IP1

Vector-Valued Nonsymmetric and Symmetric Jack and Macdonald Polynomials

For each partition τ of N there are irreducible representations of the symmetric group $\mathcal{S}_N$ and the associated Hecke algebra $\mathcal{H}_N(q)$ on a real vector space V_τ whose basis is indexed by the set of reverse standard Young tableaux of shape τ. The talk concerns orthogonal bases of V_τ-valued polynomials of $x \in \mathbb{R}^N$. The bases consist of polynomials which are simultaneous eigenfunctions of commutative algebras of differential-difference operators, which are parametrized by κ and (q,t) respectively. These polynomials reduce to the ordinary Jack and Macdonald polynomials when the partition has just one part (N). The polynomials are constructed by means of the Yang-Baxter graph. There is a natural bilinear form, which is positive-definite for certain ranges of parameter values depending on τ, and there are integral kernels related to the bilinear form for the group case, of Gaussian and of torus type. The material on Yang-Baxter graphs and Macdonald polynomials is based on joint work with J.-G. Luque.

Charles F. Dunkl
University of Virginia
cfd5z@virginia.edu

IP2

Two Variable q-Polynomials

Abstract not available at this time.

Mourad Ismail
University of Central Florida
mourad.eh.ismail@gmail.com

IP3

On the Asymptotic Behavior of a Log Gas in the Bulk Scaling Limit in the Presence of a Varying External Potential

Abstract not available at time of publication.

Percy Deift
Courant Institute
New York University
deift@math1.cims.nyu.edu

IP4

Title Not Available at Time of Publication

Abstract not available at this time.

Olga Holtz
University of California, Berkeley
Technische Universitaet Berlin
holtz@math.tu-berlin.de

IP5

Integrable Probability and the Role of Painlev Functions

We will review various models in probability that are integrable in the sense that various distribution functions can be explicitly evaluated in terms of Painleve functions and their generalizations. We develop in more detail a class of stochastic growth models that belong to the Kardar-Parisis-Zhang (KPZ) uni- versality class such as the asymmetric simple exclusion process and the KPZ equation. In addition, the experiments of Takeuchi and Sano will be briefly discussed.

Craig A. Tracy
University of California, Davis
tracy@math.ucdavis.edu

IP6

Limits of Orthogonal Polynomials and Contractions of Lie Algebras

In this talk, I will discuss the connection between superintegrable systems and classical systems of orthogonal polynomials in particular in the expansion coefficients between separable coordinate systems, related to representations of the (quadratic) symmetry algebras. This connection allows us to extend the Askey scheme of classical orthogonal polynomials and the limiting processes within the scheme. In particular, for superintegrable systems in 3D, the polynomial representations of quadratic algebras are given in terms of two-variable polynomials and the two-variable analog of the Askey scheme, including the quadratic Racah algebra, will be discussed along with the limiting processes within the scheme.

Sarah Post
University of Hawaii at Manoa
Department of Mathematics
spost@hawaii.edu

IP7

A New Look at Classical Orthogonal Polynomials

There are two possible definitions of classical orthogonal polynomials:(i) they satisfy a second order differential or difference equation;(ii) (generalized) derivative of them gives again orthogonal polynomials. Both definitions are related with concrete forms of corresponding operators. We propose a new approach dealing with some abstract umbral operators. This gives a wide generalization of a notion of classical orthogonal polynomials.

Alexei Zhedanov
Donetsk Institute for Physics and Engineering, Ukraine
zhedanov@yahoo.com

IP8

Asymptotic and Numerical Aspects of Special Functions

For the numerical evaluation of special functions, asymptotic expansions are an important tool. The standard expansions can be used rather straightforwardly. The so-called uniform expansions need more attention, especially for critical values of secondary parameters in the asymptotic problem. For example, the Airy-type expansion of the Bessel function $J_\nu(z)$ can be used for large domains of the argument and order, but for the transition value $z = \nu$ special methods are needed for computing the coefficients. We mention several methods for handling this type of problem. We start with a few examples for which Maple and Mathematica have problems in the evaluation of well-known special functions, like the Kummer U-function, for medium-sized values of the parameters. We discuss recent activities in the Santander-Amsterdam project on the evaluation of special functions, in particular for certain cumulative distribution functions. We start with the incomplete gamma

functions, and we give recent results for the non central chi-squared or the non central gamma distribution, also called Marcum $Q-$function in radar detection and communication problems. This is joint work with Amparo Gil and Javier Segura (University of Cantabria, Santander, Spain).

Nico M. Temme
Centrum Wiskunde & Informatica
The Netherlands
nico.temme@cwi.nl

IP9

Multivariate Orthogonal Polynomials and Modified Moment Functionals

Multivariate orthogonal polynomials can be defined by means a measure defined on a domain on $\mathbb{R}^d$. A very important class of multivariate orthogonal polynomials is called *classical* because the measure satisfies a matrix analogue of the Pearson differential equation as well as the orthogonal polynomials are the eigenfunctions of a partial second order differential operator. In this talk, we present old and new results on classical multivariate orthogonal polynomials. In particular, some classical multivariate orthogonal polynomials and some useful modifications will be studied, as well as their impact into the useful properties of the orthogonal polynomials. We study the so–called Uvarov modification obtained by adding to the measure one or a finite set of mass points. Recently, Christoffel modification in several variables, that is, the modification obtained by multiplying the measure times a polynomial, has been studied in the frame of linear relations.

