework is demonstrated by an application to the NGSIM data set collected by the US Highway Administration and the PeMS data set collected by the California Department of Transportation. By obtaining a Koopman mode decomposition of the data sets, we are able to accurately reconstruct our observed dynamics, distinguish any growing or decaying modes, and obtain a hierarchy of coherent spatiotemporal patterns that are fundamental to the observed dynamics. Furthermore, it is demonstrated how the Koopman Mode Decomposition (KMD) can be utilized to accurately forecast traffic dynamics by obtaining a decomposition of a subset of the data, that is then used to predict a future subset of the data.

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CP3

Multi-scale Analysis of Coherent Structures in Wall Turbulence

Chaotic systems such as turbulent flows can give rise to dynamic coherent structures which occur and evolve over a spectrum of scales. Detailed analyses of certain classes of structures are complicated by the broadband nature of the flow which modulates each manifestation of a structure and which renders the isolation of individual features a challenging task. We introduce a topological algorithm derived from computer vision to capture the time evolution of coherent features and explore the underlying physical mechanisms. The algorithm applies morphological operations to condense the structure of the turbulent flow field into a discrete graph described by sets of nodes and links. The low-dimensional geometric information is stored in a database. Application of topological filtering criteria allows the identification and analysis of equivalent dynamical processes across multiple scales. The method is employed in the study of hairpin vortices in wall-bounded turbulence. Application to direct numerical simulations data of a turbulent boundary layer allows the first time-resolved sampling and exploration of the hairpin process in natural flow. A dynamical averaging procedure yields rich statistical results which lead to a refined characterization of the hairpin process. The results also provide a sharpened understanding of the underlying causality of events.

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CP3

The unpredictable behaviors involved in a traffic system Locating Exact Coherent Structures using Dy-

namic Mode Decomposition

A Koopman expansion is a spectral decomposition for observables of a dynamical system which yields a linear representation of the nonlinear evolution. Koopman eigenfunctions can delineate invariant sets of the underlying dynamical system, or pick out Fourier components of periodic orbits. Under certain circumstances Dynamic Mode Decomposition (DMD) can accurately determine a subset of the Koopman eigenvalues, eigenfunctions and modes which constitute such an expansion. We will explore this connection by analytically deriving the full Koopman decomposition for (i) a model ODE system describing a collapse onto a limit cycle and (ii) the viscous Burgers equation. Time permitting, we will also discuss the results of applying DMD to short trajectories from a turbulent Couette flow in a small box.

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$\mathbf{CP3}$

Data-driven Koopman Analysis of Tropical Climate Space-time Variability

We study the nonlinear high-dimensional dynamics of the Earth's tropical atmosphere-ocean system. For that, we apply a recently developed technique for feature extraction and mode decomposition of spatiotemporal data generated by ergodic dynamical systems. The method relies on constructing low-dimensional representations (temporal patterns) of signals using eigenfunctions of the Koopman operator governing the evolution of observables in ergodic dynamical systems. This operator is estimated from timeordered data through a Galerkin scheme applied to basis functions computed via the diiffusion maps algorithm. We apply this technique to a variety of tropical atmosphere and ocean datasets and extract a multiscale hierarchy of spatiotemporal patterns spanning many timescales. In particular, we detect for the first time, without prefiltering the input data, modes of variability acting on timescales of days (propagation of organized convection) to decades (ocean circulation adjustment). We discuss the salient properties of the coherent structures in this hierarchy of modes and demonstrate how the activity of certain types of high-frequency atmospheric waves is modulated by lowfrequency ocean variability.

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CP3

Mixing and Lagrangian Coherent Structures in Two-dimensional Rayleigh-Benard Convection with Periodic Perturbations

In this talk, we investigate the mixing structure in a twodimensional Rayleigh-Benard convection with periodic per-

turbations from a Lagrangian point of view. To analyze the mixing structure associated with the transport of fluid particles, the Lagrangian Coherent Structure (LCS) is numerically detected by computing Finite-Time Lyapunov Exponents (FTLE), and the KAM tori are analyzed by using Poincare maps. In particular, we clarify some topological similarity appeared in the Forward and Backward FTLE fields and the image of the Poincare maps in a log time computation, and also show how the chaotic and non-chaotic regions are changed in the long time integration. Furthermore, we analyze the periodic and quasi-periodic orbits on KAM tori, and finally show the global structure of the Lagrangian transport of fluid particles in cells associated with the KAM tori. Main results are given as: (1) fluid particles which are surrounded by countless KAM curves are transported periodically inside a convectional cell; (2) fluid particles in a non-chaotic region behave as one vortex as a whole, which moves periodically inside a cell and maintains its mixing rate low; (3) such global structures are invariant when the proportion of the velocity parameter and the angular frequency of the perturbation, the wave number of the horizontal cell pattern, and the amplitude of the perturbation are equal.

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$\mathbf{CP4}$

Lump Solutions to a $(2\!+\!1)\text{-}\mathsf{Dimensional}$ Fifth-order KdV-like Equation

The fifth-order Kortewegde Vries equation(KdV) is an important higher order extension of the famous KdV equation in fluid dynamics. In this work, a (2+1)-dimensional fifth-order KdV-like equation is introduced through a generalized bilinear equation with the prime number p = 5. The new equation possesses the same bilinear form as the standard (2 + 1)-dimensional fifth-order KdV equation. Then lump solutions, which are rationally localized in all directions in the space, are constructed for the derived equation using symbolic computation with Maple. We get free parameters in the resulting lump solutions, of which we can get a non-zero determinant condition ensuring analyticity and rational localization. Moreover, a particular class of lump solutions with special choices of the free parameters are generated and plotted as illustrative examples.

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$\mathbf{CP4}$

Existence of Stationary Fronts in a System of Two Coupled Wave Equations with Spatial Inhomogeneity

The aim of this talk is to discuss the existence of stationary

fronts to a system of two coupled nonlinear wave equations with spatial inhomogeneity. The spatial inhomogeneity has finite length. When taking one of the two dependent variables to be the zero solution the coupled equations reduce to a single sine-Gordon equation with spatial inhomogeneity. For this equation, the existence of stationary fronts is known. Through a numerical and dynamical systems approach it is possible to determine the bifurcation at which the second component becomes non-zero. The bifurcation curves are given in terms of the three parameters, namely the coupling strength of the equations, the length of the inhomogeneity and the strength of it.

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$\mathbf{CP4}$

Multiscaled Solitary Waves

It is analytically shown how competing non-linearities yield multiscaled structures for internal solitary waves in stratified shallow fluids. These solitary waves only exist for large amplitudes beyond the limit of applicability of the KdV equation or its usual extensions. Multiscaling phenomenon exists or does not exist for almost identical density profiles. Trapped core inside the wave prevents appearance of such multiple scales within the core area. The structural stability of waves of large amplitude is briefly discussed.

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$\mathbf{CP4}$

The Semi-classical Sine-Gordon Equation with Pure Impulse Initial Data, Universality at the Gradient Catastrophe

We consider a class of solutions with pure impulse initial data below critical value such that within small time only librational-type waves are generated and the solutions should decay when $|x| \to \infty$ [Buckingham, Miller, Memoirs of the AMS 225]. In a neighbourhood of a certain gradient catastrophe point that contains both modulated plane waves and localized structures or "spikes", the asymptotic behaviour of the solutions can be universally described by analyzing a Riemann-Hilbert problem related to Painlevé I equation Tritronquée solutions. It is a well-known fact that the solution to Painlevé equations have poles. In fact we show the locations of the poles are directly linked to where the "spikes" happen. In suitable scaling limit, we are able describe the first correction of the solution (compared to before breaking happens) using Painlevé I Tritronquée solution away from the "spikes", and then modify the Riemann-Hilbert problem to describe the "spike" shapes. Notice that this result is universal in the sense that the local asymptotics is not sensitive to the initial condition as long as it falls into a large class; it is only the space-time location of the transition that depends on the initial data. Our technique is the Deift-Zhou steepest descent method related to an approach of [Bertola, Tovbis, Communications on Pure and Applied Mathematics, LXVI, 0678-0752] to universality for the focusing nonlinear Schrödinger equation.

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$\mathbf{CP4}$

Soliton Solutions of Certain Reductions of the Matrix Nonlinear Schrodinger Equation

We will present soliton solutions for two novel reductions of the matrix nonlinear Schrodinger equation which are integrable, and which are the analog of the modified Manakov system with mixed signs of the nonlinear coefficients, i.e., a nonlinearity in the norm which is of Minkowski type, instead of Euclidean type. We will classify one soliton solutions, discuss regularity conditions and investigate special solutions including double pole solutions, bound states, and two soliton solutions.

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$\mathbf{CP4}$

On the Existence of the Highest Traveling Solitary Wave Solution to Whitham's Equation

We consider traveling solitary wave solution to Whitham's equation of the form $cu-K*u-u^2 = 0$, where c > 1. First, we construct a local solution curve, bifurcating from the trivial solution (0, 1), using a version of the center manifold theorem for nonlocal equations. Next, we prove that the hypotheses for the analytic global bifurcation theorem are met. Finally, we exclude the possibility that the curve is closed, using the qualitative knowledge on the bounded solutions that the center manifold reduction provides, and prove that the limiting curve is in fact the highest traveling solitary wave solution.

Tien T. Truong

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CP4

Asymptotics for the Focusing Integrable Discrete Nonlinear Schrödinger Equation

The focusing integrable discrete nonlinear Schrödinger equation can be solved by inverse scattering. Eigenvalues appear in quartets. In the reflectionless case, it admits a (multi-)soliton solution under generic assumptions. Phase shifts are determined by the eigenvalues. We consider what happens if the reflection coefficient does not vanish identically. We can show that the soliton resolution conjecture is valid in this case. Namely, the solution is asymptotically a sum of 1-solitons. In |n/t| < 2, the phase shifts depend on the eigenvalues and the reflection coefficient. If |n/t| is not less than 2, they are independent of the reflection coefficient. The proof is based on the method of nonlinear steepest descent due to Deift and Zhou. Details can be

found in arXiv:1512.01760 [math-ph].

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CP5

Numerical Study of Psuedo Poiseuille Taylor Couette Flow Structures in a High Speed Motor Narrow Gap Region

This paper will present results for a numerical Computational Fluid Dynamics (CFD) study of the flow field developed within the narrow gap region of a high speed (30,000 rpm) motor. The flow field is deemed a pseudo Taylor-Couette-Poiseuille flow since a very small diameter pipe the inlet and exit of the motor are used to supply heat transfer cooling air (200 Liters/min) to the motor. The effects of inlet air speed, rotor rotating speed and parasitic heat transfer rate on the development and decay of the Taylor-Couette cells formed in the small gap region of the motor are presented. The CFD results are correlated against experimental test data. The CFD model uses the k-w SST turbulence model, Mach j 0.3 incompressible flow, and Knudsen ii 0.0001 continuum flow. The small gap of 2 mm and high speed of 30,000 rpm sets up structures with Taylor number = 724, Reynolds number = 1600. Effects of combined heat transfer and fluid momentum effects on the wash out of the Taylor-Couette cells is presented. Comparison of CFD to linearized stability theory for wave-speed of the structures is also presented.

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$\mathbf{CP5}$

Necessary Conditions for Streaming Instability of 3D Internal Gravity Wave Beams

Internal gravity waves are ubiquitous in stratified fluids, such as the oceans, where they play a major role in mixing the abyssal sea. The generation of internal waves through tides is fairly well understood. In contrast, the fate of internal waves is still debated. We study a mechanism of internal wave beam breakdown through weakly nonlinear wave-wave interactions, known as streaming instability (Dauxois et al, Annual Review of Fluid Mech., 2018). Strikingly often, internal wave solutions to the linearized equations also happen to solve the nonlinear equations exactly. Prominent examples are 2D monochromatic internal wave beams in the absence of viscosity. If weakly viscous, then the nonlinear terms of such beams are typically nonzero, but small. With a perturbational approach, these small non-linear terms may be treated as Reynolds stresses, which induce a mean flow. The induced mean flow can become strong if the weak forcing through wave-wave interactions is persistent over a long time. Eventually, nonlinear wave-mean interactions may lead to a breakdown of the wave beam itself. This study investigates under which circumstances the induced mean flow may become very strong. Strong induced mean flow is a necessary condition for streaming instability. We classify different processes that may lead to strong induced mean flow, which turn out to be fundamentally different in 3D configurations as compared to well-studied 2D idealizations.

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$\mathbf{CP5}$

When Is the L^2 Energy of the 1D Damped Klein-Gordon Equation Decaying?

We study time decay of the energy of the 1D damped Klein-Gordon equation. We give an explicit necessary and sufficient condition on the continuous damping functions $\gamma \geq 0$ for which the energy $E(t) = \int_{-\infty}^{\infty} |u_x|^2 + |u|^2 + |u_t|^2 dx$ decays exponentially, whenever $(u(0), u_t(0)) \in H^2(R) \times H^1(R)$.

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$\mathbf{CP5}$

Spectral Stability of Finite-Amplitude Gravity Waves in the Atmosphere

Atmospheric gravity waves play an important role in numerical weather prediction. The overturning and breaking of these waves is of particular interest as it is associated with significant energy and momentum transport. Therefore, our work focuses on the stability of waves with large amplitudes. The corresponding vector-valued dissipative modulation equations describing the nonlinear waves turn out to possess solutions of mixed type: both fronts and pulses are possible traveling wave solutions. Spectral stability analysis reveal criteria for wave breaking. The results are tested and supported by numerical simulations.

Mark Schlutow, Erik Wahlen

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$\mathbf{MS1}$

A Phase Field Crystal Model for Active Binary Mixtures

We consider a minimal continuum approach to microscopically model binary mixtures of interacting active and passive particles. This is an extension of an active Vacancy Phase Field Crystal (VPFC) model that we recently introduced for interacting active particles. We use this model to numerically study how a small amount of activity can enhance crystallization in passive systems. The results are in quantitative agreement with experiments and with the prediction of agent-based simulations. We address the opposite case as well: how a few passive particles affects the dynamics of an active bath. Moreover we analyze which patterns and states emerge in this new mixed system.

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$\mathbf{MS1}$

Modelling Defect Structures and Pattern Formation in 2 Dimensional Materials and Heterostructures

There has been of a great deal of interest in two dimensional systems with honeycomb symmetry such as graphene, hexagonal boron nitride (hBN) and molybdenum disulfide. The interest arises from their extraordinary electronic and mechanical properties and the potential for a myriad of electronic device applications. In this talk I will discuss the development of continuum phase field crystal and related amplitude expansions for modeling these systems and the use of these models in predicting the complicated defect structures that emerge at grain boundaries, triple junctions and inversion boundaries. By combining these methods with atomistic approaches quantitative predictions for grain boundary and triple junction energies, Moire pattern structures and the influence of microstructure on thermal conductivity can be made.

<u>Ken Elder</u> Dept Physics Oakland U. elder@oakland.edu

$\mathbf{MS1}$

Dynamics and Competition of Block Copolymer Aggregates

Nanoparticle aggregates formed incopolymer/homopolymer mixtures exhibit a wide variety of morphologies. These systems are described by Cahn-Hilliard-type phase field models and their corresponding sharp interface limits. Families of equilibrium solutions are found both analytically and numerically, and are characterized by their geometry and chemical potential. The interaction of aggregate equilibria occurs by mass diffusion driven by differences in chemical potential, and is studied in the context of a LSW-type approximation. Competition may occur between aggregates both of different size and morphology, leading to a much richer picture of interaction that traditional coarsening of phase mixtures. In addition, morphological preference is shown to be size dependent, and shape instabilities may lead to topological transitions. The non-monotone relationship between size and chemical potential leads to circumstances where collections of aggregates coexist rather than compete via Ostwald ripening. In other cases, approximate dynamic scaling of size is observed, despite a lack of exact scaling properties in the underlying equations.

<u>Karl Glasner</u> The University of Arizona Department of Mathematics kglasner@math.arizona.edu

MS1

Defects in Two-fluid Models of Lipid Droplets

Lipid droplets are fat storage mechanisms that encapsulate oil within a lipid layer. Growing within the endoplasmic reticulum, they form the prototypical example of a two-fluid amphiphilic system driven by interfacial competition that drives the budding and pinch-off and remerging of lipid droplets. We present an analysis of pearling modes and suggest mechanisms for robust pearling inhibition within piecewise defined and singularly perturbed systems.

Keith Promislow

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MS2

Tsunami

People around the world know more about tsunami now than they did 15 years ago because of two events: the 2004 tsunami, which killed more than 200,000 people along the shorelines of the Indian Ocean, and the 2011 tsunami off the coast of Japan, which set of a nuclear disaster, the effects of which are still unfolding. This talk will explain how tsunami work, why they are dangerous in some places and benign in others, and what can be done to withstand future tsunami. Almost no mathematics is required to understand the basic principles of tsunami.

Harvey Segur Dept. of Applied Mathematics University of Colorado Harvey.Segur@colorado.edu

MS2

Direct Numerical Simulation of Nonlinear Water Waves

Extensive experimental investigations have compared physical water waves to solutions of classical wave equations, where these equations are derived as reduced asymptotic models of the Navier-Stokes equation, with one or more interfaces. In place of physical experiments, we consider a direct numerical simulation, by a volume-of-fluid method, of the Navier-Stokes equation. In these simulations, a 'height function' is used to improve the order of the approximation of the curvature of the interface between fluids. One advantage of direct numerical simulation is that, unlike physical experiments, all physical quantities, including viscosity, can be set to prescribed values. We report on progress in producing and measuring soliton-like waves in simulations of surface waves.

David Trubatch

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$\mathbf{MS2}$

Rudy Horne: The Mathematical Consultant of the Hidden Figures Movie

Dr. Rudy Horne brought mathematics portrayed in the Hollywood movie "Hidden Figures." This movie featured female African American mathematicians who helped NASA win in the Space Race in a racially segregated era. We will explore how Dr. Horne brought mathematics, physics, history and race to Hollywood.

Talitha Washington Howard University talitha.washington@howard.edu

$\mathbf{MS2}$

Nonlinear Waves in a Number of Discrete Dynamical Lattices

Beginning in 2006, Panos Kevrekidis and I began a series of fruitful collaborations with Dr. Rudy Horne. These collaborations took place at UMass as well as a week at IMA. I will describe briefly some of our work including examining solitary waves in discrete media with four wave mixing, plane waves and localized modes in quadratic wave guide arrays and PT-symmetry management in oligomer systems. I will also describe my personal connection with Rudy through the movie "Hidden Figures".

Nathaniel Whitaker

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$\mathbf{MS3}$

Band Degeneracies in 90-degree Rotationally Invariant, Periodic Schroedinger Operators

Schroedinger operators with periodic potentials are central to the mathematical description of waves in periodic media, with many applications in Quantum Physics, Electromagnetics and other fields. Wave propagation properties are encoded in "band structure," the collection of dispersion surfaces and associated Floquet-Bloch eigenmodes. Band degeneracies are special quasi-momentum energy pairs where consecutive dispersion surfaces intersect, and these intersections are often caused by symmetries of the potential. Among the best known degeneracies are Dirac points, conical intersections, which arise for honeycomb potentials (Fefferman and Weinstein, 2012; Fefferman, Lee-Thorp, and Weinstein 2016). In this work we study integer lattice potentials, the class of \mathbf{Z}^2 -periodic potentials which are real-valued, even and $\pi/2$ -rotationally invariant. The band structure for such potentials is proved to have spectral degeneracies at certain high-symmetry quasimomenta. We give a detailed general picture of the dispersion surfaces near such degeneracies, and applying our results, we show that the well-known conical + flat band dispersion surface profile for the tight-binding Lieb lattice model does not persist for finite-depth potentials.