Teresa E. Pérez
Universidad de Granada
Departamento de Matemática Aplicada
tperez@ugr.es

IP10

Hypergeometric Series: On Number Theory's Secret Service

A natural outcome of the theory of generalized hypergeometric functions are rational approximations to the values of Riemann's zeta functions and alike mathematical constants. In my talk I plan to outline the way it goes (hypergeometric series, hypergeometric Barnes- and Euler-type integrals) and stress on some recent achievements — the current best irrationality measures of π (due to Salikhov), of $\log 2$ (due to Marcovecchio) and of $\zeta(2)$. The final part of the talk will discuss some linear and algebraic independence results that make use of generalized hypergeometric functions.

Wadim Zudilin
University of Newcastle
wzudilin@gmail.com

IP11

Orthogonal Polynomials and the 2-Species ASEP

The asymmetric exclusion process (ASEP) is a model of particles hopping on a 1-dimensional lattice with open boundaries. The partition function of this model is related to moments of Askey-Wilson polynomials. Askey-Wilson polynomials are at the top of the hierarchy of orthogonal polynomials, and are also a limiting case of Koornwinder polynomials, which are multivariate orthogonal polynomials. Recently Eric Rains defined moments of Koornwinder polynomials at q=t, which appear to be polynomials with positive coefficients when written appropriately in the parameters of the ASEP. I'll explain joint work with Sylvie Corteel in which we show that Koornwinder moments at q=t are related to the 2-species ASEP, an exclusion process involving two different types of particles. I'll also describe complementary work of Olga Mandelshtam and Xavier Viennot providing a combinatorial description of the stationary distribution of the 2-species ASEP.

Lauren Williams
University of California Berkeley
williams@math.berkeley.edu

SP1

SIAG/OPSF Gbor Szeg Prize Announcement and Lecture

Abstract not available at time of publication..

Karl Liechty
DePaul University
Department of Mathematical Sciences
kliechty@depaul.edu

CP1

Killip-Simon Problem and Jacobi Flow on Gsmp Matrices

D. Damanik, R. Killip and B. Simon completely described the spectral properties of Jacobi matrices J_+, which are in a sence ℓ^2 perturbations of the isospectral torus of periodic Jacobi matrices. The spectrum of a periodic Jacobi matrix is a system of intervals of a very specific nature. Jointly with P. Yuditskii, we generalize this result to spectral sets, which are arbitrary finite system of intervals.

Benjamin Eichinger
Institute of Analysis, Johannes Kepler University Linz
benjamin.eichinger@jku.at

CP1

A Matrix Approach for the Semiclassical and Coherent Orthogonal Polynomials

We obtain a matrix characterization of semiclassical orthogonal polynomials in terms of the Jacobi matrix associated with the multiplication operator in the basis of orthogonal polynomials, and the lower triangular matrix that represents the orthogonal polynomials in terms of the monomial basis of polynomials. We also provide a matrix characterization for coherent pairs of linear functionals.

Lino G. Garza
Universidad Carlos III de Madrid
lggarza@math.uc3m.es

Francisco Marcellán
Universidad Carlos III de Madrid
Instituto de Ciencias Matemáticas (ICMAT)
pacomarc@ing.uc3m.es

Luis E. Garza
Departamento Matematicas,Universidad Nacional
Colombia
garzaleg@gmail.com

Natalia C. Pinzón-Cortés
Universidad Nacional de Colombia
ncpinzonco@unal.edu.co

CP1

An Explicit Family of Askey-Wilson Type Matrix-Valued Orthogonal Polynomials

We discuss an explicit set of matrix-valued orthogonal polynomials, which are eigenfunctions to a matrix-valued Askey-Wilson type q-difference operator. A study of the corresponding matrix-valued weight leads to an expression of the matrix entries of these polynomials in terms of polynomials from the q-Askey-scheme. Other properties of these polynomials as well as an outlook will be discussed. (Joint work with Noud Aldenhoven and Pablo Román.)

Erik Koelink, Noud Aldenhoven
Radboud Universiteit
e.koelink@math.ru.nl, n.aldenhoven@science.ru.nl

Pablo Román
Univesidad Nacional de Córdoba
roman@famaf.unc.edu.ar

CP1

Jacobi Polynomial Moments and Products of Random Matrices

In this talk we study global singular value distributions arising from products of complex Gaussian random matrices and truncations of Haar distributed unitary matrices. To this end, we introduce an appropriate general class of measures with moments essentially given by specific Jacobi polynomials with varying parameters. Solving the underlying moment problem is based on a study of the Riemann surfaces associated to a class of algebraic equations.