Rachael Keller

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Braxton Osting University of Utah osting@math.utah.edu Columbia University miw2103@columbia.edu

MS3

Mean Field Limits of Large Quantum Systems in Equilibrium

In this talk I will present some recent results on the mean field limits for quantum systems, where the interaction strength behaves as 1/N where N is the number of particles, in the limit of large N. I will in particularly compare bosons and fermions.

Mathieu Lewin

CNRS and University of Cergy-Pontoise, France mathieu.lewin@math.cnrs.fr

MS3

Symmetry Breaking and Branching in Density Functional Theory Models

We discuss recent results in two projects relating to nonlinear bound states in settings with Coulomb potential wells. In one project with Sarah Raynor and Gideon Simpson, motivated by some non-relativistic approximations to Dark Matter models, we analytically study branches of nonlinear bound states in 3d Schrdinger-Poisson equations with Coulomb potentials and observe that each radial linear eigenfunction leads to a set unique continuous branch of solutions that can never cross. Asymptotics and stability analysis are also done. In addition, we will discuss recent analytic and numerical work on symmetry breaking in a Local Density Approximation model from Density Functional Theory for the H_2 molecule with Mike Holst, Jianfeng Lu, John Weare, Duo Song and Houdong Hu.

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Duo Song

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MS3

Wave-packet Dynamics in Locally Periodic Media

We study the dynamics of wave-packet solutions of Schrödingers equation and Maxwell's equations in media with a local periodic structure which varies adiabatically (over many periods of the periodic lattice) across the medium. We focus in particular on the case where symmetries of the periodic structure lead to degeneracies in the Bloch band dispersion surface. We derive systematically and rigorously the 'anomalous velocity' of wave-packets due to the Bloch bands Berry curvature, and the dynamics of a wave-packet incident on a Bloch band degeneracy in one spatial dimension. Joint work with Michael Weinstein and Jianfeng Lu.

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$\mathbf{MS4}$

Computation of Singular Solutions using Hermite Methods with H-P Refinement

Hermite methods are arbitrary-order polynomial-based general-purpose methods for solving time dependent PDEs. Noteworthy properties of Hermite methods are high order of accuracy in both space and time combined with the ability to march in time with $c\Delta t \leq h$, for any order of accuracy. The essential description of Hermite methods is as follows:

- (i.) The degrees of freedom are tensor-product Taylor polynomials at the cell vertices for multi-indices α , $0 \le \alpha_j \le m$, $U_k^{\alpha}(t_n) \approx \frac{h^{\alpha}}{\alpha!} D^{\alpha} u(\mathbf{x}_k, t_n)$.
- (ii.) The cell polynomial is the Hermite interpolant of the vertex polynomials; that is $D^{\alpha}P = U_k^{\alpha}(t_n)$ at all vertices x_k . This yields a tensor-product polynomial of degree 2m + 1 in each coordinate.
- (iii.) *P* is evolved locally at the cell center to produce the required data on the staggered grid.

This talk will present the basic elements of Hemite methods and their application to problems with singular solutions. In particular we will present how H-P refinement can be leveraged to achieve accurate approximations with minimal computational effort.

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Thomas M. Hagstrom Southern Methodist University Department of Mathematics thagstrom@smu.edu

MS4

New Integrals of Motion and Singularities in 2D Fluid Dynamics with Free Surface

We study the problem of 2D incompressible fluid dynamics with free surface, we assume fluid to be ideal and the flow is potential. Following the conformal mapping technique we reformulate the problem to surface variables and demonstrate the existence of previously undiscovered constants of motion associated with singularities in the analytic continuation of conformal map and complex potential. In numerical simulations we recover the analytic structure of the surface shape and observe simple poles and branch point singularities of the square-root type. We use the Alpert-Greengard-Hagstrom method to recover the location, type and magnitude of the singularities. We show how the approach of square-root type singularities may be responsible for the breaking of waves in the ocean, following the nonlinear stage of modulational instability.

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MS4

Loss of Phase and Universality of Stochastic Interactions Between Laser Beams

We show that in nonlinear propagation of laser beams, the shot-to-shot variation of the nonlinear phase shift increases with distance, and ultimately becomes uniformly distributed in $[0, 2\pi]$. Therefore, if two beams travel a sufficiently long distance before interacting, it is not possible to predict whether they would intersect in- or outof-phase. Hence, if the underlying propagation model is non-integrable, deterministic predictions and control of the interaction become impossible. Because the relative phase between the two beams becomes uniformly distributed in $[0, 2\pi]$, however, the statistics of these stochastic interactions are universal and fully predictable. These statistics can be efficiently computed using a novel universal model for stochastic interactions, even when the noise distribution is unknown.

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$\mathbf{MS4}$

Computing Quasi-periodic Water Waves

We present a framework for computing quasi-periodic water waves with spectral accuracy. The method uses conformal variables to rapidly evolve families of solutions of the free-surface Euler equations that are required to fit together (by a shooting method) to form a single quasiperiodic solution. Some of the new solutions are hybrid traveling-standing waves that return to a spatial translation of their initial condition at a later time. Others are nonlinear superpositions of several standing waves with irrationally related periods. Many examples will be given to illustrate the types of behavior that can occur.

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MS5

Nonlinear Resonances: An Exploration of NLS on

Alexander Dyachenko

a Simple Open Metric Graph

There is an ongoing effort to understand the nonlinear Schrödinger (NLS) equation on metric (quantum) graphs. When studying existence of standing waves on graphs with a non trivial topology, some peculiar bifurcation diagrams have been discovered even in the simplest examples. Namely, a whole branch of solutions may disappear under arbitrarily small perturbation of the geometry of the graph. It appears that the key to understanding these diagrams lie in the extension of the notion of resonances to nonlinear equations. We argue that metric graphs are the perfect models to study nonlinear resonances: on one hand, there are interesting phenomena to explain and, on the other, the almost-one dimensional character of graphs simplifies both numerical experimentation and rigorous analysis. We use the simple example of a Y-branching graph with one infinite edge and with NLS on the finite edges only. We describe the bifurcation diagram of stationary solutions as we change the ratio of the edge lengths. The drastic change in the diagram is explained by continuing the solutions for complex vales of the energy (or frequency) parameter and locating the "disappearing" branches away from the real axis. Based on a joint work with Diego Noja (Milano Bicocca).

Gregory Berkolaiko Texas A&M University berko@math.tamu.edu

MS5

Eigenvalues of the NLS Equation on Compact Graphs

In this talk, I will discuss how to compute the number of real eigenvalues of a standing wave of the nonlinear Schrödinger (NLS) equation on a compact quantum graph. By linearising the NLS equation about the standing wave, I will show how to use the Maslov index, as well as the spectrum of two associated operators in order to get a count of the number of real eigenvalues. This result is similar to a classical one of Jones, and Grillakis,Shitah and Strauss about standing wave solutions to the NLS equation on the line.

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Yuri Latushkin Department of Mathematics University of Missouri-Columbia latushkiny@missouri.edu

Selim Sukhtaiev Rice University ss168@rice.edu

$\mathbf{MS5}$

Nodal Deficiency, Quantum Graphs and the Maslov Index

We discuss the Maslov index in the study of eigenvalues and eigenfunctions for second order elliptic operators. Hopefully, we will discuss some new results in the areas of nodal domains and quantum graphs.

Jeremy L. Marzuola Department of Mathematics University of North Carolina, Chapel Hill marzuola@math.unc.edu

$\mathbf{MS5}$

On Limits of Quantum Graph Operators with Shrinking Edges

In this talk I will discuss continuity of eigenvalues and eigenfunctions of self-adjoint Schrödinger operators on metric graphs with respect to edge lengths. The standard results in this direction address only the case of strictly positive edge lengths. I will show that most of these results can be carried over to the case of zero limiting lengths. This talk is based on joint work with G. Berkolaiko and Y. Latushkin.

<u>Selim Sukhtaiev</u> Rice University ss168@rice.edu

MS6

A Uni-directional Model for Nonlinear Waves in Water of Finite Depth

A weakly nonlinear evolution model is derived for surface gravity waves in water of finite depth under the assumption of uni-directional wave propagation. A systematic asymptotic expansion is performed in spectral space to the third order in wave steepness. Some applications of the model and its numerical solutions will be discussed.

Wooyoung Choi Dept of Mathematics New Jersey Institute of Tech wychoi@njit.edu

MS6

Two-dimensional Stability of Solitary Waves on a Linear Shear Current

This work considers the linear stability of two-dimensional motion of solitary waves propagating in permanent form on a vertically sheared current of constant vorticity. We use a fully nonlinear and weakly dispersive model which can be applied to large-amplitude motion, and focus on stability of steady solutions near the critical value of vorticity at which the wave slope becomes infinity at crest. It is shown that numerically accurate evaluation of the linear stability requires suitable treatment of exponentially decaying behavior of solutions in the outskirt of solitary waves.

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MS6

A Lagrangian for Water Waves with Application to the Stability of Stokes Waves

Abstract not available at time of publication.

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W. Kendall Melville Scripps Institution of Oceanography UCSD kmelville@ucsd.edu

$\mathbf{MS6}$

Computing Travelling Wave Solutions to Euler's Equations

In this talk, we introduce the models describing travelling waves with a variety of conditions at the interface. We outline the procedure to find solutions to these models, including the numerical difficulties involved and how they can be overcome. We then proceed to show solutions for these waves in different parameter regimes, comparing and contrasting several models.

Olga Trichtchenko University of Washington olga.trichtchenko@gmail.com

MS7

Degenerate FCH Functional and Defects in Amphiphilic Structures

Amphiphilic structures such as cell membranes and lipid vesicles play essential roles in biological applications. In this talk, we will first introduce the functionalized Cahn-Hilliard (FCH) model for the free energy of amphiphilic mixtures. The FCH model admits local minimizers corresponding to amphiphilic bilayers, filaments, micelles, and other defect structures. We will describe the geometric motion of bilayers, filaments, and their competition. To capture the coexistence and localization of amphiphilic structures, we introduce a degenerate FCH functional as a modified model for the free energy of amphiphilic mixtures. We prove that the degenerate FCH functional admits geometrically localized minimizers, which correspond to lipid bilayers. In addition, we identify the leading order profile of the bilayers under the assumption that the geometrically localized minimizers have bounded variations along the tangential directions.

<u>Shibin Dai</u> University of Alabama sdai4@ua.edu

$\mathbf{MS7}$

Defects in Bilayer Interfaces in the Multicomponent Functionalised Cahn-Hilliard Equation

We study a multi-component extension of the functionalised Cahn-Hilliard (fCH) equation, which provides a framework for the formation of patterns in fluid systems with multiple amphiphilic molecules. The assumption of a length scale dichotomy between two amphiphilic molecules allows the application of geometric techniques for the analysis of patterns in singularly perturbed reaction-diffusion systems. For a generic two-component system, we show that solutions to the four-dimensional connection problem provide the leading order approximation for solutions to the full eight-dimensional barrier problem, which can be obtained through a perturbative expansion in the layer width. Based on previous results on the one-component fCH equation, we discuss the control of pearling instabilities in the multi-component setting.

<u>Frits Veerman</u> University of Edinburgh fveerman@ed.ac.uk

MS7

Phase Field Crystal Models of Graphene Formation on Crystalline Substrates

In this talk I will review some phase field crystal models of hexagonal and honeycomb crystal lattice growth from either a melt or a vapor phase. I will describe some numerical methods for efficient and stable numerical approximation of the models and will present some numerical results.

<u>Steven M. Wise</u> Mathematics Department University of Tennessee swise@math.utk.edu

MS8

Scattering and Attenuation of Solitary Waves in a Fragmented Sea Ice

A numerical model for the attenuation of nonlinear ocean waves propagating through fragmented sea ice is proposed. This model solves the full nonlinear equations for potential flow coupled with an elastic thin-plate formulation for the ice cover. Distributions of ice floes can be directly specified in the physical domain by allowing the coefficient of flexural rigidity to be spatially variable. Dissipation due to ice viscosity is also taken into account by including diffusive terms in the governing equations.

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MS8

Mathematical Modelling of Wave Transmission by Overwashed Sea-ice Floes

The inter-facial region between open and ice covered oceans is known as the marginal ice zone. Its distinct physical property is the presence of thin and long sheets of sea ice, otherwise known as floes. As ocean waves interact with these floes they alter the ice cover, and, in turn, the floes attenuate the waves. The canonical model for how floes attenuate the waves is based on how water waves interact with a floating elastic plate. It uses linear potential water wave theory to model the water waves and thin elastic plate theory to model the floes. It predicts that wave attenuation is linear with respect to incident wave amplitude. A sizable amount of recent work has been conducted on validating this wave-plate interaction model in laboratory wave tanks. These experiments have found that if the surrounding waves are sufficiently large water washes onto the surface of the plate — a process known as overwash. They find that when this occurs the attenuation becomes nonlinear, and the waves transmitted by the plate are significantly less than those predicted by existing linear models (which do not include the effects of overwash). In this presentation I will discuss these waves tank experiments and present a theoretical model of overwash. I will show how, and why, the overwash creates strong nonlinear wave attenuation, and discuss the implications of this work with respect to wave-sea ice modelling.

David M. Skene

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$\mathbf{MS8}$

Mathematical and Experimental Perspectives on Modelling Wave-induced Collisions and Rafting of Ice Floes

At high latitudes, ocean surface waves strongly impact the sea ice cover through a variety of dynamic and thermodynamic processes. In regions with medium to high concentrations of sea ice, wave-induced collisions and rafting (overlapping) of sea ice floes have been identified as potentially significant contributors to wave-ice dynamics. In my talk, I will discuss the efforts to model these floe-floe interactions. A series of laboratory experiments are employed to identify different collision and rafting behaviours over a range of wave forcing parameters. These results are then compared against a mathematical model to determine the model's range of validity and identify dominant physical mechanisms that the model does not capture.

Lucas J. Yiew Nanyang Technological University lucas.view@ntu.edu.sg

MS9

Dark Solitons: From 1D to 2D and 3D with Some Quantum Touches

In the present talk, we will revisit some principal excitations in self-repulsive Bose-Einstein condensates, namely dark solitons in single-component systems, and dark-bright solitons in multi-component systems. Upon introducing them and explaining their existence and stability properties in 1d, we will extend them both in the form of stripes and in that rings in two-dimensions, presenting an alternative (adiabatic-invariant based) formulation of their stability and excitations. We will explore their filamentary dynamics, as well as the states that emerge from their transverse (snaking) instability. Then, we will consider these structures even in three dimensions, in the form of planar, as well as spherical shell solitons and generalize our adiabatic invariant formulation there. Finally, time permitting, we will give some glimpses of how some of these dynamical features in 1d and 2d generalize in a multi-orbital, timedependent quantum setting.

Panayotis Kevrekidis University of Massachusetts kevrekid@math.umass.edu

MS9

Long-range Schroedinger Dynamics

We want to justify certain model equations proposed in the biophysics literature for charge transport on polymers like DNA and protein, so we consider a general class of discrete nonlinear Schroedinger equations on lattices, and prove that in the continuum limit, the limiting dynamics are given by a nonlinear Schroedinger equation (NLS) with a fractional Laplacian. In particular, a range of fractional powers arise from long-range lattice interactions in this limit, whereas the usual NLS with the non-fractional Laplacian arises from short-range interactions. We also obtain equations of motion for the expected position and momentum, the fractional counterpart of the well-known Newtonian equations of motion for the standard Schroedinger equation, and use a numerical method to suggest that the nonlocal Laplacian introduces decoherence, but that effect can be mitigated by the nonlinearity. Joint work with Gigliola Staffilani, Enno Lenzmann, Yanzhi Zhang, Peter Hislop, Stefano Olla, and Jeffrey Schenker.

Kay Kirkpatrick

University of Illinois at Urbana-Champaign kkirkpat@illinois.edu

$\mathbf{MS9}$

The Concept of a Phonon in Boson Dynamics in a Trap

Phonons are, roughly speaking, elementary collective excitations in a large quantum-mechanical system. In this talk, I describe the modeling of phonons for weakly interacting Bosons inside a trap (external potential) in the light of recent understanding of corrections to mean field dynamics. A central theme is the non-unitary transformation of the many-body Hamiltonian by use of the pair-excitation kernel, K. This kernel satisfies an integro-differential equation with coefficients depending on the condensate wave function, which in turn solves the Gross-Pitaevskii equation. By this description, phonons emerge via the spectrum of a one-particle Schroedinger-type operator that depends on K.

Dionisios Margetis University of Maryland, College Park dio@math.umd.edu

MS9

Rigorous derivation of nonlinear Dirac equations for wave propagation in honeycomb structures

We show how to rigorously obtain nonlinear equations of Dirac type as an effective description for slowly modulated, weakly nonlinear waves in honeycomb lattices. Both, local and nonlocal Hartree-nonlinearities are discussed and connections to closely related earlier results are pointed out.

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MS10

Fast and High Order Computation of Axisymmetric MHD Equilibria via Conformal Mapping

The equilibrium magnetic configuration in toroidally axisymmetric magnetic confinement fusion experiments is given by the Grad-Shafranov equation, a two-dimensional, second-order, elliptic semilinear partial differential equation for the magnetic flux function. In order to solve this equation on arbitrary domains relevant to magnetic fusion and with high order accuracy, we proceed in three steps. We first transform the semilinear Grad-Shafranov equation into a semilinear Poisson equation. We then map the problem to a semilinear Poisson equation on the unit disk, relying on the Kerzman-Stein integral equation for the conformal mapping. Finally, we use a Fourier representation for the angle dependence and an integral representation for the radial dependence to obtain a fast and spectrally accurate Poisson solver on the unit disk. We demonstrate that combining these three steps, we have a fast, high order convergent Grad-Shafranov solver for general geometries. Importantly, high order convergence is also achieved for the first and second derivatives of the solution, which play a crucial role in the physical properties of the magnetic equilibrium. However, we also show that our solver is inherently subject to the phenomenon of crowding, which degrades accuracy for highly elongated domains.

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MS10

Direct Computation of Singular Solutions of Evolutionary PDEs

Singular solutions are a prominent feature of many evolutionary partial differential equations arising in nonlinear wave theories. As such, there have been many numerical calculations of such solutions, often built around sophisticated spatially adaptive grids but relatively straightforward simulations in time. We consider the analysis of such approaches as well as alternatives based on problem reformulations which admit direct computations of a singularity.

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$\mathbf{MS10}$

Transverse Instability and Filamentation of Electron Plasma Waves via Direct 2+2D Vlasov Simulations

Transverse instability can be viewed as initial stage of electron plasma waves (EPWs) filamentation. We performed direct 2+2D Vlasov-Poisson simulations of collisionless plasma to systematically study the growth rates of oblique modes of finite-amplitude EPW depending on its amplitude, wavenumber, angle of the oblique mode wavevector relative to the EPW's wavevector and the configuration of the trapped electrons in the EPW. Simulation results are compared to the predictions of theoretical models.

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Harvey Rose New Mexico Consortium Inc. hrose@newmexicoconsortium.org

$\mathbf{MS10}$

Critical Points in Strichartz Functionals

We study an infinite dimensional dynamical system naturally associated with the study of minimizing/maximizing functions for the Strichartz inequalities for the Schrödinger equation. There exist infinitely many critical points which include eigenstates of the quantum mechanical harmonic oscillator. We also investigate stability of these critical points and relate their stability to the properties of the corresponding Hamiltonian system.

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MS11

Calculating the Maslov Index for Traveling Waves in a Singularly Perturbed System

The Maslov index is a powerful tool in the stability analysis of nonlinear waves. As a generalization of Sturm-Liouville theory, it provides the ideal result of stability analyses; spectral information is encoded in qualitative properties of the wave itself. Although theorems exist relating the Maslov index to stability, calculating the index is difficult in practice. In this talk, we provide a framework for calculating the index for traveling waves in singularly perturbed reaction-diffusion equations. The key insight is (the general fact) that the index is encoded in the twisting of an unstable manifold for the traveling wave equation. Using Fenichel theory, the index can then be calculated by following this manifold around phase space. We perform this calculation for a doubly-diffusive FitzHugh-Nagumo equation and prove that fast traveling waves are stable.

Paul Cornwell

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MS11

A Symplectic Perspective on Constrained Eigenvalue Problems

In recent years, much progress has been made in applying the Maslov index to spectral problems for selfadjoint differential operators on multidimensional domains. In this talk, I will apply this methodology to constrained eigenvalue problems, where the differential operator is restricted to a subspace (not necessarily of finite codimension). In particular, I will show how the constrained Morse index can be computed in terms of the Maslov index, then use this to prove a constrained Morse index theorem, which says the Morse index for the constrained problem equals the number of constrained conjugate points. Applications to the nonlinear Schrödinger equation will be discussed. (This talk represents joint work with Jeremy Marzuola.)

<u>Graham Cox</u> Pennsylvania State University Department of Mathematics gcox@mun.ca

MS11

The Maslov Index for Linear Hamiltonian Systems on [0,1] and Applications to Periodic Waves

Working with general linear Hamiltonian systems on [0,1], and with a wide range of self-adjoint boundary conditions, including both separated and coupled, I will discuss a general framework for relating the Maslov index to spectral counts. As an example of the general framework, I will analyze the spectrum of linear operators obtained when Allen-Cahn equations and systems are linearized about stationary periodic solutions.

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MS11

Abstract Boundary Value Problems, Maslov Index, and Applications

We describe relations between the Maslov index and the counting function for the spectrum of selfadjoint extensions of abstract symmetric operators related to abstract boundary triples. Applications are given to multidimensional Schrödinger operators on periodic and star-shaped domains.

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MS12

On Resonant Wave Interactions of Gravitycapillary Waves

We revisit resonant wave interactions of gravity-capillary waves in deep water. By examining the resonance conditions, we identify explicitly a region in parameter space, where resonant triads can be always found. Then we investigate special resonant triads that exchange no energy during their interactions and find a two-dimensional wave fie

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MS12

Surface Waves over Highly Irregular Topographies

Particular interest is given to the three-dimensional (3D) problem of (2D) surface water waves propagating over large-amplitude, non-smooth, bottom topographies. A brief introduction is made for the numerical construction of a 3D Dirichlet-to-Neumann operator for linear water waves in the presence of highly irregular topographies. Then we present a 2D (weakly nonlinear) Boussinesq-type system in the presence of highly variable ridge-like topographies. Extending the conformal mapping technique to 3D, where the Laplacian is no longer invariant, we generalize the terrainfollowing Boussinesq system presented in Nachbin (SIAP 2003). Work in collaboration with David Andrade (IMPA).

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MS12

Nonlinear Surface and Internal Wave Interactions

Nonlinear interactions between surface and internal waves in a two-layer system with a free surface are studied using a Hamiltonian system written explicitly in four canonical variables. Using a pseudo-spectral method, the system is solved numerically for wave fields of both narrow- and broad-band spectrum. Special attention is paid to the evolution of wave spectra and the energy transfer between the surface and internal wave modes.

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MS12

Experimental Investigation of Gas-liquid Interaction in Wavy Two-phase Pipe Flow

We present an experimental study of stratified gas-liquid pipe flow conducted at the Hydrodynamics laboratory, University of Oslo. The experimental setup consists of a 31 meter long, 10 cm diameter pipe, the test fluids are air and water at atmospheric conditions. Simultaneous twophase particle image velocimetry (PIV) is used to evaluate two-dimensional velocity fields along the center plane of the pipe. In addition, wave statistics are acquired using conductance wave probes. The main feature of this flow is portrayed by a complex interplay between a turbulent airflow above propagating interfacial waves. At selected flow rate conditions, the waves are governed by non-linear interactions (overtone resonance and sub-harmonic resonance). They also dissipate energy through a weak form of wave breaking, i.e. spilling or micro- breaking. Analysis of PIV velocity fields reveals that the airflow separates above steep waves. Such separation events are characterized by shedding of coherent structures on the lee-ward side of a wave crest. The combination of non-linear interfacial mechanisms and airflow separation is believed to contribute to the interfacial drag, which in turn affects the pressure drop of the system. The latter is an important modeling parameter in engineering applications that encounter gasliquid flow transport in pipes, e.g. petroleum and nuclear industries.

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MS13

The floe size effect on gravity waves propagating through multiple floes

The thin elastic plate theory is a reasonable model for long waves propagating through a continuous ice sheet with relatively uniform thickness. However, it is unclear whether the theory may also be applied to a fragmented ice cover by introducing an equivalent elasticity. This study presents a combined laboratory and numerical investigation of floe size effect on gravity waves propagating through multiple ice floes. We analyzed the celerity and attenuation data of two laboratory experiments and the parallel numerical solutions obtained by the Matched Eigenfunction Expansion Method (MEEM). The resulting equivalent elasticity can be several orders of magnitude lower than the intrinsic elasticity of ice. The reason for this low equivalent elasticity is the reduction of elastic energy stored by the ice floes, due to the existence of many free edges. Wave attenuation from the laboratory data is compared with numerical results. Significant discrepancy of attenuation between laboratory and numerical data indicates other attenuation mechanisms beyond wave reflections are important to consider. By expanding the numerical experiments to field scale, we obtain a general empirical relation between the equivalent elasticity and floe size, mechanical property of ice, and wavelength. Such formula is potentially useful in relating an observed fragmented ice cover to its elastic property in the wave-in-ice models.

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Andrei Tsarau 2) Norwegian University of Science and Technology n/a

Karl-Ulrich Evers Arctic Technology n/a Clarkson University n/a

MS13

Novel Field and Laboratory Observations of Waveice Interactions in Various Sea Ice Types

Interaction between water waves and sea ice is important for ice breaking, ice drift, and safety of human operations. In the past 3 years, our group has gathered field and laboratory data about those phenomena, and compared them with available models. Our field data comes from a series of experiments in Svalbard. We performed experiments of waves in landfast ice, grease ice, and large drifting ice floes. Our work focused on three aspects: developing affordable data loggers for increasing the volume of measurements, working towards developing new processing methods of the data, and assessing commonly used models for wave propagation under sea ice. All three aspects will be presented, with a special emphasis on measurements of wavelength and directional wave spectrum. While we are interested in the phenomena happening in the wild, controlled experiments in wave tanks are an efficient method to observe more of the details of waves-ice interactions. We have performed 3 series of laboratory experiments: in a large wave flume using model ice (plastic sheets), in a small wave tank using salt water and grease ice, and in a large wave flume using continuous ice. The data collected, both in the field and laboratory, helped develop a new model for describing wave attenuation by sea ice, which will be discussed in regards to other available models.

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MS13

Bounds on the Effective Viscoelasticity of an Ice Covered Ocean

Wave-ice interactions in the polar oceans comprise a complex but important set of processes influencing sea ice extent, ice pack albedo, and ice thickness. In both the Arctic and Antarctic, the ice floe size distribution in the Marginal Ice Zone (MIZ) plays a central role in the properties of wave propagation though the it. Ocean waves break up and shape the ice floes which, in turn, attenuate various wave characteristics. Recently, continuum models have been developed which treat the MIZ as a two-component composite of ice and slushy water. The top layer has been taken to be purely elastic, purely viscous or viscoelastic. At the heart of these models are effective parameters, namely, the effective elasticity, viscosity, and complex viscoelasticity. In practice, these effective parameters, which depend on the composite geometry and the properties constituents, are quite difficult to determine. To help overcome this limitation, we employ the methods of homogenization theory, in a quasi-static, fixed frequency regime, to find a Stieltjes integral representation for the complex viscoelasticity. This integral representation involves the spectral measure of a self adjoint operator and provides bounds on the effective viscoelasticity. The bounds themselves depend on the moments of the measure which then depend on the geometry. This work has the potential to provide simple parameterizations of wave properties which take into account floe concentration and geometry.

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MS13

Physics, Mathematics and Reality of some Viscoelastic Wave-in-ice Models

Ice covers are mixtures of various forms of solid, water (with or without salt), and air. They constantly evolve under thermal, chemical, and mechanical forcing. They are heterogeneous under all scales. To study how such materials influence the gravity wave propagation requires simplifications. A number of viscoelastic models have been proposed to describe an ice cover. All admit that ice covers may store and dissipate energy simultaneously. These models quantify the change of wave dispersion and its attenuation. Three typical models are discussed here, each attributes the storage and dissipation in a different way. The behavior of these models and comparisons with observations are presented.

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$\mathbf{MS14}$

Effect of Wind and Viscosity on Ocean Waves: Numerical and Analytic Approaches

The nonlinear Schrödinger equation (NLS) is the bestknown model for propagation of deep-water gravity waves. It predicts well the existence and propagation of breathers, solitons and the statistical properties of smooth sea-states. In order to consider the effect of wind forcing and viscous damping, homogeneous gain or loss are conventionally included. The NLS can be generalized by adding high-order terms to model broader bandwidth waves [K. B. Dysthe, Note on a Modification to the Nonlinear Schrdinger Equation for Application to Deep Water Waves, Proc. R. Soc. A, **369** (1979)], likewise inhomogenous (frequency dependent) corrections provide a realistic description of wind and viscosity [J. D. Carter and A. Govan, Frequency downshift in a viscous fluid, Eur. J. Mech. B, 59 (2016); D. Eeltink et al., Spectral up- and downshifting of Akhmediev breathers under wind forcing, Phys. Fluids **29** (2017)]. We will show that a low-dimensional truncation [A. Armaroli et al., Recurrence in the high-order nonlinear Schrdinger equation: A low-dimensional analysis, Phys. Rev. E 96 (2017)] allows us to describe the non-linear behavior of modulational instability under different wind forcing strengths. We will also discuss the impact of dispersive forcing/damping on the occurrence of rogue waves. Finally we will discuss the model limitations, by proposing a modified viscous dispersion relation and its implementation in a propagation equation.

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MS14

The Bidirectional Whitham Equations

In 1978, Hammack and Segur performed a series of tightlycontrolled laboratory water-wave experiments. The experiments were conducted in a long, narrow tank with relatively shallow undisturbed water and a wave maker at one end. Among many other things, they showed that analytic and asymptotic results obtained from the KdV equation compared favorably with measurements from the experiments. In 2016, Trillo et al. conducted new experiments and demonstrated that the Whitham equation models those experiments more accurately than does the KdV equation. In this talk, we will compare measurements from the Hammack and Segur experiments with numerical simulations of the St. Venant, KdV, Serre, and Whitham equations. We will focus on various forms of the Whitham equation including the Whitham equation with surface tension and the bi-directional Whitham equations.

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MS14

Breather Evolution in Shawllow Water

Breathers or envelope solitons correspond to a family of unstable and modulated wave trains, solutions of the Nonlinear Schrödinger equation. These envelope solitons are commonly used in various fields of physics, like optics, plasma, solids or surface waves. A family of breathers widely describes in the literature is the Akhmediev breathers (Akhmediev 1986, 1993). These solutions are periodic in space with an infinite period in time. Theoretically these solutions exist only for values of kh > 1.363, with h the water depth and k the wavenumber. For values lower than this threshold, solitons become stable and perturbations are no longer amplified. These solutions are extensively studied because they are good approximations of modulated surface waves observed in the ocean. The aim of this work is to propagate Akhmediev breathers over a variable bathymetry with a 1/200 slope. By selecting appropriate wave periods, wave trains evolve first in water depth where kh > 1.363, and then up to the shore for values where kh < 1.363. In the first part, the wave train is unstable, and the perturbation increases up to the focusing point. Therefore, when the wave train reaches the focusing distance, the amplitude of some waves of the group are much larger than the waves of the initial group. These waves can be identified as rogue waves. The question is to know if the rogue waves propagating in the shallow water region still exist or disappear.

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MS14

On the Orbital Stability of Elliptic Solutions to Focusing NLS

The focusing Nonlinear Schrodinger Equation admits a class of standing wave solutions, expressed in terms of elliptic functions. The linear stability of these solutions with respect to subharmonic perturbations (i.e., perturbations whose period is a multiple of the solution period) is well understood using techniques which rely on integrability. In this talk I show that the linear stability results can be strengthened to orbital (nonlinear) stability by using the conserved quantities of NLS to construct a Lyapunov functional.

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MS15

Interplay between the IST and the Unified Transform: Applications to PDEs on Graphs and Nonlocal Reductions

The so-called Unified Transform (or Fokas method) has emerged as a systematic framework to tackle initialboundary value (IBV) problems for linear and integrable nonlinear PDEs. Using the nonlinear Schrdinger (NLS) equation as main example, I will show various results using the ideas of that method. Firstly, using an appropriate matrix form of the Lax pair formulation, one can generalize the Unified Transform to formulate IBV problems on stargraphs in a way completely analogous to initial-boundary value problems on the half-line. I'll briefly mention how more general graphs could be tackled. Secondly, as a consequence of this framework I will show a converse to the result of Biondini, Fokas, Shepelsky linking the Unified transform with linearizable (integrable) boundary conditions to ISM with special symmetries on the scattering data: conversely, the ISM is a special case of my matrix Unified Transform with certain linearizable boundary conditions. In a sense, this provides further justification for the terminology "Unified" Transform. Finally, as a consequence of this result, I will show that the nonlocal reduction introduced by Ablowitz and Musslimani can be viewed as a standard local reduction applied to a matrix problem on the half-line. In fact, from the point of view of the reduction group theory, the nonlocal NLS and the standard NLS are the only two possible Z_2 reductions of the form I will consider.

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MS15

Zakharov-Shabat Scattering Problems with Nonzero Background

We present various results regarding the Zakharov-Shabat scattering problem with periodic boundary conditions.

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MS15

Interactions between Solitons and Radiation in Modulationally Unstable Media

The interaction between solitons and radiation in modulationally unstable media is studied by means of the focusing nonlinear Schrödinger (fNLS) equation with nonzero boundary conditions at infinity and with initial data allowing for the presence of a discrete spectrum. In particular, the long-time asymptotic behavior of the fNLS solution is characterized via the Deift-Zhou nonlinear steepest descent method for oscillatory Riemann-Hilbert problems. The growing jumps involved in the Riemann-Hilbert problem, which are the signature of modulational instability, are regularized via the Deift-Zhou method by appropriate deformations which are associated with different kinds of asymptotic behavior of the solution on the xt-plane.

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MS15

Inverse Scattering Transform and Soliton Solutions for Matrix Nonlinear Schrodinger Equations

We will discuss the Inverse Scattering Transform (IST) for two novel reductions of the matrix nonlinear Schrodinger equation which are integrable, and which are the analog of the modified Manakov system with mixed signs of the nonlinear coefficients, i.e., a nonlinearity in the norm which is of Minkowski type, instead of Euclidean type.

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MS16

Light Dynamics in Twisted Photonic Structures

In this talk we discuss properties of light propagation in photonic media where a topological property of twisting the structure in the direction of propagation induces novel dynamics. We present results in two scenarios: twisted nonlinear arrays and a twisted photonic crystal fiber.

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$\mathbf{MS16}$

Stokes Waves over a Constant Vorticity Flow

Abstract not available at time of publication.

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MS16

Slow Light Pulses in Two-level Media

In ruby crystals, slow light pulses were observed, and described using two-level Maxwell-Bloch equations with high polarizability damping. We compute that two regimes exist, depending on the ratio of medium-polarizability and level-inversion damping. When this ratio is moderate, soliton-like pulses exist. Damping decreases their amplitudes and speed. A precursor of radiation coexists, and dominates for strong damping and large damping ratio. Starting slowly, it accelerates to the speed of light.

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MS16

Instability and Noise in Nonlinear Optical Resonators

Modelocked pulses are of fundamental importance in a large number of laser and resonator applications. Dissipative solitons are the most important example, but a wide variety of other structures have been investigated in the past 20 years, including dark solitons, bright and dark soliton molecules, soliton crystals, and cnoidal waves or Turing rolls. For almost all of these systems, the nonlinear Schrdinger equation is a lowest-order model. Two examples are the Haus modelocking equation and the cubicquintic equation that are used to model passively modelocked lasers, and the Lugiato-Lefever equation that is used to model microresonators. The first question that arises is identifying stationary solutions of these equations. Once they are found, the next question that arises is determining the parameter regime in which they exist stably. Once this parameter regime is determined, it is often of interest to determine the impact of noise on these stationary solutions. While dynamical methods that can address these questions have been available in principle since the early 20-th century, effective computational methods to address these questions have only recently been developed. In this contribution, we describe our progress in developing these computational methods.

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MS17

Conservation Law Geometry and the Evolution of Nonlinear Waves

The study of the slow evolution of wavetrains is able to provide a surprising amount of information regarding their evolution. This talk will demonstrate the link between conservation law criticality of the wave (often associated with stability boundaries for the system) and the emergence of nonlinear PDEs governing its wavenumber. These equations present a qualitative picture into the evolution of wavetrains at the onset of instability. A coupled nonlinear Schrödinger equation is used to illustrate the theory, demonstrating how dark and bright solitary waves emerge and can lead to extreme wave events.

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MS17

One-dimensional BGK Model

We rigorously show that the solutions to one-dimensional BGK model decay to a subclass of the grossly determined solutions. In particular, we determine the spectrum and generalized eigenfunctions of the associated nonselfadjoint operator and derive the associated generalized Fourier transform and Parseval's identity.

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MS17

Homoclinic Snaking and its Structural Stability in Discrete Systems

Homoclinic snaking, i.e. a snaking structure in the bifurcation diagrams of localised solutions, has been observed in various continuous and discrete systems. While homoclinic snaking in continuous systems is due to a pinning effect between fronts and periodic states. In discrete systems, it is caused by the pinning of fronts to the imposed lattices. While homoclinic snaking in continuous systems have been analysed rather thoroughly, the phenomenon is almost unstudied in discrete systems. Using a so-called one-active-site approximation, we construct a theoretical study of snaking in 1D and 2D systems. Using the approximation we can explain geometrically the mechanism behind the homoclinic snaking. The occurrence of the snaking can be predicted as well.