Thorsten Neuschel
Department of Mathematics, KU Leuven
Leuven, Belgium
thorsten.neuschel@wis.kuleuven.be

CP2

Special Functions As Non-Uniform Steady-State Solutions of a Mathematical Model Describing Fluid-Solute Transport in Peritoneal Dialysis

Recently a mathematical model for fluid and solute transport in peritoneal dialysis has been proposed (R. Cherniha et al, Int. J. Appl. Math. Comp. Sci., 2014, V.24, P.837-851). The model is based on a nonlinear system of partial differential equations for fluid and solute concentrations. The model was simplified in order to obtain exact formulae for steady-state solutions, hence, exact solutions involving Bessel and hypergeometric functions for the fluid and solute fluxes are constructed. Biomedical interpretation of mathematical results is provided.

Roman Cherniha
School of Mathematical Sci. University of Nottingham
Roman.Cherniha@nottingham.ac.uk

Jacek Waniewski
IBIB of Polish Acad. Sci.
Warsaw, Poland
jwaniewski@ibib.waw.pl

CP2

Approximate Controllability of An Impulsive Neutral Differential Equation with Deviating Argument and Bounded Delay

In this paper we prove the approximate controllability of an impulsive neutral differential equation with deviated argument and control parameter included in the nonlinear term. We use Schuader fixed point theorem and fundamental assumptions on system operators to prove the result; thereby removing the need to assume the invertibility of a controllability operator which fails to exist in infinite dimensional space if the generated semigroup is compact. We also give an example to illustrate our result. In this paper we study the approximate controllability of the following problem

$$\begin{aligned}
\frac{d[x(t)+g(t,x_t)]}{dt} &= A[x(t)+g(t,x_t))]+Bu(t)+f(t,x(a(x(t),t)),u(t)),\\
x_0(\theta) &= \phi(\theta), \theta\in[-r,0]\\
x(t_k^+)-x(t_k^-) &= I_k(x(t_k)),\ k=1,...,m,
\end{aligned}$$

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Sanjukta Das
Indian Institute of Technology, Roorkee,
sanjukta_das44@yahoo.com

Dwijendra Pandey
Indian Institute of Technology, Roorkee
dwijfma@iitr.ernet.in

CP2

On the Study of Solution of Lorenz System Using Generalized Mittag-Leffler Function

In this talk, generalized Mittag-Leffler function of Prabhakar (1971) and Shukla *et al.* (2009) are applied to solve approximate and analytical solutions of nonlinear fractional differential equation system such as Lorenz system. The obtained results were compared with the results of Homotopy perturbation method and Variational iteration method.

Ranjan K. Jana
S. V. NATIONAL INSTITUTE OF TECHNOLOGY (SVNIT)
SURAT, GUJARAT, INDIA
rkjana2003@yahoo.com

CP2

Hypergeometric Functions and Chebyshev Polynomials: Explicit Solutions of 2-D Free Boundary Problems in Groundwater Hydrology

Steady, phreatic, Darcian flows in hillslope hydrology [1] are generalized to bedrocks with faults, karst or zonal fracturing. By conformal mappings and solution of Hilberts boundary value problems, characteristic functions (derivatives of complex potential/coordinate) are expressed as combinations of hypergeometric functions with three overlapping convergence domains. If a part of impermeable substratum is a control variable the kernel of the Cauchy-type integral is expanded into a series of Chebyshev polynomials. Accuracy of integrations and series truncations is studied. 1. Kacimov A.R., Obnosov Yu.V., Abdalla, O.,

Castro-Orgaz O. (2014) Groundwater flow in hillslopes: analytical solutions by the theory of holomorphic functions and hydraulic theory. Applied Mathematical Modelling, doi:10.1016/j.apm.2014.11.016.

Anvar Kacimov
Sultan Qaboos University
anvar@squ.edu.om

Yurii Obnosov
Kazan Federal University, Russia
Institute of Mathematics and Mechanics
yobnosov@kpfu.ru

CP3

Approximation by a Generalization of the Jakimovski-Leviatan Operators

We introduce a generalization of the Jakimovski–Leviatan-Kantorovich Type operators constructed by A. Jakimovski and D. Leviatan (1969) and the theorems on convergence and the degree of convergence are established. Furthermore, we study the convergence of these operators in a weighted space of functions on a positive semi-axis.

Didem Aydin Ari
Kirikkale University
Kirikkale/ Turkey
didemaydn@hotmail.com

Sevilay Kirci Serenbay
Baskent University
Ankara / Turkey
sevilaykirci@gmail.com

CP3

Integrals Involving Powers of K

Let K denote the complete elliptic integral of the first kind. We investigate integrals of the form

$$\int_0^1 f(x)K(x)^n dx,$$

where f is an algebraic function. We show that, for all natural n, there exist classes of f such that this integral may be evaluated in closed form; e.g.

$$\int_0^1 K(x)^3 dx = \frac{3\Gamma(1/4)^8}{1280\pi^2}.$$

These results are proven using Eisenstein series and Fourier series, and are connected to multiple sums arising from chemical lattices.