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MS18

Nonlinear Kinetic Non-stationary Waves in Vlasov Plasmas: Past and Present

The study of nonlinear electron plasma waves (EPW) goes back to Buneman, Dawson and Whitham in the 1950s, following the seminal work of Landau, Case, van Kampen, Balescu, Lenard and Bernstein on the kinetic theory of linear plasma waves. As amplitudes of driving fields are increased, one progresses from linear waves to plasma echos (multiple linear waves beating), to trapped particle states (single mode nonlinear detuning saturation) such as nonlinear electron plasma waves (EPW), and then their destruction through mechanisms such as the side-band (or the trapped particle) instability. We will present examples of these physical states, their interactions and interdependencies. Theoretical and computational methods to capture these dynamical states will be described. Beyond linearly resonant waves, there exist nonlinear kinetic waves that do not have an infinitesimal amplitude limit, nor a fluid/low order moment description. KEEN waves are one example: Kinetic Electrostatic Electron Nonlinear Waves. We will describe how they are formed self-consistently by modifying the distribution function that can host wave structures, how they are are multi-mode, non-stationary, and ultimately composed of condensates of a small number of phase space partitions, each with its own dynamics. Interactions between EPW and KEEN waves will also be described, as well as their generalization to KEEPN waves in pair plasmas. Work sponsored by AFOSR FA9550-15-C-0036 and DOE FES SC0018283.

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MS18

Modern Algorithms for PIC Simulation of Laserplasma Interactions (LPI)

Leap-frog based explicit algorithms, either "energy conserving" or "momentum conserving", do not conserve energy discretely. Time-centered fully implicit algorithms can conserve discrete energy exactly,¹ which is desirable for long-term simulations, but introduce large dispersion errors in the light-wave modes, regardless of timestep sizes. This can lead to intolerable simulation errors where highly accurate light propagation is needed. In this study, we selectively combine the leap-frog and Crank-Nicolson methods to produce a low-dispersion, exactly energy-and-chargeconserving PIC algorithm. Specifically, we employ the leap-frog method for Maxwell equations, and the Crank-Nicolson method for particle equations. Such an algorithm admits exact global energy, local charge conservation, and preserves the dispersion properties of the leap-frog method for the light wave. The algorithm has been implemented in a code named iVPIC, based on the VPIC code² developed at LANL. We will present numerical results that demonstrate the properties of the scheme with sample test problems (e.g. Weibel instability run for 10^7 timesteps, and LPI applications).³

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MS18

Landau Damping and Collisions in Stable and Unstable Initial Velocity Distribution

In 1960 George Backus studied Collisionless Linearized Plasma Oscillations. Backus discussed results of Van Kampen, who solved the problem by superposing normal modes. Backus claims this implicitly introduced stability and the modes used were incomplete. In this talk we will investigate the completeness properties of normal modes and examine what happens when collisions are introduced to this technique.

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MS18

The Evolution of Non-Linear Steady-states in a 1D, 1λ , Driven, Electron Plasma Wave in a Vlasov-

 $^{^1 {\}rm Lapenta}$ and Stefano. Phys Plasmas 18.7 (2011): 072101 $^2 {\rm https://github.com/losalamos/vpic}$

³This work was performed under the auspices of the U.S. DOE by the Los Alamos National Security, LLC Los Alamos National Laboratory and was supported by the DOE Office of Fusion Energy Science.

Fokker-Planck-Poisson System

Using a Vlasov-Fokker-Planck code based on the sphericalharmonic decomposition of the electron velocity distribution function $(e^{-}$ VDF), we describe the evolution of a driven electron plasma wave (EPW) in a uniform plasma in one spatial dimension. We find nonlinear steady-state solutions when the bounce frequency, ω_B , is of the same order as the collisionless (Landau) damping rate, γ_{LD} , and much greater than the electron-electron collision frequency, ν_{ee} , i.e. $\omega_B \sim \gamma_{LD} \gg \nu_{ee}$. The EPW amplitude exhibits transient oscillations due to an interplay between electronelectron collisions and the non-linear frequency shift. The oscillations are repeated for some number of cycles, N_{osc} , which depends on γ_{LD}/ν_{ee} , until the system settles into a steady-state. We will also highlight differences in the nonlinear plasma response when the collisions are described by the linearized Fokker-Planck operator versus the more simplified one first suggested by Lenard-Bernstein and adopted by Zakharov and Karpman among many others. We will extract the energy found in the resulting plasma waves as a function of drive amplitude, drive mode number and collision frequency in various asymptotic distinguished limits between the bounce frequency, linear Landau damping rate and the collision rate.

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MS19

Wave-induced Stress and Breaking of Sea Ice in a Coupled Hydrodynamic Discrete-element Waveice Model

In this paper, breaking of a continuous sea ice sheet by waves is analyzed with a coupled sea ice-wave model. The sea ice module is a discrete-element bonded-particle model, in which ice is represented as cuboid 'grains' floating on the water surface that can be connected to their neighbors by elastic joints. The joints break if instantaneous stresses acting on them exceed their strength. The wave module is based on an open-source version of the Non-Hydrostatic WAVE model (NHWAVE). The two modules are coupled with boundary conditions for pressure and velocity, exchanged at every wave model time step. In a series of two-dimensional (one vertical and one horizontal dimension) simulations with varying sea ice properties and incoming wavelength it is shown that wave-induced stress reaches maximum values at a certain distance from the ice edge, dependent on ice properties (thickness, elastic modulus), but not on the length of the incoming waves. Consequently, both regular and random (Jonswap spectrum) waves break the ice into floes with almost identical sizes. When sea ice properties are spatially variable, a narrow, approximately normal floe-size distribution is produced. In the last part of the paper, results of simulations with free-slip and partial-slip ice-water boundary conditions are compared in order to get insight into the role of friction on wave propagation into the ice and the resulting breaking patterns.

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MS19

The Evolution of Scaling Laws in the Sea Ice Floe Size Distribution

Abstract not available at time of publication.

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MS19

Modelling Wave-induced Ice Breakup In the Marginal Ice Zone

The intensification of wave climate in the Southern Ocean tends to enhance the positive icealbedo feedback. Waves break up the sea ice increasingly deeper within the outer margin of the Antarctic sea ice cover, referred to as the marginal ice zone (MIZ). During the melting season, this process accelerates sea ice loss and acts as a controlling mechanism for sea ice extent. Although much attention has been given to study the effect of the MIZ on the propagation of ocean waves, very little is known about the impact of waves on the morphology of the sea ice. The latter is principally governed by the break-up of bent sea-ice floes as a result of wave interaction. I will discuss a sub-grid scale process-informed model describing in a coupled framework (i) wave scattering by the sea ice and (ii) ice breakup caused by the waves. I will show that the model is able to simulate a positive wave/ice feedback, though which waves gradually march forward in the MIZ by breaking up the sea ice cover and therefore reducing attenuation from scattering.

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MS19

Seasonal and Interannual Variations of Floe Size Distribution in the Pan-Arctic Ice-Ocean Modeling and Assimilation System

The Arctic sea ice cover consists of floes of varying thicknesses and sizes, particularly in the marginal ice zone (MIZ). To better describe the state of sea ice, both an ice thickness distribution (ITD) and a floe size distribution (FSD) are needed. We have developed a FSD theory that is coupled to the ITD theory of Thorndike et al. [1975] to simulate the evolution of FSD and ITD jointly [Zhang et al., 2015, 2016]. The FSD theory includes a FSD conservation equation which takes into account changes in FSD due to ice advection, thermodynamic growth, lateral melting, ridging, and opening. It also includes mechanical redistribution of floe size due to ice fragmentation induced by winds, waves, and currents. The formulation of the mechanical redistribution is essential to simulate FSDs that follow a power law as observed by satellites and aerial surveys. The FSD theory has been implemented in the Panarctic IceOcean Modeling and Assimilation System (PI-OMAS). Model calibration and validation are conducted using TerraSAR-X and MODIS derived FSDs and a range of ice thickness observations. Here we will examine the seasonal and interannual variations of FSD in PIOMAS over

the period 1979 to the present after a review of the powerlaw obeying FSD theory. We will investigate the role of ice fragmentation and lateral melting in altering FSDs in the MIZ. We will also examine how changes in FSD impact the ice mass balance in the Arctic Ocean.

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MS20

Bidirectional Fully Dispersive Models for Water Waves over an Uneven Bottom

Long wave approximation is applied to the Hamiltonian formulation of water waves problem

$$\eta_t = rac{\delta \mathcal{H}}{\delta \Phi}, \qquad \Phi_t = -rac{\delta \mathcal{H}}{\delta \eta}$$

We show that changing only the total energy \mathcal{H} and meanwhile staying in the same framework of accuracy, one can arrive to different Whitham-Boussinesq systems. Some of them have been given attention to in recent years and some of them are new. In fact, all these systems are different in the sense of well-posedness and numerical stability. Though they all approximate the Euler system of an inviscid potential fluid similarly, i. e. they are fully dispersive in linear part and in nonlinear part they are approximately equivalent to the standard Boussinesq nonlinearity. These systems are suitable to describe water problems with variable depth topography. They can combine gravitational and capillary or elastic effects. Their accuracy is checked by comparing with solutions of the Euler system given by the conformal mapping technique and with laboratory experiments.

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MS20

KdV Equations and Wave Breaking in Undular Bores

A river bore is a transition between different flow depths which is generally caused by tidal forces and travels upstream. Similar flows can also be realized in controlled environments such as wave flumes, and a number of studies have been conducted in order to understand some of the main features of bores. In particular, in Favre's work (Ondes de Translation, Dunod, Paris, 1935) a dedicated series of laboratory experiments is described which is aimed at classifying different types of bores. In the relatively simple situation of a wave flume, one may without loss of generality assume that the upstream flow depth is the undisturbed depth of the tank, say h_0 , and the incident depth is $a_0 + h_0$. Defining the bore strength by the ratio a_0/h_0 , it was found by Favre that there are three main bore types. If the bore strength is below 0.28, the flow is laminar and oscillations of the free surface start to develop. Since in this case, none of the wave are breaking, this case is termed the purely undular bore. If the ratio a_0/h_0 exceeds 0.28, then the leading wave behind the transition front starts to break, and while the flow still features oscillations, there is some turbulence associated with the breaking waves. If the ratio exceeds 0.75, a fully turbulent bore appears. The main purpose of this lecture is to explore whether the ratio 0.28 can be found using some fairly simple wave models such as the KdV equation.

Henrik Kalisch

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MS20

Relationships between Pressure, Wave-height and Bathymetry

In this talk, I will describe a new wave to view relationships between the pressure at the bottom of a fluid, the shape of the bathymetry, and the surface elevation of a wave for steady flow. Given a measurement of any one of these physical quantities (pressure, bathymetry, or surface elevation), a numerical representation of the other two quantities is obtained via a nonlocal nonlinear equation obtained from the Euler formulation of the water-wave problem without approximation. From this new equation, a variety of different asymptotic formulas are derived. The nonlocal equation and the asymptotic formulas are compared with both numerical data and physical experiments.

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MS21

The Instabilities of Elliptic Solutions of Integrable PDEs

The elliptic solutions of the various integrable equations have been examined and their spectral instabilities are characterized. In addition, we can characterize the subharmonic perturbations with respect to which the solutions are (nonlinearly) orbitally stable. The focusing NLS equation and the sine-Gordon equation are used as examples.

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MS21

The Semiclassical Defocusing Davey-Stewartson II Equation

Studying the Cauchy problem for the defocusing Davey-Stewartson II equation in the semiclassical limit using inverse scattering techniques requires the asymptotic analysis of a linear elliptic equation in the plane to construct the scattering data. We will describe some partial results and ongoing challenges related to this problem.

<u>Peter D. Miller</u> University of Michigan, Ann Arbor millerpd@umich.edu

MS21

Small-dispersion Limits for Focusing NLS with Periodic BC

We study the small dispersion limit of solutions of the focusing NLS equation with small dispersion and periodic boundary conditions using suitable WKB expansions for the associated scattering problems.

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Gino Biondini State University of New York at Buffalo Department of Mathematics biondini@buffalo.edu

MS21

Numerical Inverse Scattering for Step-like Potentials

We consider the problem of computing the inverse scattering transform for the Schrödinger equation on \mathbb{R} when the potential q(x) satisfies $\lim_{x\to+\infty} q(x) \neq \lim_{x\to-\infty} q(x)$. From the theoretical side, we build on the work of Cohen and Kappeler (1985) and Andreiev et al. (2016). In particular, we demonstrate how the use of both left and right reflection coefficients is necessary, in contrast to decaying initial data. Applications to dispersive shock waves in the KdV equation are discussed.

<u>Thomas Trogdon</u> <u>University of California Irvine</u> ttrogdon@uci.edu

MS22

New Methods for Diffusive Wave Field Theory

This talk will demonstrate how the unified transform method developed by Fokas and collaborators can be extended to provide new solution methods for a class of boundary value problems arising in an area known as diffusive wave field theory. The new techniques will be demonstrated in the context of specific example applications. [Joint work with Jordan Hauge].

Darren G. Crowdy, Jordan Hauge Imperial College London d.crowdy@imperial.ac.uk, minitutorial title: dynamic mode decompositions a

MS22

Dispersive Quantization in Nonperiodic Domains

We apply the UTM and related methods to the surprising and as yet poorly understood phenomenon of dispersive quantization, a.k.a. the Talbot effect. This refers to the remarkable observation that solutions to certain classes of dispersive wave equations possessing discontinuous initial data exhibit a continuous but fractal profile at irrational (relative to the length of the interval) times but "quantize" at rational times into a piecewise continuous profile that is constant. By interpolating between, on the one hand, periodic boundary conditions and the given fixed boundary conditions, at what stage and how do the dispersive quantization effects disappear? Preliminary results indicate that quantization is still present in (at least) the case of energyconserving non-periodic boundary conditions. In general, we would like to know how the boundary conditions affect the behavior of the solution.

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MS22

Nonlocal Problems for Linear Evolution Equations

Linear evolution equations, such as the heat equation, are commonly studied on finite spatial domains via initialboundary value problems. In place of the boundary conditions, we consider "multipoint conditions', where one specifies some linear combination of the solution and its derivative evaluated at internal points of the spatial domain, and "nonlocal' specification of the integral over space of the solution against some continuous weight.

David Smith

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MS23

Bifurcation to Locked Fronts in a Two Component Reaction-diffusion System

We study invasion fronts and spreading speeds in two component reaction-diffusion systems. Using Lins method, we construct traveling front solutions and show the existence of a bifurcation to locked fronts where both components invade at the same speed. Expansions of the wave speed as a function of the diffusion constant of one species are obtained. The bifurcation can be sub or super-critical depending on whether the locked fronts exist for parameter values above or below the bifurcation value. Interestingly, in the sub-critical case the spreading speed of the system does not depend continuously on the coefficient of diffusion.

Matt Holzer

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MS23

Selection and Bifurcations of Depinning Fronts Outside the Homoclinic Snaking Region in the Planar Swift-Hohenberg Equation

In this talk, I will present a new numerical continuation algorithm to investigate invading depinning fronts near the homoclinic snaking region in the planar Swift-Hohenberg equation. These fronts typically select a far-field wave number and travel at a non-constant propagation speed making them difficult to set up the boundary value problem. We find that these fronts have a rich bifurcation structure. In particular, we find that fast depinning fronts may regain transverse stability in the Swift-Hohenberg equation with a cubic-quintic nonlinearity.

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MS23

Run-and-Tumble Processes: Equidistribution, Ripples, Clusters, and Blowup

Motivated by the observation of rippling patterns and fruiting bodies in colonies of Myxobacteria, we study a simplest' modes for dynamics of populations, where agents run either left or right with constant speed, and reverse direction depending on local population densities in a nonlinear fashion. One roughly observes three qualitative regimes in global behavior: "equidistribution", "ripples and waves", and "blowup". We describe and present several results supporting a conjecture, which distinguishes between those regimes in terms of spatially constant equilibria. In particular, A global Lyapunov function establishes equidistribution for a class of tumbling rates, and one can find ripples, waves, and stationary clusters for tumbling rates that do not belong to this class. Lastly, we show finite-time blowup in a third scenario.

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MS24

LoKi and Valhalla Simulations of Nonlinear Kinetic Plasma Structures including Collisions

We consider the capacitor model for resonance absorption in steep density profile plasmas. We treat resonantly driven electron plasma waves (EPW) far into the nonlinear regime in a plasma with a Gaussian density profile near and somewhat removed, below the peak density. We examine the kinetic nonlinear plasma response that includes trapping and wave breaking. We are especially interested in the wave breaking regime and how it influences the local distribution function evolution at resonance. This is a challenging computational problem because a minority of particles are hurled large distances at very large velocities atop a family pf particles that are trapped, which means that a uniform grid modeling is not ideal to represent these dynamics. We show simulation results with the Vlasov-Maxwell code LOKI, which does utilize a uniform grid and highorder integration techniques. We also sketch how Valhalla could be used, in its AMR form, to render such simulations more efficient. Our results will be compared to regular PIC codes, such as BRPY of U Alberta, and to quiet PIC methods, such as SFK of LLNL and its adaptive generalization, PASTEL, of PRI. Work performed under AFOSR FA9550-15-C-0036, DOE FES SC0018283, and by LLNL under Contract DE-AC52-07NA27344.

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MS24

Vlasov and Particle Simulations of Nonlinear Plasma Waves in the Presence of Self-generated Magnetic Fields

The existence of a large amplitude, nonlinear electron plasma waves unavoidably involves trapping some fraction of plasma electrons. The dynamics of these waves is thus related to the behavior of trapped particles. Here we study the effects of magnetic fields on the generation and longevity of nonlinear electron plasma waves. We will compare the effects of an externally applied (spatially nonuniform) magnetic field to those generated self-consistently (by either the Harris or Weibel instability). We will consider three distinct regimes: $\omega_{ce} \ll \omega_{pe}, \ \omega_{ce} \approx \omega_{pe}$, and $\omega_{ce} \gg \omega_{pe}$. Our results will primarily be drawn from simulations studies using both Vlasov-Maxwell and marcoparticle methods. We will compare electrostatically driven modes to full electromagnetic treatments. Our discussion will highlight the numerical challenges posed by these problems and their relation to the features and drawbacks of the simulation methods. This work support by the US DoE under grants DE-SC0018363 and DE-SC0018283 and by AFOSR grant FA9550-15-C-0036.