James G. Wan
Singapore University of Technology and Design
james_wan@sutd.edu.sg

CP3

Reproducing Kernels of Spherical Monogenics

On the space of spherical harmonics there exists a reproducing kernel that is given by a Gegenbauer polynomial. By going over to complex variables, one obtains a reproducing kernel expressed as a Jacobi polynomial. In this talk, the space of hermitian monogenics, which is the space of polynomial bihomogeneous null-solutions of two complex conjugated Dirac operators, is considered. The reproducing kernel for this space is obtained and expressed in terms of Jacobi polynomials.

Michael Wutzig, Hendrik De Bie, Frank Sommen
Ghent University
michael.wutzig@ugent.be, hendrik.debie@ugent.be, frank.sommen@ugent.be

CP3

New Index Transforms with the Product of Bessel Functions.

In this talk we discuss new index transforms with the product of Bessel functions as kernels. In particular, we will investigate a generalization of the Lebedev transform with the square of the Macdonald function. Mapping properties are studied, inversion formulas are proved. As applications, initial value problems for higher order partial differential and difference equations are solved.

Semyon Yakubovich
University of Porto
Faculty of Sciences, Dept. of Mathematics
syakubov@fc.up.pt

CP4

Representations for the Parameter Derivatives of Orthogonal Polynomials in Two Variables

The aim of this paper is to obtain the parameter derivative representations in the form of

$$\frac{\partial P_{n,k}(\lambda;x,y)}{\partial\lambda} = \sum_{m=0}^{n-1}\sum_{j=0}^{m} d_{n,j,m}P_{m,j}(\lambda;x,y)+\sum_{j=0}^{k} e_{n,j,k}P_{n,j}(\lambda;x,y)$$

for some families of orthogonal polynomials in two variables with respect to their parameters by using the parameter derivatives of the classical Jacobi polynomials $P_n^{(\alpha,\beta)}(x)$ where λ is a parameter and $0 \le k \le n$; $n,k = 0,1,2,\ldots$. Furthermore, the orthogonality properties of the parametric derivatives of these polynomials are discussed.

Rabia Aktas
Ankara University
raktas@science.ankara.edu.tr

CP4

Sums of Squared Baskakov Functions

The complete monotonicity of sums of squares of generalized Baskakov basis functions is proved by deriving the corresponding result for certain hypergeometric functions. This implies its logarithmic convexity. Thereby a recent conjecture of Ioan Rasa is established. Moreover in the central Baskakov case the limit distribution of the zeros is computed for large values of a parameter. As a byproduct new representations for the complete elliptic integral of the first kind are established.

Wolfgang Gawronski
Department of Mathematics
University of Trier
gawron@uni-trier.de

Ulrich Abel
Technical University of Applied Sciences Mittelhessen
Germany

ulrich.abel@mnd.thm.de

Thorsten Neuschel
Department of Mathematics, KU Leuven
Leuven, Belgium
thorsten.neuschel@wis.kuleuven.be

CP4

On the Horizontal Monotonocity of the Gamma Function

The behaviour of the gamma function along vertical lines in the right half plane has been studied closely but very little on the modulus of the gamma function along horizontal lines in the upper half plane. We show that $|\Gamma(t + ia)|$ is monotone in t as soon as the imaginary part a exceeds a threshold value. We give a proof of this and indicate some sharp lower bounds for a beyond which this monotonic behaviour holds.

Gopala Krishna Srinivasan
Indian Institute of Technology Bombay
gopal@math.iitb.ac.in

CP5

Orthogonal Polynomial Interpretation of Δ-Toda and Volterra Lattices

The correspondence between dynamics of Δ-Toda and Δ-Volterra lattices for the coefficients of the Jacobi operator and its resolvent function is established. A method to solve inverse problem –integration of Δ–Toda and Δ-Volterra lattices – based on Padé approximants and continued fractions for the resolvent function is proposed. The main ingredient are orthogonal polynomials which satisfy an Appell condition, with respect to the forward difference operator Δ. It is shown that the Δ-Volterra lattice is related to the Δ-Toda lattice by Miura or Bäcklund transformations.

Ana Foulquie
Universidade de Aveiro
foulquie@ua.pt

Ivan Area
Universidad de Vigo
area@dma.uvigo.es

Amilcar Branquinho
Universidade de Coimbra
ajplb@mat.uc.pt

Eduardo Godoy
Universidad de Vigo
egodoy@dma.uvigo.es

CP5

State of the Art Visualizations of Complex Function Data

The NIST DLMF contains function visualizations that would have been difficult, if not impossible, to envision when Abramowitz and Stegun was published. We have improved quality and accessibility using mesh generation, approximation theory, and the latest 3D web graphics technology to capture key features such as zeros, poles, and branch cuts. We demonstrate our redesign of the function visualizations using WebGL, a JavaScript API for rendering 3D graphics in a web browser without a plugin.