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MS24

Driven Kinetic Nonlinear Plasma Waves in Multiple Dimensions Controlled by Externally Generated Transient Magnetic Fields

Nonlinear kinetic plasma structures are often studied using purely longitudinal excitations, such as the ponderomotive force generated by the optical mixing of crossing laser beams. The most common high frequency wave so generated is the electron plasma wave (EPW). The nonlinear stability of such waves in the kinetic regime is critically dependent on a physical mechanism known as (particle orbit) trapping. In higher dimensions, the dynamics of trapping is far more complex. Waves that would be undamped in 1D can suffer side losses and decay in more than one dimension. Deciphering the phase space structures of driven waves in higher dimensions is the principle objective of this work. A secondary objective is to ascertain the feasibility of maintaining or re-attaining stability via the use of externally imposed magnetic(B) fields. Since particle orbits tend to circulate around strong B fields, it is reasonable to expect that stable configurations can result even when the ponderomotive force is transversely localized and thus susceptible to trapping side losses. B fields could expand the range of parameters where finite amplitude waves persist without damping. We will consider examples from High Energy Density Physics where fast time scale laser pulses can be used to drive EPWs and to generate B fields as well through mechanisms such as Weibel instability. The combined application of which might allow new stable configurations for plasma wave structures far from equilibrium.

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MS24

Shape Function Kinetics (SFK) Simulations of Electron Plasma Waves and Keen Waves

Simulations of electron plasma waves (EPW) and KEEN waves using the shape function kinetics (SFK) method are presented. SFK uses discrete particles described by shape functions of compact support. The particle positions and shape evolve in response to internal velocity spread and external forces. Remapping is necessary to maintain accuracy. A mesh-free remapping method using expectationmaximization and a mixture model enhanced via Anderson acceleration is introduced. This method is compared to a complimentary technique, PASTEL, Phase-Space Adaptive Sparse Tiling and Effective Lagrangian. Simulations of standard test problems, numerically-demanding KEEN waves, and resonantly-driven EPWs in a nonuniform density profile where trapped particles and wave breaking play significant roles, illustrate the advantages of the method. Phase space adaptivity is required to efficiently simulate KEEN and resonantly-driven nonlinear plasma waves because a minority of particles with intricate orbits play an essential role in both phenomena. SFK holds the promise of efficient, noise-free, and non-dissipative generalization to multiple dimensions. This work was performed under the auspices of the U.S. DOE by LLNL under Contract DE-AC52-07NA27344 and sponsored by grants from AFOSR FA9550-15-C-0036 and DOE DE-SC0018283

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MS25

Dynamics and Structure of Active Fluids under Confinement

We numerically study the dynamics and structure of a simple apolar active fluid under confinement through an active liquid crystal model. At the micro scale, we consider a suspension comprises microparticles that are are nonmotile but mobile, and exert active dipolar stresses on the ambient liquid. We show that such a system can be described by a coarse-grained liquid crystal model that is derived from a microscopic Doi-Onsager kinetic theory. Then we use the macro model to study the collective dynamics of the apolar fluid confined in straight, circular and bi-concave geometries. We also explore the soft confinement effect by encapsulating a dense active suspension in a droplet whose motion is driven by the internal active nematic flows.

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MS25

The Role of Slip and van der Waals Forces in Fiber Coating

Viscous thin liquid films flowing down a vertical fiber can exhibit interesting dynamics via droplet formation driven by a Rayleigh mechanism with the presence of gravity. Inspired by recent experiments by Sadeghpour et al. (2017), we vary the nozzle geometry to parameterize the family of traveling wave solutions in the Rayleigh-Plateau regime, whose propagation velocity is slightly greater than the values predicted by existing models. Here we present a modified lubrication model to resolve this discrepancy by including additional slip and van der Waals forces. In particular, we show that slip and van der Waals forces promote the traveling wave propagation, which agrees with available experimental data. In addition, we show that the proposed model better captures the bifurcation between Rayleigh-Plateau and isolated droplets regimes by numerically investigating the instability of traveling wave solutions which happens for thin fibers.

Hangjie Ji

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MS25

Signatures of Slip in Thin-film Flows

In this talk the evolution of a liquid polymer film from its initial rupture to the subsequent dewetting dynamics of liquid rims into a polygonal pattern, their destabilisation into droplets and satellite droplets will be discussed. The large-scale pattern of this process is controlled by the local interaction of polymer chains in a boundary layer at the interface with a solid or another liquid surface. As a paradigm for other more complex systems, it will be shown how these local interactions give rise to the formation of a coherent structure at the meso-scale that can be characterized in terms of an *effective slip* and how this gives rise to a host of interesting mathematical challenges to capture the evolution of the film on the large scale.

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MS25

Transiently Networked Solutions

Complex (visco-elastic) fluids generally consist of structural elements at the mesoscale ranging from long chain polymers which transiently entangle to actual space filling transient networks. The mesoscale configurations of these fluids are affected by the macroscale flow and vice versa. Modeling of these transiently network fluids at the mesoscopic level has the advantage that macroscale closure approximations are avoided, and that local properties of the elements such as their stretch, orientation and concentration - and particularly local nonlinearities, are accounted for. In this talk, we present a stochastic dumbbell model of wormlike micellar solutions in which the beads of chains of dumbbells connect and disconnect continuously forming junctions of up to f beads. In this analysis the topology of the strands is tracked in time allowing for a full local analysis. Flow curves resulting from distinct breakage and reforming energy functions are simulated and the macroscale predictions are presented.

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Lin Zhou

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MS26

Rigorous Numerical Verification of Properties of Traveling Waves

We discuss recent progress in computer assisted techniques to prove properties about traveling waves. In particular, we discuss rigorous numerical verification techniques that depend on the use of interval arithmetic to prove stability of traveling waves. Some algorithms that work well using doubles fail when carrying out interval arithmetic computations. We describe our recent work in developing algorithms that work with interval arithmetic. Topics may include work done in collaboration with J.D. Mireles James, Bjorn Sandstede, Kevin Zumbrun, or others.

<u>Blake Barker</u>

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MS26

On the Stability of Fronts with Marginally Unstable Essential Spectrum

We study a planar front solution for a class of reactiondiffusion equations in the case when the essential spectrum of the linearization about the front touches the imaginary axis. The spectrum of the wave is stabilized by an exponential weight. For perturbations that belong to the intersection of the exponentially weighted space with the original space without a weight, we use a bootstrapping argument to show that initially small perturbations to the front remain bounded in the original norm and decay algebraically in time in the exponentially weighted norm.

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MS26

Center Manifolds for a Class of Degenerate Evolution Equations and Exponential Decay of Kinetic Shocks

We construct center manifolds for a class of degenerate evolution equations including the steady Boltzmann equation and related kinetic models, establishing in the process existence and behavior of small-amplitude kinetic shock and boundary layers. Notably, for Boltzmann's equation, we show that elements of the center manifold decay in velocity at near-Maxwellian rate, in accord with the formal Chapman-Enskog picture of near-equilibrium ow as evolution along the manifold of Maxwellian states, or Grad moment approximation via Hermite polynomials in velocity. Our analysis is from a classical dynamical systems point of view, with a number of interesting modifications to accommodate ill-posedness of the underlying evolution equation.

Alin Pogan

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Kevin Zumbrun Indiana University USA kzumbrun@indiana.edu

MS26

On the Ground States for the Schroedinger Equation under Magnetic Trap Potential

We consider the ground states of the Schroedinger equation in \mathbb{R}^n , subject to magnetic traps and p^{th} power nonlinearity. In particular, we show that all waves are orbitally stable in the regime 1 and we discuss fairly general instability by blow up results in the case $<math>p > p_n > 1 + \frac{4}{n}, \omega > 0$, where ω is the spectral parameter of the wave.

<u>Atanas Stefanov</u> University of Kansas stefanov@ku.edu

MS27

Nonlinear Schrödinger Equation and the Universal Description of Dispersive Shock Waves

The Nonlinear Schrödinger (NLS) equation and the Whitham modulation equations both describe slowly varying locally periodic wavetrains. The commonalities and differences between the two descriptions have been widely discussed in the literature. In this talk I will show how the defocusing NLS equation combined with the dispersive shock wave (DSW) fitting method provide a universal description of DSWs in integrable and non-integrable systems.

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MS27

Solitary/Linear Wave-mean Flow Interaction

The Whitham modulation equations are a system of first order, quasi-linear partial differential equations that describe the slow variation of a nonlinear, periodic, traveling wave's parameters. The consideration of the one-phase Whitham equations in the small wavenumber or small amplitude regime leads to an efficient, accurate description of the coupling between a mean, hydrodynamic flow and a solitary wave or linear wavepacket, respectively. Generically, the Whitham equations in these regimes can be diagonalized in terms of Riemann invariants. Hydrodynamic reciprocity enables the non-trivial extension of this description to a compressive mean flow, i.e., one with dispersive shock waves, which would otherwise require a degenerate two-phase modulation description. In this talk, some recent results in this direction and their physical implications will be presented.

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MS27

Shock Dynamics of Phase Diagrams: Maxwell Relations, Collective Phenomena and Integrability

A phase transition denotes a drastic change of state of a thermodynamic system due to a continuous change of parameters. Inspired by the theory of nonlinear conservation laws and shock waves we develop an approach to phase transitions based on the solution of Maxwell relations. This theory provides an exact mathematical description of discontinuities of order parameters and phase transitions via nonlinear integrable PDEs, it allows to classify universal classes of equations of state and interpret the occurrence of critical points in terms of the dynamics of nonlinear shock wave fronts. The approach is shown at work in the case of mean field magnetic and fluid models, nematic liquid crystals and random graphs.

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MS27

The Dispersive Riemann Problem in Nonlinear Fiber Optics

A classical problem for nonlinear hyperbolic equations, named after Riemann, is the evolution of a step-like initial condition. We show that optical fibers offer the opportunity to experimentally address such a problem in the regime where the propagation is accurately described by the defocusing nonlinear Schrödinger equation. At leading order, the decay of the step is ruled by a non-dispersive hyperbolic 2x2 model (shallow water equations) and hence occurs toward an emerging wave pair. Each wave can be of rarefaction type or a shock wave, respectively, and the pair is typically connected by a constant state. The presence of dispersion, however, turns the shocks into dispersive (oscillating) shock waves. In this regime, one can predict the existence of different scenarios of step decay by resorting to Whitham modulation theory. In our fiber optics experiment, by suitably controlling the input step in power and instantaneous frequency, equivalent to density and velocity of the photon fluid, we are indeed able to access different regimes and characterize the phase transitions between them. The experiment also constitutes, to the best of our knowledge, the first fully quantitative test of Whitham theory. Also the connection to typical problems of fluid dynamics such as the dam breaking or the piston problem are highlighted. In this context, the presence of vacuum points or cavitating states assumes a particularly relevant role.

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MS28

Initial-boundary Value Problems for a Class of Nonlocal PDEs

We implement the unified transform method to study initial-boundary value problems for a class of non-local evolution PDEs. After formulating the general theory, we discuss several examples, comparing the results with those arising for standard evolution PDEs.

Gino Biondini

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MS28

Initial and Boundary Value Problems for Nonlinear Evolution Equations

In this talk we shall discuss recent progress on the wellposedness of the initial and boundary value problem for several important partial differential equations, including the Boussinesq, the nonlinear Schrödinger (NLS), the Korteweg-de Vries (KdV), and the Camassa-Holm (CH) equations. For these equations, we shall present wellposedness results in low (Sobolev) and high (analytic) regularity data spaces.

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MS28

Stokes Flow in Polygonal Channels

Motivated by modelling challenges arising in microfluidics, we consider low-Reynolds-number flow in a twodimensional channel with different widths in the upstream and downstream directions. The channel geometry is approximated by a polygonal domain with angled edges at 'transition' points. We consider a pressure-driven flow with different inlet and outlet velocities related via the flux balance condition and obtain semi-analytical solutions using new transform methods. Our aim is to examine how different parameter choices affect the resulting flow.

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MS28

Numerical Inverse Scattering For the Sine-Gordon Equation

The sine-Gordon equation is a nonlinear PDE which appears in differential geometry, superconductivity and a number of other applications. It is known to be integrable and was solved by Kaup using the Inverse Scattering Transform (IST). In 2012, Trogdon, Olver and Deconinck implemented the IST for the Korteweg-de Vries and modified Korteweg-de Vries equations. The same idea is applied to the sine-Gordon equation, extended so as to account for the extra singularity appearing in the IST for the sine-Gordon equation. In this paper we implement the Inverse Scattering Transform for the sine-Gordon equation using the ISTPackage developed by Trogdon and the RHPackage by Olver. Our numerical experiments show that the method is spectrally accurate. Since x and t are parameters in the method, one computes only at the point of interest and time stepping is not required as opposed to traditional numerical methods for time evolution equations.

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MS29

Domain Walls in Landau-Lifschitz Equations with Spin Torque Term

Abstract not available at time of publication.

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MS29

An Action Functional Approach to Localized Patterns and its Application to the Spatial Heterogeneity

We treat existence and stability problem of stationary pulse solutions in a singularly perturbed three-component FitzHugh-Nagumo model. Combining geometric singular perturbation techniques and an action functional approach, we obtain explicit conditions determining the pulse width and their stability information. The stationary problem, corresponding to Euler-Lagrange equation of an action functional, is reduced to a problem finding critical points of the function with respect to the parameters. The pinned solutions in the presence of spatial heterogeneity are also investigated by computing their pinning distance to the discontinuity. The analytical results are under-pinned with numerical simulations.

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MS29

Anomalous Scaling of Hopf Bifurcation Thresholds of Localized Spot Patterns in 2-D

For a singularly perturbed two-component reactiondiffusion system in a bounded 2-D domain admitting localized multi-spot patterns, we provide an anomalous scaling threshold for the reaction-time constant τ that determines stability to temporal oscillations of the spot amplitudes. In the limit of large inhibitor diffusivity, the linear stability is determined by the spectrum of a class of nonlocal eigenvalue problems (NLEPs). In a certain parameter regime, we show from a new parameterization of the NLEP that no Hopf bifurcations leading to temporal oscillations in the spot amplitudes can occur for any $\tau \sim \mathcal{O}(1)$. Instead, by deriving a new modified NLEP appropriate to the regime $\tau \gg 1$, we show that a Hopf bifurcation will occur at some $\tau = \tau_H \gg 1$, where τ_H has an anomalous scaling law in the activator diffusivity $\varepsilon^2 \ll 1$.

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MS29

The Stability of Hotspots for a Three-component Reaction-diffusion Model of Urban Crime with Focused Police Patrol

The existence and stability of localized patterns of criminal activity is studied for a three-component reaction-diffusion model of urban crime. Such patterns, characterized by the concentration of criminal activity in localized spatial regions, are referred to as hotspot patterns and they occur in a parameter regime far from the Turing point associated with the bifurcation of spatially uniform solutions. Singular perturbation techniques are used to construct steady-state hotspot patterns in a one dimensional domain, and a new class of nonlocal eigenavalue problem with two nonlocal terms is derived that determines the stability of these hotspot patterns to $\mathcal{O}(1)$ time-scale instabilities. In the context of this model we discuss optimal strategies for the police deployment to destabilize the crime hotspots. In the absence of police, we show that the basic crime model

admits solutions where new hotspot patterns can nucleate from a quiescent background. Global bifurcation diagrams are computed numerically, and the results are compared with full numerical simulations.

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MS30 Bifurcations on Quantum Graphs

We consider some bifurcation problems in quantum graphs, i.e. systems in which a PDE is solved along the edges of a graph, with consistency conditions imposed at the vertices. We compare these results to simpler problems on discrete graphs.

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MS30

On-site and Off-site Bound States of the Discrete Nonlinear Schroedinger Equation and the Peierls-Nabarro Barrier

We construct several families of symmetric localized standing waves (breathers) to the one-, two-, and three-dimensional discrete nonlinear Schroedinger equation (DNLS) with cubic nonlinearity using bifurcation methods about the continuum limit. Such waves and their energy differences play a role in the propagation of localized states of DNLS across the lattice. The energy differences, which we prove to exponentially small in a natural parameter, are related to the Peierls-Nabarro Barrier in discrete systems, first investigated by M. Peyrard and M.D. Kruskal (1984). These results may be generalized to different lattice geometries and inter-site coupling parameters. Finally, we discuss the local stability properties of these bound states. This is joint work with Michael I. Weinstein.

Michael Jenkinson

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MS30

Different Dynamical Behaviors of Dispersive Waves from Weakly Nonlinear to Strongly Nonlinear

Regimes

Strongly nonlinear dispersive waves are challenging to study. We use two examples to show the different dynamical behaviors of dispersive waves from weakly nonlinear to strongly nonlinear regimes. For beta Fermi-Pasta-Ulam systems, we show that in their long time dynamics often there is an embedded effective linear stochastic structure with renormalized dispersion relation, which controls spatiotemporal structures and energy transport. Scaling exponent of momentum and heat spread are different in weakly and strongly nonlinear regimes. For the coupling between internal and surface waves, we show that in strongly nonlinear regimes surface waves are located at the leading edge of a large-amplitude internal wave seen in various experimental observation. This asymmetric behavior is induced by the class 3 triad resonance between internal waves and surface waves. In weakly nonlinear regimes, we do not observe such spatiotemporal behavior of surface waves.

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MS30

Seeking an Effective Dispersion Relation in Solutions to the NLS and Measuring Effective Nonlinearity

The linear part of the Nonlinear Schrödinger Equation (NLS) $(iq_t = q_{xx})$ has dispersion relation $\omega = k^2$. We don't expect solutions to the fully nonlinear equation to behave nicely or have any kind of effective dispersion relation like this. However, I have seen that solutions to the NLS are actually weakly coupled and are often nearly sinusoidal in time with a dominant frequency, often behaving similarly to modulated plane waves. In fact, these highly nonlinear solutions eventually end up behaving more and more linearly.

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MS31

Nonlinear Dynamics in Honeycomb Optical Lattices

By arranging micrometer sized dielectrics in a honeycomb pattern called optical graphene, researchers are able to reproduce in optics the remarkable properties seen in carbon based graphene. In particular, Dirac points, or conical degeneracies, are found in the dispersion bands, allowing for relativistic like effects. This talk will present results on the interaction between Dirac points and the presence of edges, non-linearity, and PT-symmetric gain-loss terms in the lattice. We will address methods for determining the existence of topologically protected edge modes as well as how PT symmetric perturbations modify the Dirac cone structure.

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MS31

Dynamics and Noise Performance of Periodicallystationary Pulses in Fiber Lasers

Fiber lasers operating in the dispersion-managed and similariton regimes generate periodically-stationary pulses that undergo large changes each round trip and which form the basis of frequency combs for applications to time and frequency metrology. Using a lumped rather than an averaged system model, we develop an efficient computational method to determine periodically-stationary solutions. We assess the stability of these solutions and we quantify the noise performance of the system in terms of the variances of the central time and phase of the pulse, and the width of lines in the frequency comb. Specifically, we derive a formula for the comb linewidths in terms of the timing and phase variances, and we show that for fiber lasers operating in the similariton regime the minimum linewidths occur at a small negative value of round trip dispersion.

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MS31

Non-linearity Induced Non-reciprocity and Topological Transitions in Optical Metamaterials

Reciprocity is a fundamental principle in optics requiring that signal transmission between two points in space is the same for both transmission directions. Breaking reciprocity is important in the design of isolators and circulators and also in topological insulators. To this end one option is using nonlinearities combined with spatial asymmetries. This approach has been mainly applied to isolators, yet even in this simple case, the reported designs have poor performance regarding isolation and transmission. We identify the reason for this trend, show how nonlinear isolators can be improved and demonstrate how nonlinearities can be used for the design of more complex structures, such as circulators and topological insulators. We show that time-reversal symmetry imposes a fundamental bound between asymmetry and transmission resulting in a trade-off between transmission and isolation intensity range for isolators with a single nonlinear resonator. This trade-off can be overcome by using multiple resonators allowing to achieve transmission close to unity over a desired range of input intensities. We also show that systems with multiple resonators can lead to even more exciting functionalities, such as circulators and topological insulators. Our results show the great possibilities of nonlinear resonators for nonreciprocal devices and open paths for exciting research in this emerging research field.

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MS31

Resonant vs Non-resonant Collapse Events of Optical Beams

It is well-known that the high-intensity gaussian beam tends to collapse in free space under the effect of selffocusing. Nonlinear Schrodinger Equation in critical dimension is the mathematical model that correctly describes it. Collapse is a fundamental phenomenon and is wellstudied. It has an important application in the area of filamentation, collapse leads to ionization of the air and creation of plasma channel. In this talk, I will describe new types of collapse events of two-color beams being both at resonance and non-resonance.

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MS32

Spectral Stability of Inviscid Roll Waves

In this talk, we will discuss the stability of roll-waves, which are well-known hydrodynamic instabilities associated with the destabilization of constant flow consisting of a periodic series of shocks separated by smooth monotone wave profiles, advancing down an incline at constant speed. Modeling such waves as discontinuous traveling wave solutions of the inviscid St. Venant equations, we will present results of a recent systematic analytical and numerical stability study based on a periodic Evans-Lopantinski determinant, analogous to the periodic Evans function of Gardner in the (smooth) viscous case. This is joint work with Pascal Noble (Toulouse), L. Miguel Rodrigues (Rennes), Zhao Yang (Indiana) and Kevin Zumbrun (Indiana).

Mathew Johnson

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MS32

Stability of Traveling Waves in a Model for a Thin Liquid Film Flow

We consider a model for the flow of a thin liquid film down an inclined plane in the presence of a surfactant. The model is known to possess various families of traveling wave solutions. We use a combination of analytical and numerical methods to study the stability of the traveling waves. We show that for at least some of these waves the spectra of the linearization of the system about them are within the closed left-half complex plane.

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MS32

Front Solutions of Modified Rosenzweig-MacArthur Model

We investigate traveling wave solutions in diffusive modified Rosenzweig-MacArthur predator-prey model in two situations. One situation is when the prey diffuses at the rate much smaller than that of the predator. In the second situation both the predator and the prey diffuse very slowly. Both situations can be captured as singular perturbations of the associated limiting systems.

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MS32

Stability of Standing Waves for a Nonlocal NLS Equation

We consider the ground states of the Schrödinger equation with trapping potential (and spectral parameter $\omega > -n$). In particular, our interest is in the stability properties of these waves. We show that for $p < 1 + \frac{4}{n}$, $\omega > -n$ the waves are orbitally stable, while for $p > p_n = 1 + 4\frac{1+\sqrt{n^2+1}}{n^2} > 1 + \frac{4}{n}$ and some $\omega_{p,n} < 0$, one has instability by blow up, whenever $\omega > \omega_0$.

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MS33

Universal Behavior of Modulationally Unstable Media with Non-zero Boundary Conditions

This talk is divided into three parts. First, I will briefly describe the inverse scattering transform for the focusing nonlinear Schrodinger (NLS) equation with nonzero boundary conditions at infinity, and then I will present the long-time asymptotics of pure soliton solutions on the nonzero background. Second, I will describe in detail the properties of the asymptotic state of the modulationally unstable solutions of the NLS equation, including the number of oscillations and the local structure of the solution near each peak, showing in particular that in the long-time limit the solution tends to an ensemble of classical (i.e., sech-shaped) solutions of the NLS equation. Third, I will show that a similar asymptotic state is shared among a broad class of systems of NLS-type possessing modulational instability.

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MS33

Internal Waves in the Ocean - what we know, and what we don't

Internal waves in the ocean are a fascinating object to study, with time scales from minutes to days and length scales from tens of meters to kilometers. Particularly interesting is the seemingly universal spectral energy density of internal waves given by the celebrated Garrett and Munk spectrum of internal waves concocted in the seventies. A brief review of wave turbulence formalism will lead to the open questions about internal waves and possible solutions. Taking full advantage of the fact that the internal waves interaction is not local in phase space leads to alternatives closures to describe the spectral energy density of internal waves.

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MS33

Thermalization in one dimensional chains

Using Wave Turbulence Theory we estimate the time scale of equipartition of a number of one dimensional chains such as the FPU and the Nonlinear Klein Gordon. We show that for weak nonlinearity and in the thermodynamic limit the equipartition time scales like $1/epsilon^2$, with ϵ the strength of the nonlinearity. For a finite number of masses, we predict that the time scale of equipartition is much larger, i.e. $1/\epsilon^4$. Our theoretical results are confirmed by massive numerical simulations.

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MS33

Rogue Wave Formation Modeled by the Focusing Nonlinear Schroedinger Equation

Abstract not available at time of publication.

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MS34

Transitions from Localised Multiple-spike Waves to Bumps in Spatially-extended Networks of Integrate-and-Fire Neurons

I will discuss how interfacial dynamics can be used to perform bifurcation analysis of travelling waves in networks of integrate-and-fire neurons. As an example I will consider a well-known event-driven network of spiking neurons, proposed by Laing and Chow. In this setting, I will use interfacial dynamics to construct and investigate stability of travelling waves whose profiles are localised and possess an arbitrary number of spikes. The origin of these coherent structures is an open problem. I will show that this model can not support stationary, spatially heterogeneous states, that is, one can not define stationary bumps in the usual manner. I will then provide numerical evidence that disconnected branches of travelling bumps with an arbitrary number of spikes exist, and originate at suitably defined grazing points; such grazing points correspond to travelling waves with an increasing number of spikes, a well-defined width, and decreasing propagation speed, suggesting that a "bump solution can be recovered asymptotically, as the number of spikes in a localised travelling wave solution tends to infinity.

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MS34

Traveling Waves in a Stochastic Nagumo Equation

We consider reaction-diffusion equations that are stochastically forced by a small multiplicative noise term. We show that spectrally stable travelling wave solutions to the deterministic system retain their orbital stability if the amplitude of the noise is sufficiently small. By applying a stochastic phase-shift together with a time-transform, we obtain a semilinear sPDE that describes the fluctuations from the primary wave. We subsequently develop a semigroup approach to handle the nonlinear stability question in a fashion that is closely related to modern deterministic methods.

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MS34

Stability of Hamiltonian Lattice Waves in the Highenergy Limit

Abstract not available at time of publication.

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MS34

Moving Inhomogeneities in Oscillating Chemical Reactions

It is well known that impurities in oscillating chemical reactions act as source defects and generate target patterns. If we think of the impurity as having strength ϵ , then these solutions are characterized by an asymptotic wavenumber that is smaller than ϵ beyond all orders. This is again true in the case of moving inhomogeneities, only now we don't see target patterns but a 'sonic' cone due to the doppler effect. This result was previously obtained by studying the phase evolution of these patterns. Here, we consider instead the complex Ginzburg-Landau equation in two dimensions as a model equation. Among the challenges in the analysis we have that a regular perturbation in ϵ does not provide the correct ansatz, and that the linearization about the steady state has a zero eigenvalue embedded in the essential spectrum. These two points make finding solutions via perturbation theory difficult, but can be resolved if: i) we use matched asymptotics, and ii) we consider weighted Sobolev spaces that make the linear operator Fredholm.

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MS35

Wave Model for Nematic Liquid Crystal

In this talk, we discuss some recent progress on a wave model for the nematic liquid crystal. The solution for this system in general has finite time gradient blowup.

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$\mathbf{MS35}$

Stokes Expansions and Asymptotic Models of Water Waves

The study of irrotational incompressible Euler equations has been a long tradition in the fluid community. When the free surface was taken into account, both the theoretical study and robust numerical schemes become very challenging, especially for the case of deep water (Euler equations on a fluid domain with infinite depth). In the numerical side, various methods used to compute the Dirichlet-to-Neumann map (which is highly related to the water wave equations) proposed by W. Craig et al (1993) and M.J. Ablowitz et al (2006, 2008) involve highly illconditioned intermediate calculations (while the difficulties can be overcome by implementing multiple-precision arithmetic). The boundary integral collocation method and the transformed field expansion method are then introduced to avoid catastrophic cancellation of digits in the intermediate results; however, carrying out those methods in the threedimensional case seems difficult. Therefore, the search for good asymptotic models for water waves become appealing for it might provide models that can be easily implemented and at the same time provide accurate enough evolution of the free surface. B. Akers et al (2010) propose a quadratic approximation of the water wave equation; however, the derivation of such a model seems not rigorous. In this talk, I will present how the Stokes expansions can be used to derive asymptotic models up to any order.

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MS35

On the Dynamics of Ferrofluids: A Relaxation Limit from the Rosensweig Model Towards Equilibrium

This talk presents the global existence of weak solutions of the Rosensweig model of ferrofluids (cf. Rosenweg (1985)) on a Lipschitz domain D, time interval [0, T], T < 1, with suitable initial and boundary conditions using DiPerna-Lions' theory of compressible fluids. In addition, the relaxation to equilibrium limit $\varepsilon \to 0$ is investigated using the relative entropy method. If the limiting system has a Lipschitz continuous solution, we can show a convergence rate in ε , if the limiting system has only a weak solution, we obtain strong convergence of a subsequence in L^2 .

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MS36

Variation of Inverse Cascade Spectrum for Gravity Waves due to Condensate

Results of numerical simulation of inverse cascade of surface gravity waves are considered. This experiment was performed in the framework of pseudo-spectral code on a discreet grid of wave vectors. Wave vectors grid is characteristic not only for numerical experiments, but for any observation in wave tanks and other strongly limited in size experimental setups. Deviation of the measured spectrum of inverse cascade from the one predicted by the Wave Turbulence theory is observed. The spectrum's slope make's us lean toward existence of a new universal updated spectrum for waves in inverse cascade. An attempt to explain the observed spectrum analytically is presented.

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MS36

Zonal Flows: A Quasilinear Foothold to Gaining Insight

Zonal flows, which consist of a regular spatially alternating pattern of flows forming spontaneously in fluid systems, are of fundamental physical interest. They occur in diverse systems such as planetary atmospheres, with the banded flows of Jupiter being a prominent example, and in magnetically confined plasmas. In laboratory plasmas that are aimed at fusion-energy production, zonal flows have been linked to the suppression of detrimental turbulent transport, and have therefore attracted significant attention. While simulations are essential for quantitative studies of realistic devices, developing a mathematical theory of zonal flow is important for gaining physical insight. Recent work based on combining a quasilinear approach with a statistical formulation has brought a fresh look at zonal flow. In this theory, zonal flows grow as a coherent structure due to the symmetry-breaking zonostrophic instabilitythe broken symmetry being statistical homogeneity. Just beyond the stability threshold, the system obeys the Real Ginzburg-Landau equation, linking the study of zonal flow to the wider field of pattern formation. These findings have been confirmed with numerics. From this quasilinear approach, a conceptual framework involving the growth and saturation of zonal flow has emerged that can inform future studies.

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MS36

Efficient Numerical Methods for Initial Value Problem with Randomness

We present a novel numerical approach for the study of nonlinear PDEs with random initial conditions. The naïve approach to compute the statistics of these random dynamics, the Monte-Carlo approach, is vastly inefficient, and often impractical. Recent years have seen the growth of the Polynomial-Chaos Expansion (PCE), a spectrallyaccurate algorithm for the computation of statistical moments. The PCE is, however, also inadequate, both for the efficient computation of distributions, as well as for non-smooth quantities of interest. Our newly developed methods solve both these problems, and therefore open a new road to the study of noise and randomness in nonlinear wave equations. We apply our numerical approach to describe phase randomness and solitons interactions in the Nonlinear Schrödinger equation (NLS), and to the Burgers equation.

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MS36

Appearance of Traveling-standing Waves in Deep Water

We study evolution of a finite amplitude monochromatic wave in deep ocean taking into account gravity but not capillar effects. We simulate one period of Stokes wave to allow for superharmonics, and avoiding modulational instability at wavelengths longer than the initial spatial period of monochromatic wave. We investigate the possibility of generation of traveling–standing waves, or Stokes waves in Euler equations in the long time limit.

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MS37

Optical Mode Stability and Dynamics in Nonlinear Twisted Pt-symmetric Structures

We present theoretical and numerical work on the stability properties of light propagating in a multi-core fiber array where the axis of each core follows a helical path. Adding a twist to this optical system induces a phase factor in the nearest neighbor coupling constants, thus adding a new degree of freedom to a system that has nonlinearity and loss/gain strength representing a discrete Parity-Time (PT) property. This new degree of freedom adds a new venue to control the transition from unbroken to broken PT-symmetric phases, modifying the tunneling dynamics among the guiding cores. We utilize mathematical modeling and numerical simulations to specifically show how nonlinearity, coupling, geometric twist, and balanced gain/loss relate to existence and stability of six nonlinear optical modes modeled by the Discrete Nonlinear Schrdinger Equation.

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MS37

Integrable Models for Ultral-short Pulse Propagation

In this talk, we will discuss the modeling for ultra-short pulse propagation in nonlinear optics. The complex short pulse (CSP) and the coupled complex short pulse equation will be proposed and their various solutions will be solved. Their integrable discretizations will also be constructed.

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MS37

Deterministic Triggering of Cavity Solitons in a Microresonator

Generation of Kerr comb, especially the deterministic generation of a fully coherent comb with single cavity soliton in a microresonator, has attracted much attention in the past years. A wavelength scanning continuous wave pump laser is typically used to excite the Kerr comb. But the cavity soliton number obtained in the micro-cavity is uncontrollable. In the wavelength scanning, the intracavity energy and hence the temperature of the microresonator are greatly enhanced and then experience a sharp drop when Kerr comb is excited. Such a significant energy drop in a hot cavity is hard to control in experiment. In this talk, we introduce a novel method to deterministically excite cavity solitons in a microresonator. In contrast to the traditional wavelength scanning method, we use a continuous wave pump laser with a fixed wavelength. We model the cavity dynamics with a sophisticated model including the nonlinear and thermal effects. With a single shot picosecond pulse trigger, we can deterministically trigger a single cavity soliton in a microresonator. Comparing with the energy drop in the wavelength scanning method, the energy variation is greatly reduced since we excite the cavity soliton from a cold cavity. By injecting trigger pulses one by one, we can excite multiple cavity solitons in the microresonator.

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MS38

Kink Dynamics in a Parametric ϕ^8 Model

We explore a variant of the ϕ^8 model. We are interested in the interaction between different kinks that can co-exist as in the case described by Khare, Christov, Saxena. So far, in lower-order field theories, the kinks and anti-kinks being scattered necessarily have the same energy as the theory can have only 2 or 3 degenerate minima. Here, for the first time, two kinks of the same type as well as two kinks of different types can exist having equal or unequal energies. Thus, we study how the parameters in the potential term affect their scattering. Even if we choose the parameters such that all the kinks have the same energy, we explore if their interaction energy will stay the same.

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MS38

One-dimensional Periodic Solutions in a Threecomponent Reaction-diffusion System

Periodic patterns occur ubiquitously in nature, but the mechanism behind the formation of periodic patterns away from onset is not well understood. In this talk we consider the mechanism behind the generation of periodic stationary solutions in a singularly perturbed reaction-diffusion system. The system has one fast nonlinear component, interacting with two slow components. We investigate the existence and bifurcations of families of one-dimensional periodic solutions in this system. It will be shown how changes in the slow manifold and changes in the fast dynamics lead to an intriguing sequence of self-replicating patterns of large amplitude periodic waves. We will conclude this talk with a discussion about extensions to twodimensional patterns and travelling waves.

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MS38

Gelfand Type Problem for Turbulent Jets

Gelfand problem is one of the canonical problems in the theory of non-linear parabolic and elliptic partial differential equations (PDEs). This problem naturally arises in the Frank-Kamenetskii theory of thermal explosion (autoignition) and describes an initial stage of evolution of a temperature field in reactive materials and mixtures. In this talk I will present a generalization of Gelfand problem for analysis of autoignition of reactive turbulent jets. I will present both derivation of this new model and its analysis. The latter is performed using a combination of rigorous, formal asymptotic and numerical techniques. It will be shown that similar to the classical Gelfand problem an autoignition in jets occur exclusively owing to the absence of self-similar temperature distribution which, in mathematical terms, leads to loss of regularity (blow-up) of underlying PDE. The detailed analysis of self-similar temperature profiles will be presented and a sharp characterization of an autoignition event in terms of principal geometric and physical parameters of the problem will be given. This a joint work with U.G. Hegde and M.C. Hicks of NASA Glenn Research Center.

Peter Gordon

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MS38

Snaky Structures of Localized Patterns with Oscillatory Tails and Collision Dynamics in Heteroge-

neous Media

We consider the dynamics of traveling pulses with oscillatory tails for the three-component reaction diffusion system of the FitzHugh-Nagumo type; one activator and two inhibitors. We numerically explore the snaky structures of standing and traveling pulses with oscillatory tails and their interrelation among those global branches and the background state when some appropriate parameters are changed. We also study the collision dynamics between the traveling pulse and the heterogeneity of bump type in the media. It turns out that all the outputs after collision are included in the set of the global branches emanating from the background state as height of the bump is varied. This is a joint work with Takeshi WATANABE.

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MS39

Strong Interaction Between Plants Induces Circular Barren Patches: Fairy Circles

Fairy circles consist of isolated or randomly distributed circular areas devoid of any vegetation. They are observed in vast territories in southern Angola, Namibia and South Africa. In this talk, we analyze on the formation of fairy circles, and we interpret them as localized structures with a varying plateau size as a function of the aridity. Their stabilization mechanism is attributed to a combined influence of the bistability between the bare state and the uniformly vegetation state, and non-local coupling that models the competition between plants. We show that the proposed mechanism is model-independent.

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MS39

Water Transport in Dryland Ecosystems: Shaping Banded Vegetation Patterns

Regular spatial patterns in the vegetation growth of dryland ecosystems are thought to arise through selforganization in response to water scarcity. This behavior has been qualitatively reproduced by reaction-advectiondiffusion systems that model various interactions between the plants and their environment. The increasing availability of remote sensing data may allow us to test certain predictions made by such models of these potentially vulnerable ecosystems and inform improvements. As a first step, incorporating topographic information into modeling efforts has the potential to provide deeper insight into the role that water transport plays in the vegetation dynamics. A simple modeling framework that incorporates the influence of topography on water flow provides qualitative agreement with satellite imagery and topographic surveys. Further improvements to the water transport model include capturing plant feedbacks on surface/subsurface water transport in a more realistic way.