Bonita V. Saunders, Brian Antonishek, Qiming Wang
National Institute of Standards and Technology (NIST)
bonita.saunders@nist.gov, brian.antonishek@nist.gov,
qiming.wang@nist.gov

Bruce Miller
Applied and Computational Mathematics Division
National Institute of Standards and Technology
bruce.miller@nist.gov

CP5

Extension of Generalized Mittag-Leffler Density and Processes

Shukla and Prajapati [J. Math. Anal. Appl. No 336(2007),797-811] extended the Mittag-Leffler function and defined as $E^{\gamma,q}_{\alpha,\beta}(z) = \sum_{n=1}^{\infty} \frac{(\gamma)_{qn}}{\Gamma(\alpha n+\beta)} \cdot \frac{z^n}{n!}$, where $\alpha, \beta, \gamma \in \mathbf{C}, \Re(\alpha) > 0, \Re(\beta) > 0, \Re(\gamma) > 0, q \in (0, 1) \cup \mathbf{N}$, and the function $E^{\gamma,q}_{\alpha,\beta}(z)$ converges absolutely for all $z \in \mathbf{C}$ if $q < \Re(\alpha) + 1$ (entire function of order $\Re(\alpha)^{-1}$) and for $|z| < 1$ if $q = \Re(\alpha)+1$. In this paper, we obtain some properties of Mittag-Leffler density, Structural representation of the Mittag-Leffler variable and Mittag-Leffler Stochastic Process by using $E^{\gamma,q}_{\alpha,\beta}(z)$.

Pratik V. Shah
C.K.Pithawalla College of Engg. & Technology,
Surat - 395007 , Gujarat , INDIA.
pratikshah8284@yahoo.co.in

Ajay Shukla
S.V.National Institute of Technology,
Surat 395007, Gujarat, India.
ajayshukla2@rediffmail.com

CP5

Associated Polynomials, Markov's Theorem, and First-Hitting Times of Birth-Death Processes

A classical result of Karlin and McGregor states that for a birth-death process on the nonnegative integers with corresponding birth-death polynomials $\{Q_n\}$, the Laplace transform of the first-hitting time T_{ij} satisfies

$$E[e^{sT_{ij}}] = \frac{Q_i(s)}{Q_j(s)}, \quad s < 0,$$

provided $i < j$. In a recent paper (J. Theor. Probab. 25 (2012) 950-980) Gong, Mao and Zhang, using the theory of Dirichlet forms, established representations for $E[e^{sT_{ij}}]$ with $i > j$, featuring associated polynomials of $\{Q_n\}$. It will be shown in the talk that these results may also be obtained with the help of Markov's theorem.

Erik A. Van Doorn
University of Twente
e.a.vandoorn@utwente.nl

CP6

Ring of Integrals Operators

we define an ring of linear operators whose kernels are essentially Bessel functions. These operators act on spaces of entire functions of exponential type and are endowed with a complete and explicit symbolic calculus that we will described. We introduce a new symbols to obtain a new class polynomials.

Miloud Assal
Institut Supérieur des Mathématiques Appliquées
Miloud.assal@fst.rnu.tn

CP6

Stable Regions of Turán Expressions

Consider polynomial sequences that satisfy a first-order differential recurrence. We prove that if the recurrence is of a special form, then the Turán expressions for the sequence are weakly Hurwitz stable (non-zero in the open right half-plane). A special case of our theorem settles a problem proposed by S. Fisk that the Turán expressions for the univariate Bell polynomials are weakly Hurwitz stable. We obtain related results for Chebyshev and Hermite polynomials, and propose several extensions involving Laguerre polynomials, Bessel polynomials, and Jensen polynomials associated to a class of real entire functions. This is joint work with Lukasz Grabarek (University of Alaska) and Mirkó Visontai (Google Reseach).

Matthew Chasse
Rochester Institute of Technology
mncsma@rit.edu

CP6

New Characterizations of Leonard Pairs

Leonard pairs are pairs of linear transformations that act on each other's eigenspaces in an irreducible tridiagonal fashion. They are related to the Askey scheme of orthogonal polynomials, distance-regular graphs, and the representation theory of Lie algebras. In this talk, we will discuss two new characterizations of Leonard pairs using certain sequences of parameters.

Edward Hanson
State University of New York at New Paltz
hansone@newpaltz.edu

CP6

A Classification of the Lowering-Raising Triples.

Let F denote a field, and let V denote a vector space over F with positive dimension $d+1$. By a decomposition of V we mean a sequence of one-dimensional subspaces whose direct sum is V. Let $\{V_i\}_{i=0}^d$ denote a decomposition of V. A linear transformation $A \in \text{End}(V)$ is said to lower $\{V_i\}_{i=0}^d$ whenever $AV_i = V_{i-1}$ for $1 \leq i \leq d$ and $AV_0 = 0$. The map A is said to raise $\{V_i\}_{i=0}^d$ whenever $AV_i = V_{i+1}$ for $0 \leq i \leq d-1$ and $AV_d = 0$. A pair of elements A, B in $\text{End}(V)$ is called lowering-raising (or LR) whenever there exists a decomposition of V that is lowered by A and raised by B. A triple of elements A, B, C in $\text{End}(V)$ is called LR whenever any two of A, B, C form an LR pair. We classify up to isomorphism the LR triples on V. We discuss how LR triples are related to the quantum group $U_q(sl_2)$.