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MS39

Periodic Vegetation Patterns and Stability Boundaries

In semi-arid regions a prominent form of vegetation pattern are bands, whose idealisation are periodic stripe patterns in the plane. The selection of such patterns is naturally related to stability properties. Studies of a conceptual class of models motivated a number of numerical as well as analytical investigations on the nature of stability regions and boundaries in prototypical systems. This talk discusses some of these more abstract results, which can be applied to other models. This is based on joint work with a group of collaborators including Eric Siero, Bjrn de Rijk, Arjen Doelman and Frits Veerman.

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MS39

Multistability and Effects of Grazing on Desertification

Many ecosystems exhibit spatial patterns. An important example is banded vegetation in drylands. These patterns are multistable, meaning that for a given set of parameters, a whole range of patterns (with different wavelengths) is stable. In this talk we make a comparison between remotely sensed data and model results. What can be inferred from an observed ensemble of vegetation patterns on varying hill slopes? We end by including a more realistic model term for grazing, and study how this alters the desertification process.

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$\mathbf{MS40}$

Formation of rogue waves in continuous and discrete models: Theory and Computation

In this talk, we will discuss the dynamics of rogue waves in nonlinear Schrödinger (NLS) equations and discrete variants thereof. Initially, we will consider NLS equations with variable coefficients which can be converted into their integrable siblings by utilizing suitable transformations. Then, the Peregrine soliton will be fed to the transformation employed. Using direct numerical simulations, the formation of such soliton solutions will be presented. Subsequently, and in the realm of atomic Bose-Einstein Condensates (BEC), the IBVP with Gaussian wavepacket initial data for the scalar (NLS) will be discussed where some novel features will be presented. In particular, it will be shown that as the width of the relevant Gaussian is varied, large amplitude excitations strongly reminiscent of Peregrine solitons or regular solitons appear to form. This analysis will be complemented by considering the Salerno model interpolating between the discrete NLS (DNLS) and Ablowitz-Ladik (AL) models where similar phenomenology is observed. The findings presented in this talk might be of particular importance towards realizing experimentally extreme events in BECs.

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MS40

Nonlinear Excitations in Magnetic Lattices with Long Range Interactions

In this talk, I will discuss the experimental and theoretical study of nonlinear excitations in lattices with long rang interactions. In particular, solutions that are spatially localized and periodic in time (so- called breathers) are explored in a chain which has algebraically decaying interactions. It has been previously established [Flach, PRE 58, R4116 (1998)] that lattices with such long rang interactions feature breather solutions where the spatial decay of the tail shows a crossover from exponential to algebraic decay. In this talk, this problem is revisited in the setting of a chain of repelling magnets with a mass defect. The existence of breathers that exhibit a crossover is verified both numerically and experimentally.

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MS40

Waves and Obstacles in Square Lattices

We study dynamical systems posed on a square lattice, with a special focus on the behaviour of basic objects such as travelling corners and travelling waves under (potentially large) perturbations of the wave and the underlying spatial lattice. Such travelling structures satisfy functional differential equations of mixed type, which can be seen as generalizations of delay equations in the sense that both delays and advances in the arguments are allowed.

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MS40

Spatial Localization in in Homogeneous FPU Lattices

We study energy localization in a quartic FPU model with spatial inhomogeneity corresponding to a site-dependent number of interacting neighbors. Lattices with a site dependent number of interacting neighbors can have linear normal modes that are strongly localized in the regions of high connectivity and there is evidence that some of these localized modes persist in the weakly nonlinear regime. The argument uses normal forms to detect invariant subspaces spanned by spatially localized modes. We present examples from 1- and 3-D lattices. The 3-D lattices are small amplitude approximations of elastic lattice models of protein vibrations, and we present results form two protein types. We also present 1-D examples where oscillations remain localized over long times.

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MS41 Snaking in Dimensions 1+epsilon

The Swift-Hohenberg equation is a widely studied partial differential equation which is known to support a variety of spatially localized structures. The one-dimensional equation exhibits spatially localized steady-state solutions which give way to a bifurcation structure known as snaking. That is, these solutions bounce between two different values of the bifurcation parameter while ascending in norm. The mechanism that drives snaking in one spatial dimension is now well-understood, but recent numerical investigations indicate that upon moving to two spatial dimensions the related radially-symmetric spatially-localized solutions take on a significantly different snaking structure which consists of three major components. We apply a dimensional perturbation in an effort to use well-developed methods of perturbation theory and dynamical systems to understand this new bifurcation structure. In particular, we are able to identify key characteristics that lead to the segmentation of the snaking branch and therefore provide insight into how the bifurcation structure changes with the spatial dimension. In this presentation we will focus on results pertaining to only one of the three major components of the perturbed snaking structure, as well as discuss the difficulty of extending these results to the other two.

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MS41

Diffusive Stability of Planar Wave Trains in Reaction-diffusion Systems Against Nonlocalized Perturbations

Planar wave trains are traveling wave solutions, whose

wave profiles $p: \mathbf{R}^2 \to \mathbf{R}^n$ are periodic in one spatial direction and constant in the transverse direction. They can be constructed by trivially extending a wave train on the real line. However, planar wave trains do not necessarily have the same stability properties as their underlying 1-dimensional counterparts: there is a sign criterion that determines whether spectral stability is 'inherited'. Both in the 1- and 2-dimensional setting spectral stability yields diffusive stability against localized perturbations. In this talk, I show that in the planar case one can in fact allow for nonlocalized perturbations. I will explain the importance of 2 spatial dimensions for closing the nonlinear iteration argument. At the moment, an analogous result seems not to exist for the 1-dimensional setting, although nonlocalized perturbations affecting the *phase* of the wave train can be dealt with.

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MS41

Stability of Spiral Waves in Cardiac Dynamics

Ventricular fibrillation in the heart is often caused by the formation and breakup of spiral waves in cardiac tissue. Clinically, spiral wave breakup has been linked to the formation of alternans, an oscillation in the action potential duration. We seek to understand how and why alternans develop, and if stable alternans patterns exist. To investigate these questions, we analyze spectral properties of spirals on bounded disks formed in reaction-diffusion systems. Moreover, I will address difficulties that arise if the reaction-diffusion system has one or more variables without diffusion: in this situation, the eigenvalues exhibit unexpected accumulation points, far from the asymptotically predicted absolute spectrum curves.

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MS41

A Pattern Formation Mechanism for Control of Synapse Density During C. Elegans Growth

We propose a novel mechanism for Turing pattern formation that provides a possible explanation for the regular spacing of synapses along the ventral cord of C. elegans during development. The model consists of two interacting chemical species, where one is passively diffusing and the other is actively trafficked by molecular motors; we identify the former as the kinase CaMKII and the latter as the glutamate receptor GLR-1. We use linear stability analysis to derive conditions on the associated nonlinear interaction functions for which a Turing instability can occur. We find that the dimensionless quantity γ , the ratio of switching rate and diffusion coefficient to motor transport velocity, must be sufficiently small for patterns to emerge. One consequence is that patterns emerge outside the parameter regime of fast switching where the model effectively reduces to a two component reaction-diffusion system. Furthermore, these patterns are also maintained during domain growth. We analyze selection and stability of these patterns in one and two dimensions.

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MS42

Nonlocal Interaction Models in Biological Systems

Biological pattern formation has been extensively studied using reaction-diffusion and agent based models. In this talk we will discuss nonlocal pattern forming mechanisms in the context of bacterial colony formation and surface striping on animals with an emphasis on arrested fronts. This will lead to a novel nonlocal framework to understand the interfacial motion in biological systems. We will then use this approach to model an interesting bacterial phenomenon, and to understand simple microscopic requirements for flat stripe solutions to persist in nature.

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MS42

Dynamics of Religious Group Growth and Survival

We model and analyze the dynamics of religious group membership and size. A group is distinguished by its strictness, which determines how much time group members are expected to spend contributing to the group. Individuals differ in their rate of return for time spent outside of their religious group. We construct a utility function that individuals attempt to maximize, then find a Nash Equilibrium for religious group participation with a heterogeneous population by solving a free boundary problem. We then model dynamics of group size by including birth, death, and switching of individuals between groups. Group switching depends on the strictness preferences of individuals and their probability of encountering members of other groups. We show that in the case of only two groups one with finite strictness and the other with zero there is a clear parameter combination that determines whether the nonzero strictness group can survive over time, which is more difficult at higher strictness levels. At the same time, we show that a higher than average birthrate can allow even the highest strictness groups to survive. Finally, we consider cases of several groups, gaining insight into strategic choices of strictness values and displaying the rich behavior of the model.

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MS42

Dynamic Implicit Solvent Model: Understanding Solvent Stokes Flow and Interface Dynamics for Biomolecules

The interface between protein solute and aqueous solvent exhibits complex geometries, and can undergo conformational changes by combined influences from electrostatic force, surface tension, and hydrodynamic force. To understand the role of solvent Stokes flow in this process, we develop a Dynamic Implicit Solvent Model (DISM). Based on this model, we first analytically study the linear stability of a cylindrical solute-solvent interface, where the asymptotic dispersion reveals a power law. Moreover, we develop a computational method to simulate the solvent Stokes flow and interface motion. The key components of our fluid solver are virtual node method, pressure Poisson equation, specially designed boundary condition, Schur complement and least square techniques. 3D numerical tests will demonstrate the convergence of our method, and show that this new approach can capture interesting dynamics of protein conformational change.

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Bo Li

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MS42

Mean-field Models for Chemotaxis

This talk will focus on my works on mean-field models for chemotaxis based on kinetic theory, including pathway based mean-field models, augmented Keller-Segel model for E. coli chemotaxis, and an asymmetric model for biological aggregation. I will give mathematical derivation of the mean-field models by taking some proper moment closure of kinetic biological systems. Building biological mechanism in the models are essential to capture some interesting swarming phenomena, for example, phase-delayed traveling wave (memory effect) and soliton solution (asymmetric sensing). Connections to the chemotaxis model proposed in [G. Si, T. Wu, Q. Quyang and Y. Tu, Phys. Rev. Lett., 109 (2012), 048101] will be also discussed.

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MS43

Discontinuous Galerkin Methods for the Nonlinear Shallow Water Equations with Horizontal Temperature Gradients

Shallow water equations with horizontal temperature gradients, also known as the Ripa model, are used to model flows when the temperature fluctuations play an important role. These equations admit steady state solutions where the fluxes and source terms balance each other. We will present well-balanced discontinuous Galerkin methods for the Ripa model, which preserve the lake-at-rest and isobaric steady states as well as the more general moving water equilibrium. The key idea is the recovery of well-balanced states, source term approximations, and appropriate approximations of the numerical fluxes. We will also discuss the positivity preserving property of our methods. Numerical examples will also be presented to verify the well-balanced property, high order accuracy, and good resolution for both smooth and discontinuous solutions.

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MS43

Exact Solutions and Integrability for Nonlinear Development of Kelvin-Helmholtz Instability for Counterflow of Superfluid and Normal Components of Helium II

A relative motion of the normal and superfluid components of Helium II results in Kelvin-Helmholtz instability (KHI) at their common free surface. If the limit of small density and large viscosity of the normal component the system is exactly integrable because it is reduced to solution of two decoupled complex Burgers equations in weak nonlinearity case [P. M. Lushnikov, Exactly Integrable Dynamics of Interface between Ideal Fluid and Light Viscous Fluid, Physics Letters A, v. 329, pp. 49-54 (2004)]. In fully nonlinear case and zero viscosity of normal component we found the exact solutions for the nonlinear stage of the development of that instability. Contrary to the usual KHI of the interface between two fluids, the dynamics of Helium II free surface allows decoupling of the governing equations and their reduction to the Laplace growth equation which has the infinite number of exact solutions [P. M. Lushnikov and N. M. Zubarev, Exact solutions for nonlinear development of Kelvin-Helmholtz instability for counterflow of superfluid and normal components of Helium II, arXiv:1710.10684 (2017)].

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MS43

Sparse Grid Techniques for Kinetic Plasma Simulation

The generation and propagation of nonlinear waves plays a prominent role in kinetic plasma phenomena. Simulating these systems is extremely challenging due to the highdimensionality of the kinetic PDEs to be solved. We will discuss the use of the sparse grid combination technique as a method for circumventing the curse of dimensionality in this context. The combination technique uses a linear combination of grids each of which is fine in some coordinate directions and coarse in others to mimic a single grid which is fine in every direction. The method has drawbacks most notably, it is very sensitive to the coordinate system on which the grid is based but we will show that kinetic equations for plasmas, and especially strongly magnetized plasmas, are in many ways ideally suited to overcome these drawbacks. We will present work on using sparse grids with high-order continuum finite-volume schemes, with an eye toward integration in the tokamak edge simulation code COGENT. We will present both theory and results from test problems. This material is based on work supported by the U.S. Department of Energy, Office of Science, Office of Advanced Scientific Computing Research, Applied Mathematics Program under Contract DE-AC5207NA27344 at Lawrence Livermore National Laboratory.

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MS43

Solitary Wave-hydrodynamic Flow Interaction in Bi-directional Systems

Solitary waves, sometimes referred to as solitons, are localized traveling pulses that are omnipresent in dispersive fluid-like media. We consider the interaction of a solitary wave with a large scale, spatially extended hydrodynamic flow that results from a hydraulic transition in the far-field boundary conditions. The hydraulic transition generically gives rise to a rarefaction waves (RWs) or dispersive shock wave (DSWs). Under the scale separation assumption of nonlinear wave (Whitham) modulation theory, the highly nontrivial nonlinear interaction between the soliton and the evolving hydrodynamic wave is described in terms of simple wave solutions to an asymptotic reduction of the Whitham partial differential equations. The simple wave solutions of the reduced Whitham system result in expressions for the effect of the hydrodynamic interaction on the solitary wave. The expressions for the solitary wave's amplitude and trajectory give conditions for when the wave is trapped inside or transmitted through the hydrodynamic flow.

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MS44

Time Domain Modeling of Active and Optically Tunable Materials

This talk will focus on numerical modeling of active and optically tunable materials in time domain, with details of numerical implementation and non-standard high order numerical schemes. Active media has been the focus of many studies in nanophotonics due the opportunity to compensate loss in metallic elements and due to the broad spectrum of applications such as nanolasers, spasers, and light emitting diodes. We present accurate experimentcalibrated multiphysics numerical model, where the active medium is described quantum mechanically by a set of rate equations that accounts for the atomic transitions through a multi-level system, while the light propagation is described classically with Maxwells equations. The second part of the talk is devoted to numerical modeling of optically controlled photonic devices. The idea of replacing electronic logic devices with nanoscale dynamic photonic components is not new and has being developed in the literature over the recent years. So far, there is no reliable design methodologies and tools for simulation of nanoscale dynamic photonic elements along with non-equilibrium dynamics of electrons and lattice ions. We present a numerical model of light propagation in a solid state medium, where the response is computed based on microscopic nonequilibrium thermodynamic model. The model is based on a self-consistent solution of electron and hole transport equations, semiconductor Bloch equations and Maxwell equations.

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MS44

Nonlinear Wave in Left-handed Transmission Lat-

 \mathbf{tices}

We consider the model of nonlinear left-handed electrical lattice that mimic left-handed materials. Utilizing a multiple scales expansion, we transforms the model into a nonlinear Schrodinger equation. We exam possible nonlinear waves that can be supported by the lattice and hence in a left-handed material.

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MS44

On the Discrete Solitons and Vortices in Nonlinear Two-dimensional Lattices

Solitons and vortices are considered on the two-dimensional lattice in the framework of the discrete nonlinear Schrödinger equation. Such lattices include e.g. square or hexagonal (honeycomb) lattices and we also investigate those symmetric with respect to simultaneous parity (\mathcal{P}) and time reversing (\mathcal{T}) transformations. The existence and stability of such configurations is analyzed in the limit (anti-continuum limit) of weak coupling between the lattice sites, which may correspond to the large propagation constant in optics. In particular, we find that soliton configurations can possibly be spectrally stable with respect to small perturbations in both Hamiltonian and \mathcal{PT} symmetric cases. On the other hand, though stable vortex solutions are common in the Hamiltonian case, the vortex configurations tend to be unstable in the \mathcal{PT} -symmetric case. Our analytical predictions are found to be in good agreement with numerical computations.

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MS45

Reaction-diffusion-advection Models of Vegetation Stripes on Sloped Terrain

Reaction diffusion models have been successful in modeling the formation of vegetation stripe patterns. Such models are frequently singularly perturbed, reflecting the fact that water diffuses faster than plant biomass, and stripe patterns typically arise as wavetrains or localized pulses in an associated singularly perturbed traveling wave ODE. Sloped terrain is modeled through the addition of advection terms representing the downhill flow of water. We focus on a general class of reaction-diffusion-advection models in the 'Klausmeier' limit, in which the advection is assumed large and the effect of water diffusion is neglected. We prove the existence of traveling stripe patterns and a variety of homoclinic bifurcation phenomena using geometric singular perturbation theory, blow-up desingularization techniques, and Lin's method. We also discuss implications for the stability of the resulting patterns.

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MS45

Assessing Controls on Hydrologic Connectivity, Plant Water Availability and Degradation Risk in Drylands with Lagrangian Modeling and Isotope Tracers

Dynamical systems modeling has enhanced understanding of the mechanisms that form vegetation patterns, and the implications of changing pattern morphology. However, models are still challenged by issues of equifinality, and by the representation of ecohydrological processes. Pattern morphology is not a unique indicator of process, as different models (e.g. Turing processes and fluid instabilities) may produce morphologically indistinguishable patterns. This challenges modelers to consider broader criteria for model success than matching pattern form. Conversely, pattern morphology as observed in real landscapes includes phenomena that are not explained by models. For example, natural patterns exhibit greater heterogeneity than those produced by idealized models. Finally, dynamical systems models that are amenable to analytical treatment rely on continuous-time differential equations, which represent stochastic processes with long-term averages. Storms and overland water flow occur at rapid timescales relative to biomass growth, and it is unclear how time averaging alters the modeled distribution of water relative to explicitly describing these processes. We use the shallow water equations to simulate within-storm dynamics, and synthesize multiple storm simulations to assess the longer term hillslope outcomes. This approach is intended to address the time-scale separation between storms and biomass response, and assess the assumptions underlying models of pattern formation.

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MS45

Advection and Autocatalysis in Banded Vegetation Patterns

We motivate and analyze a simple model for the formation of banded vegetation patterns. The model incorporates a minimal number of ingredients for vegetation growth in semi-arid landscapes. It allows for comprehensive analysis and sheds new light onto phenomena such as the migration of vegetation bands, their alignment with contour lines, and the interplay between their upper and lower edges.

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MS46

Properties of Some Breather Solutions of a Nonlocal Discrete NLS Equation

We present results on breather solutions of a discrete nonlinear Schrödinger equation with a cubic Hartree-type nonlinearity that models laser light propagation in waveguide arrays that use a nematic liquid crystal substratum. We study energy minimizing breathers, showing existence as well as symmetry and monotonicity properties. We also show the existence of shelf-type breather solutions (i. e. solutions that decay in one direction and asymptote to a nonzero value in the other direction) in the local and nonlocal interaction cases, and justify computations of the essential spectrum of the linearization around these solutions.

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MS46

Interactions between Solitons and Dispersive Shocks in Focusing Media

We study, numerically and analytically, interactions between solitons and dispersive shocks in modulationally unstable media.