Paul M. Terwilliger
Math Department, University of Wisconsin-Madison
terwilli@math.wisc.edu

CP7

A Q-Extension of a Reduction Formula of Watson

In this paper, we give a reduction formula for the q-integral

$$I_n = \int_0^1 x^{\alpha}(qx;q)_{\beta}\, D^n_{q^{-1}}\left[x^{\gamma}(q^{\beta+1}x;q)_{\delta}\right] d_qx, \quad n = 0, 1, 2, ..$$

where α, β, γ, and δ are complex numbers. The reductions formula is a three term recurrence relations of I_n. The representation of I_n in terms of basic hypergeometric series will be investigated. This is a q-analogue of the work introduced by W. N. Bailey in 1953.

Zeinab Mansour
King Saud University Facuty of Science, Mathematics Dep.
Riyadh P.O. Box 2455 Riyadh 11451 Saudi Arabia
zeinabs98@hotmail.com

Maryam AL Towaileb
Kingdom of Saudi Arabia
P.O.Box 22459, Riyadh 11495
mtowaileb@ksu.edu.sa

CP7

Some Q-Continued Fractions and Their Connections with Lambert Series and Mock Theta Functions

Mock theta function is last gift to Mathematical world by S.Ramanujan,that he quoted in his last letter to G.H. Hardy on January 12,1920. Lambert series is a well known series used by Ramanujan to prove many of his identities. Following Ramanujan, many mathematicians like George Andrews and B.C Berndt also used these series to prove many of Ramanujan identities. In the present work, by making use of Andrews and Hickerson identities, we have established certain new q-continued fractions for the ratio of Lambert series and also for the ratio of combination of mock theta function of order six.

Mohan Rudravarapu
Government Polytechnic,Srikakulam
Andhra Pradesh,India
mohanrudravarapu@gmail.com

Pankaj Srivastava
Motilal Nehru National Instituite of Technology,Allahabad
drpankaj23@gmail.com

CP8

On Verblunsky Coefficients Related to a Particular Class of Carathéodary Functions

Schur algorithm provides a procedure to obtain Szegö polynomials from Carathéodary functions defined on the unit disk by means of PC-fractions. The Verblunsky coefficients obtained from these Szegö polynomials are useful in characterizing various interesting properties of these polynomials. In this work, a class of Verblunsky coefficients related to a particular class of Carathéodary functions available in the literature are considered. The corresponding Chain sequences and minimal parameter sequences related to these Verblunsky coefficients are investigated. Using a well-known procedure, the corresponding para orthogonal polynomials and Szegö polynomials are reconstructed which lead to interesting observations. Various related applications for these coefficients are obtained and compared with the results exist in the literature.

Swaminathan Anbhu
I.I.T. Roorkee
mathswami@gmail.com, swamifma@iitr.ernet.in

CP8

On Zeros of a Class of Real Self-Reciprocal Polynomials

We investigate the distribution, the simplicity and the monotonicity properties of the zeros around the unit circle and real line of the real self-reciprocal polynomials given by $R_n(z) = 1 + d(z + z^2 + \cdots + z^{n-1}) + z^n$, where d is a real number and $n > 0$. The sequence of polynomials $\{R_n\}$ satisfies a three term recurrence relation, this recurrence relation is used to get further results.

Cleonice F. Bracciali
UNESP - Universidade Estadual Paulista
Dept of Applied Mathematics
cleonice@ibilce.unesp.br

Vanessa Botta, Junior Pereira
UNESP - Universidade Estadual Paulista
botta@fct.unesp.br, junior.gusto@hotmail.com

CP8

Orthogonal Polynomials with As Their Weight Functions

Abstract. The orthogonal polynomials are used very often in applied mathematics M. Podisuk and his colleague presented some kinds of orthogonal polynomials in I, II, III and IV. In this paper, the orthogonal polynomials in the closed interval [0,1] with the weight functions of the form will be illustrated.

Maitree -. Podisuk
Kasem Bundit University
podisuk@gmail.com

CP8

Discrete Orthogonal and q-Orthogonal Polynomial Sequences in the Extended Hahn Classes with Simple Recurrence Coefficients

In [1] we found explicit formulas for the coefficients a_n and b_n of the recurrence $p_{n+1}(x) = (x - b_n)p_n(x) - a_n p_{n-1}(x)$ for all the elements in the extended Hahn classes of discrete orthogonal and q-orthogonal polynomial sequences. Such coefficients are rational functions of n, in the first case, and of q^n, in the second case, which are determined by the initial terms b_0, b_1, b_2, a_1, a_2. We consider the cases that yield simple rational functions. In addition to the well-known classical sequences we obtain many other examples, such as $b_n = b_0$ and $a_n = a_1 n^2$ or $a_n = 3n^4 a_1 (4n^2 - 1)^{-1}$, in the discrete case. We also find the Bochner-type operator for each example. [1] L. Verde-Star, Linear Alg. Appl. 440 (2014) 293–306.

Luis Verde-Star
Universidad Autonoma Metropolitana
verde@xanum.uam.mx

CP9

Generalized Hurwitz Matrices: Criteria of Total Positivity

Given an infinite-dimensional generalized Hurwitz matrix $H_M(f) = \{a_{Mj-i}\}_{ij}$, defined by a polynomial $f(x) = a_0 x^n + a_1 x^{n-1} + \ldots + a_n$, $a_i \in \mathbb{R}$ and a positive integer $M \leq n$, we provide a necessary and sufficient criterion for its total positivity. Our criterion is given in the form of a finite number of determinantal conditions. We also consider a factorization of a totally positive generalized Hurwitz matrix using a generalization of the classical Euclidean algorithm.

Olga Y. Kushel
Technische Universitaet Berlin
kushel@mail.ru

Olga Holtz
University of California-Berkeley
holtz@math.berkeley.edu

Sergey Khrushchev
Kazakh-British Technical University
svk_49@yahoo.com

Mikhail Tyaglov
Shanghai Jiao Tong University
tyaglov@sjtu.edu.cn

CP9

Canonical Vector-Polynomials for Complex Order Modified Bessel Functions

The canonical vector-polynomials for the approximation of modified Bessel functions of the second kind with complex order and real argument are constructed. Lanczos Tau method computational scheme is applied for the approximation of the solutions of hypergeometric type differential equations and their systems. The stable recurrent scheme is developed for the calculation of the canonical vector-polynomials coefficients. Some applications for mixed boundary value problems in wedge domains are shown.

Juri M. Rappoport
Russian Academy of Sciences
jmrap@landau.ac.ru

CP9

Products of Truncated Unitary Matrices

We study the joint singular value density of fixed or random matrices multiplied with truncated unitary matrices distributed by the induced Haar measure. We prove that the squared singular values of a fixed matrix multiplied with a truncated unitary matrix are distributed according to a polynomial ensemble, which is a special type of determinantal point process. As a corollary, we apply this result to the situation where we multiply a truncated unitary matrix with a random matrix whose squared singular values are a polynomial ensemble and show that this structure is preserved by the product. As an application we derive the joint singular value density of a product of truncated unitary matrices and its kernel of the k-point correlation function. The correlation kernel can be written as a double contour integral and using this integral representation we obtain hard edge scaling limits.

Dries Stivigny
KU Leuven
dries.stivigny@wis.kuleuven.be

CP9

Some Applications of Determninants to Orthogonal Polynomials

We show how different type of structered matrices and their determiants contribute to the theory orthogonal polynomials and related ordinary differential equations and provide new classes of orthogonal polynomials.

Mikhail Tyaglov
Shanghai Jiao Tong University
tyaglov@sjtu.edu.cn

MS1

Multivariate Orthogonal Polynomials and Integrable Systems

In this talk we discuss multivariate orthogonal polynomials in the Euclidean space and multivariate orthogonal Laurent polynomials in the unit torus. Using the Gauss-Borel factorization of a moment matrix we consider three term relations, Christoffel-Darboux formulas, Darboux transformations and the connection with a hierarchy of compatible nonlinear partial differential and difference equations of integrable type.

Manuel Manas
Departamento de Fsica Teórica II
Universidad Complutense de Madrid
manuel.manas@fis.ucm.es

MS1

Orthogonal Polynomials on the Unit Ball and Fourth Order Partial Differential Equations

In this work we study a family of mutually orthogonal polynomials on the unit ball with respect to an inner product which includes an additional spherical term. First, we will deduce connection formulas relating classical multivariate orthogonal polynomials on the ball and the new family of orthogonal polynomials. Then, using the representation of these polynomials in terms of spherical harmonics, algebraic, analytic and differential properties will be deduced.

Miguel Pinar
University of Granada
mpinar@ugr.es

MS1

Algebraic Interpretation of Multivariate q-Krawtchouk Polynomials

An algebraic interpretation of q-analogs of the multivariate Krawtchouk polynomials introduced by Tratnik will be presented. It will be shown that these polynomials in n variables arise as matrix elements of unitary "q-rotation" operators expressed as q-exponentials in $sl_q(n+1)$ generators realized in terms of the ladder operators of n independent q-oscillators. The main ideas will be illustrated by focusing on the case $n = 2$.

Vincent Genest
Centre de recherches mathématiques
Université de Montréal
vincent.genest@umontreal.ca

Sarah I. Post
Centre de Recherches Mathématiques
post@crm.umontreal.ca

Luc Vinet
Centre de recherches mathematiques
luc.vinet@umontreal.ca

Alexei Zhedanov
Donetsk Institute for Physics and Engineering, Ukraine
zhedanov@yahoo.com

MS2

Repeated Integrals of the Coerror Function, Revisited

Nonstandard Gaussian quadrature is applied to evaluate the repeated integral $\mathrm{i}^n\mathrm{erfc}\, x$ of the coerror function for $n \in \mathbf{N}_0$, $x \in \mathbf{R}$ in an appropriate domain of the (n, x)-plane. Relevant software in Matlab is provided, in particular two routines evaluating the function to an accuracy of twelve resp. thirty decimal digits.

Walter Gautshi
Purdue University
wgautschi@purdue.edu

MS2

Numerical Multivariate Polynomial Factorization

We present a method to extract the factors of a multivariate polynomial in floating point arithmetic. We establish the connection between the irreducible factors of a multivariate polynomial and the Taylor expansion of its reciprocal. Based on this connection, a multivariate generalization of the qd-algorithm is employed to compute the irreducible factors. Our iterative method does not require the input polynomial to be square-free. Moreover, this approach provides additional insight to some problems in multilinear algebra.