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MS46

Stochastic ejection of droplet solitons in magnetic memory devices

The pursuit of fast, scalable nonvolatile storage has motivated much recent study of the formation and control of localized structures, including skyrmions and dissipative solitons, in ferromagnetic materials. These structures are supported by an energetic balance among contributions from material anisotropy, exchange energy, stray or selfinduced field energy, and forcing due to external fields or current-induced spin torque. Structures that are stable under deterministic dynamics are generally only quasi-stable under the inevitable influence of thermal fluctuations or other stochastic phenomena. In this work we use the geometric minimum action method (GMAM) to compute the action associated with exits through all saddles with a single unstable manifold, including the saddles with nonzero velocity bifurcating through the drift instability and those associated with zero velocity originating in a saddle-node bifurcation in the precessional frequency. The action quantifies how exit probabilities scale with noise strength, providing an important estimate of error rates in magnetic storage devices.

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MS46

Stability and internal modes of gap soliton in the present of a weak nonlocality in periodic potentials

In this talk, we will discuss the stability and internal modes of one-dimensional gap soliton employing the modified nonlinear Schrödinger equation with a sinusoidal potential together with the present of a weak nonlocality. Using an analytical theory, it is proved that two soliton families bifurcate out from every Bloch-band edge under self-focusing or self-defocusing nonlinearity, and one of these is always unstable. Also, we study the oscillatory instabilities and internal modes of the modified nonlinear Schrödinger equation. The analytical results are in excellent agreement with numerical results.

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$\mathbf{MS46}$

Maximum Amplitudes of Hyperelliptic Solutions of the Cubic Nonlinear Schrödinger Equation

Elementary algebraic arguments, without the use of Riemann theta functions, are used to derive simple formulas for the maximum amplitudes of hyperelliptic solutions of the cubic nonlinear Schrödinger equation.

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MS47

Large Fronts in Nonlocal Equations using Conley– Floer Theory

In this talk we consider a class of dissipative nonlocal equations. Typical examples are nonlocal Allen-Cahn equations and neural field equations. After making a traveling wave ansatz one obtains a system of nonlocally coupled equations. In the spatial dynamics reformulation the nonlocal coupling entering in the time-like variable, resulting in a forward-backward delay equation. The aim of this talk is to explain how Conley–Floer index theory can be extended to these systems. This Conley–Floer index then allows us to prove existence of travelling fronts in the dissipative nonlocal equations.

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MS47

Bifurcation of Spikes From the Essential Spectrum in Nonlocally Coupled Systems

The existence and bifurcation of spatially localized solutions (spikes) in systems of equations modeling natural phenomena has been studied intensively in the area of applied mathematics, in particular pattern formation, solitary water waves and mathematical neuroscience. In the case of one unbounded spatial dimension, the method of spatial dynamics is a powerful tool to study such questions. By interpreting the steady state equation as a dynamical system in the unbounded spatial variable, one obtains compete characterization of small bounded solutions via center manifold reduction and normal form transformations. However, adaptions to higher spatial dimension are difficult, and mostly restricted to radial symmetry. Analogously, adaptions to systems with nonlocal coupling are also difficult as spatial dynamics formulations are either cumbersome or not available. Our results covers both nonlocal and non-radially symmetric situations in systems with a generic transcritical bifurcation in the nonlinearity. Relying on Fourier multiplier preconditioning, scaling and normal forms.

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MS47

Spatial Dynamics in Global Bifurcation Arguments

In two dimensions, the equations for a traveling water wave can be rewritten as a nonlinear elliptic equation in an infinite strip. Periodic waves of large amplitude can then be constructed by combining a local Lyapunov–Schmidt reduction with a global continuation argument. For smallamplitude solitary waves or fronts, however, the Lyapunov– Schmidt reduction breaks down completely and another approach is needed. In this talk we argue that the center manifold reductions in the spatial dynamics literature are a particularly convenient replacement because they provide useful information for subsequent continuation arguments. As applications of this approach we present global existence results for solitary waves with vorticity and stratification.

Miles Wheeler

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MS47

The Stability of Grain Boundaries in Swift-Hohenberg Equation

Grain boundaries as defects are commonly seen in experiments and numerical analysis, especially perpendicular ones and weakly-bent ones. In the case when the angle of the grain boundary becomes sufficiently acute, we expect such grain boundaries become unstable, yielding point defects at the interface. Our stability analysis starts with the technically easiest case–weakly bent symmetric (that is, the angle at the interface is large) grain boundaries. In this case, restricted to exponentially weighted spaces, we take advantage of the spatial dynamics scheme, which reduces the PDE into a sixth-order ODE system, to reduce the eigenvalue problem to an inhomogeneous ODE problem on the six-dimensional center manifold. For symmetric grain boundaries with small angles, the reduced ODE system from spatial dynamics not only has higher order but also contains resonant modes, complicating the analysis and potentially making the scheme adopted by the weakly bent case intractable.

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MS48

Expansion Dynamics of Bacterial Populations

Many bacterial species can swim and are capable to sense and actively follow chemical gradients. For Escherichia coli, the cellular implementation of this chemotactic response belongs to the best-characterized subjects of molecular biology. Hoever, much less is known about the collective swimming dynamics of multiple cells and their fitness consequences for growing bacterial populations. Here I discuss the collective motion of cells along self-generated chemotactic gradients. Presenting experiments and a theoretical analysis, I describe how the interplay of metabolite sensing, proliferation, metabolite uptake, and swimming leads to the spreading and growth of an initially localized population. The collective migration dynamics of cells along ring-shaped fronts is described by a modified Keller-Segel model, emphasizing the crucial role of bacterial growth and nutrient utilization. Coupled to the front propagation via pushed waves, proliferation of cells in the back drives overall population growth. By the integration of chemoattractant sensing into directed movement, the cue-driven form of range expansion described here is fast (speed of order 1 cm/h) and easily outcompetes the canonical form of range-expansion via pulled waves and Fisher-Kolmogorov dynamics.

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MS48

Geometric Control of Active Collective Motion

Recent experimental studies have shown that confinement can profoundly affect self-organization in semi-dilute active suspensions, leading to striking features such as the formation of steady and spontaneous vortices in circular domains and the emergence of unidirectional pumping motions in periodic racetrack geometries. Motivated by these findings, we analyze the two-dimensional dynamics in confined suspensions of active self-propelled swimmers using a mean-field kinetic theory where conservation equations for the particle configurations are coupled to the forced Navier-Stokes equations for the self-generated fluid flow. In circular domains, a systematic exploration of the parameter space casts light on three distinct states: equilibrium with no flow, stable vortex, and chaotic motion, and the transitions between these are explained and predicted quantitatively using a linearized theory. In periodic racetracks, similar transitions from equilibrium to net pumping to traveling waves to chaos are observed in agreement with experimental observations and are also explained theoretically. Our results underscore the subtle effects of geometry on the morphology and dynamics of emerging patterns in active suspensions and pave the way for the control of active collective motion in microfluidic devices.

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MS48

Bubble Assemblies in Ternary Systems with Long Range Interaction

A nonlocal diffuse interface model, based on the Nakazawa-Ohta density functional theory for triblock copolymers, is used to study bubble assemblies in ternary systems. The model has three parameters weighing three types of long range interaction and two parameters that fix the total area of each constituent. As the parameters vary, a large number of morphological phases appear as stable stationary states. One open question related to the polarity direction of double bubble assemblies is answered numerically. Moreover, it is shown that the average size of bubbles in a single bubble assembly depends on the sum of the minority constituent areas and the long range interaction coefficients. One further identifies the ranges for area fractions and the long range interaction coefficients for double bubble assemblies.

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MS49

High-order Accurate Conservative Finite Difference Methods for Vlasov Equation

Kinetic simulation of multi-dimensional plasma waves through direct discretization of the Vlasov equation is a useful tool to study many physical interactions and is particularly attractive for situations where minimal fluctuation levels are desired, for instance, when measuring growth rates of plasma wave instabilities. However, direct discretization of phase space can be computationally expensive, and as a result there are few examples of published results using Vlasov codes in more than a single configuration space dimension. In an effort to fill this gap we have developed the Eulerian-based kinetic code LOKI that evolves the Vlasov-Poisson system in 2+2-dimensional phase space. Here we discuss the newly developed 6thorder accurate finite difference algorithms used in the code as well as some aspects of their parallel implementation using MPI. I will also overview simulation results of basic plasma wave instabilities relevant to laser plasma interaction, which have been obtained using the code.

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MS49

Inhomogeneous Wave Turbulence as an Effective Quantum Plasma: A Study of Zonal Flows

Zonal flows (ZFs) are band-like sheared flows that are spontaneously generated by turbulence and persist as coherent nonlinear structures. They appear in various contexts, from planetary atmospheres to magnetically confined fusion plasmas, and have been actively studied for decades because of their key role in regulating turbulence and transport. Here, a new approach to studying ZFs is presented. We show that inhomogeneous wave turbulence (IWT) with ZFs can be modeled as an effective quantum plasma where the ZF velocity serves as an emergent collective field. This effective plasma can be described by a Wigner–Moyal kinetic equation, whose geometrical-optics limit also yields an improvement to the traditional wave kinetic equation. We report the first application of the Wigner-Moyal formalism to analytical and numerical modeling of ZF physics. Our results elucidate the phase-space dynamics of IWT during the formation and deterioration of ZFs and also reveal the sheer importance of "full-wave" effects missed in the traditional wave-kinetic approach. The work was supported by the U.S. Department of Energy (DOE), Office of Science, Office of Basic Energy Sciences.

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MS49

Stochastic Dynamics in Spatially Extended Magentic Systems

Models for spatially extended magnetic systems are typically N nearest-neighbor coupled unit vectors that evolve stochastically in time due to unresolved thermal effects, and sample the Gibbs distribution defined by a Hamiltonian quantifying this nearest-neighbor interaction and effects of an external field. Interesting questions arise as the number of vectors goes to infinity while scaling other parameters that effect the temperature and damping in the system. Do discrete model dynamics created by Metropolis Hastings Monte Carlo limit to deterministic harmonic map heat flow dynamics or stochastic partial differential equation dynamics? What is the dimensional effect on the mean transition time between the two metastable states induced by the external field?

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MS49

Numerical Study of the Second-order Correct Hamiltonian Model for Unidirectional Water Waves

Attention is given to the study of the propagation of longcrested wave motions with an incompressible perfect fluid in a uniform horizontal channel. When the fluid motion is irrotational, inviscid and uniform in the cross channel direction, the two-dimensional version of the Euler equations provide a good model of waves on the surface of water. However, in many practical applications, full water wave model appears to be more complex than is necessary, and consequently further approximations are often made in the shallow water regime. In this talk, we numerically study the unidirectional fifth-order (second-order correct) KdV-BBM, and examine behavior of solutions arising from longcrested waves propagation. A numerical algorithm based on the Fourier spectral method is presented, and their numerical convergence is tested. A clean solitary wave is generated numerically, and we report a sequence of numerical tests on the validation of solitary wave approximation. Utilizing the generated clean solitary waves, various numerical experiments on numerical stability, interaction, and resolution of solitary waves are performed. A comparison is made between the KdV-BBM and the fifth-order equations using scaled and unscaled models.

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MS50

Hydrodynamic Models of Two-dimensional Active Liquid Crystals

In active liquid crystals, the energy input at the scale of particles has dramatic consequences at the macroscopic scale. The general consensus has been that activity always destabilises orientational order, ultimately leading to patterns and chaos. In contrast, I will show that in the presence of a substrate this energy input can enhance the stability of the orientationally ordered phase relative to its passive counterpart. The result is dynamically stable quasi-long-range order for active apolar liquid crystals, and a singular suppression of fluctuations, leading to longrange order and normal number fluctuations, in active polar phases. Finally, I will discuss a rotating phase of chiral orientable particles and show that in this case even a twodimensional film which is not in contact with a substrate can sustain an ordered state.

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MS50 On Yield Stress of Concentrated Suspensions

A two-phase model for concentrated suspensions that incorporates a constitutive law combining the rheology for non-Brownian suspensions and granular flow is derived from the underlying microscopic balance laws. The resulting model exhibits a yield-stress behavior for the solid phase as a function of the collision pressure. This property is investigated for simple geometries such as plane Poiseuille flow, where jammed zones of finite width arise. Phase-space methods and asymptotic analysis are used to discuss the evolution to these stationary states. The stability properties of these solutions and their relation to classical Bingham-type flows are discussed.

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MS50

Well-posed Dynamics of Dense Granular Materials

Recent developments in the continuum modeling of granular materials address the long-standing problem of illposedness in the evolution equations. The $\mu(I)$ rheology, in which the friction coefficient mu depends on shear rate through the dimensionless inertial number I, introduced more than a decade ago by Pouliquen and co-authors, is a crucial step in establishing regimes in which equations for incompressible granular flow are well-posed, leaving specific thresholds in I for the onset of ill-posedness. The effect of compressibility, a physically relevant property captured by a variable packing fraction, is introduced via constitutive laws related to Critical State Soil Mechanics. The result is a set of conditions on the constitutive laws under which granular flow is well posed for all shear rates. A simpler model, in which the packing fraction is specified as a constitutive law, is shown to be ill-posed. Extensions to suspensions will also be mentioned, in which the inertial number accounts for the fluid viscosity.

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PP1

Krein Signature for Instability of $\mathcal{PT}\text{-}\mathbf{Symmetric}$ States

Krein quantity is introduced for isolated neutrally stable eigenvalues associated with the stationary states in the \mathcal{PT} -symmetric nonlinear Schrödinger equation. Krein quantity is real and nonzero for simple eigenvalues but it vanishes if two simple eigenvalues coalesce into a defective eigenvalue. A necessary condition for bifurcation of unstable eigenvalues from the defective eigenvalue is proved. This condition requires the two simple eigenvalues before

the coalescence point to have *opposite* Krein signatures. The theory is illustrated with several numerical examples motivated by recent publications in physics literature.

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PP1

Traveling Waves in Mass and Spring Dimer Fermi-Pasta-Ulam-Tsingou Lattices

The Fermi-Pasta-Ulam-Tsingou (FPUT) lattice is an infinite chain of particles connected by springs and constrained to move on a horizontal line. We present results on the existence and properties of traveling wave solutions to the lattice equations of motion for two classes of heterogeneous FPUT lattices: the mass dimer, consisting of alternating particles and identical springs, and the spring dimer, which has identical masses but alternating spring forces. For both dimers, our traveling waves are nanopterons; unlike the classical solitary wave that decays to zero at infinity, the nanopteron is asymptotic to periodic waves with extremely small amplitude. We are able to employ the same bifurcation and quantitative contraction mapping methods for both dimers, but we do encounter some interesting and subtle differences between them in the precise techniques and results.

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PP1

Vortices in Rapidly Rotating Boussinesq Convection

We study long time asymptotics in the Boussinesq approximation for rapidly rotating stably-stratified fluids in a three dimensional infinite layer with either stress-free or periodic boundary conditions. For initial data satisfying certain smallness criteria, we use self-similar variables to show that the baroclinic vorticity converges to an Oseen Vortex with algebraic rate while the baroclinic component decays to zero faster than any algebraic rate. In the case of periodic boundary conditions, we also find that the barotropic vertical velocity and thermal fluctuations converge to decaving Gaussians whose amplitudes oscillate with opposite phase of each other. We also use dispersive estimates and Lyapunov functional techniques to determine asymptotics in the rapid rotation limit for a broader class of initial data where we only require smallness in the quasi-geostrophic part of the baroclinic dynamics. We also discuss recent work on the formation of coherent structures in un-stably stratified flows, such as large scale turbulent condensates.

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PP1

Stability of Double Pulse Solutions to the 5th Order KdV Equation, a Numerical Approach

The fifth-order Korteweg-de Vries equation (KdV5) is a nonlinear partial differential equation used to model dispersive phenomena such as plasma waves and capillarygravity water waves. For wave speeds exceeding a critical threshold, KdV5 admits a countable family of double-pulse traveling-wave solutions, where the two pulses are separated by a phase parameter multiplied by an integer N. It is known that the double pulses are unstable for even N, and it has also been shown that these pulses have either a quadruplet of eigenvalues or a pair of purely imaginary eigenvalues near the origin when N is odd. Moreover, the latter case arises provided the associated eigenfunctions are square-integrable. It is not known which of these two cases arises in KdV5. We provide the results of extensive numerical computations that indicate that the eigenvalue are indeed purely imaginary. We also present numerical Krein matrix computation for periodic waves that indicate that long-wavelength double-pulse wave trains are stable.

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PP1

Pulse Solutions for the Discrete FitzHugh-Nagumo Equation with Infinite Range Interactions.

We establish the existence and nonlinear stability of travelling pulse solutions for the discrete FitzHugh-Nagumo equation with infinite-range interactions close to the continuum limit. For the verification of the spectral properties, we need to study a functional differential equation of mixed type (MFDE) with unbounded shifts. We avoid the use of exponential dichotomies and phase spaces, by building on a technique developed by Bates, Chen and Chmaj for the discrete Nagumo equation. This allows us to transfer several crucial Fredholm properties from the PDE setting to our discrete setting.

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PP1

Spatiotemporal Pattern Extraction by Spectral Analysis of Vector-valued Observables

We present a data-driven framework for extracting spatiotemporal patterns generated by ergodic dynamical systems. Our approach is based on an eigendecomposition of a kernel integral operator acting on a Hilbert space of

vector-valued observables of the system, taking values in a space of functions (scalar fields) on a spatial domain. This operator is constructed by combining aspects of the theory of operator-valued kernels for multitask machine learning with delay-coordinate maps of dynamical systems. Specifically, delay coordinate maps performed pointwise in the spatial domain induce an operator acting on functions on that domain for each pair of dynamical states, whose eigenfunctions can recover spatiotemporal patterns with a non-separable structure in the temporal and spatial coordinates. We discuss two properties of this class of kernel integral operators, namely that they have, in the limit of infinitely many delays, common eigenspaces with the Koopman operator governing the evolution of vectorvalued observables under the dynamics, and that they naturally quotient out dynamical symmetries. We present applications of this framework to the Kuramoto-Sivashinsky model, which demonstrate considerable performance gains in efficient and meaningful decomposition over eigendecomposition techniques utilizing scalar-valued kernels.

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PP1

Reduction Approach to the Dynamics of Interacting Front Solutions in a Singular Perturbed Bistable System

The dynamics of a pulse solution in a bistable reactiondiffusion system is studied analytically by reducing the partial differential equations (PDEs) into finite dimensional ordinary differential equations (ODEs). For the reduction, we apply the multiple scales method to a mixed ODE-PDE system obtained by taking a singular limit in the PDEs. The reduced equations describe the interface motion of a pulse solution formed by two interacting front solutions, in qualitatively good agreement with the pulse dynamics observed numerically for the original PDE system. Furthermore, it is found that the reduction not only facilitates analytical study of the pulse solution, especially the specification of the onset of local bifurcations, but also allows us to elucidate the global bifurcation structure behind the pulse behavior. This is a joint work with Kei Nishi and Yasumasa Nishiura.

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