Wen-shin Lee, Annie Cuyt
University of Antwerp
wen-shin.lee@uantwerpen.be, annie.cuyt@uantwerpen.be

MS2

DLMF Standard Reference Tables on Demand (DLMF Tables) (6)

Most current libraries and systems produce tables of function values with limited information about accuracy. To address this void, NIST ACMD and the University of Antwerp CMA Group are collaborating to build the DLMF Standard Reference Tables web service to provide a standard of comparison for testing numerical software by computing, on demand, special functions to user-defined accuracy with guaranteed error bounds. We will discuss the beta version recently released, plus current and proposed work.

Bonita V. Saunders

National Institute of Standards and Technology (NIST)
bonita.saunders@nist.gov

Bruce Miller
Applied and Computational Mathematics Division
National Institute of Standards and Technology
bruce.miller@nist.gov

Marjorie McClain
Mathematical and Computational Sciences Division
National Institute of Standards and Technology
marjorie.mcclain@nist.gov

Daniel Lozier
National Institute of Standards and Technology
daniel.lozier@nist.gov

Andrew Dienstfrey
National Institute of Standards and Technology (NIST)
andrew.dienstfrey@nist.gov

Annie Cuyt
University of Antwerp
annie.cuyt@uantwerpen.be

Stefan Becuwe, Franky Backeljauw
University of Antwerp, Belgium
stefan.becuwe@uantwerpen.be,
franky.backeljauw@uantwerpen.be

MS2

Near-Minimal Cubature Rules on the Disk

High-dimensional integration rules have received a lot of attention in a whole series of paper. The connection with multivariate orthogonal polynomials is usually present. We present a unifying theory, including several of the existing minimal rules, departing from the rather recent concept of spherical orthogonality. Use of the newly defined multivariate orthogonal functions leads to a structured set of Prony-structured equations from which the nodes and weights of certain minimal cubature formulae can be computed.

Irem Yaman
Gebze Technical University
iremymn@gmail.com

Annie Cuyt
University of Antwerp
annie.cuyt@uantwerpen.be

Brahim Benouahmane
Universite Hassan II
Morocco
benouah@uh2m.ac.ma

MS3

Bressoud Polynomials and the Rogers-Ramanujnan Identities

In 1983, D.M. Bressoud published An Easy Proof of the Rogers-Ramanujan Identities (J. Number Th., 16(1983), 235-241). The elegance of his proof has been noticed by many. The conclusion of the paper is devoted to a generalization of his result where he proves a polynomial identity which in the limit yields two instances of the generalized Rogers-Ramanjuan series/product identity (G.E. Andrews, Proc. Nat. Acad. Sci., 71(1974), 4082-4085). A study of this final result in Bressoud's paper reveals the Bressoud polynomials which in turn have various applications and lead to open problems.

George E. Andrews
Penn State University
Department of Mathematics
andrews@math.psu.edu

MS3

Monodromy of Hypergeometric Functions in Several Variables

The analytic continuation properties of single variable hypergeometric values is well-known nowadays. The next challenge is to obtain similar results for the class of A-hypergeometric functions. These were introduced by Gel'fand, Kapranov and Zelevinsky in the late 1980's and contain the classical Appell and Lauricella functions as special cases.

Frits Beukers
Utrecht University
f.beukers@uu.nl

MS3

q-Bessel Functions and Rogers-Ramanujan Type Identities

We show how q-Bessel functions of the first kind lead to Rogers-Ramanujan type identities. Carlotta considered a special case of this and used it to give new proofs of the Rogers-Ramanujan identities.

Mourad Ismail
University of Central Florida
mourad.eh.ismail@gmail.com

Ruimimg Zhang
Northwest A & F University
ruimingzhang@gmail.com

MS3

Mahler Measures of Hyperelliptic Families

I will overview some recent progress on the Mahler measure of two-variable polynomials corresponding to special elliptic families; these are known as Boyd's conjectures. A machinery, which was created in our joint work with Mat Rogers and further developed by Anton Mellit and François Brunault, allows one to relate the Mahler measures to the L-values of the underlying elliptic curves when the latter are parametrised by modular units. The talk will culminate with new results on Boyd's conjectures for a hyperelliptic family obtained in joint work with Marie José Bertin (Paris 6).

Wadim Zudilin
University of Newcastle
wzudilin@gmail.com

MS4

Equilibrium Measures and Their Support

Let $Q(x)$ be an admissible external field. Let ν be the signed equilibrium associated with $Q(x)$. Suppose that the positive part of ν consists of finitely many intervals, and ν

has log-concave density there. We show that the support of the equilibrium measure consists of finitely many intervals.

David Benko
Univerity of South Alabama
Mobile, AL
dbenko@southalabama.edu

MS4

Asymptotically d-Energy Minimizing Sequences of Configurations on d-dimensional Conductors

We show that the sequence of configurations on a compact set $A \subset \mathbf{R}^d$ with $\mathcal{L}_d(A) > 0$ and $\mathcal{L}_d(\partial A) = 0$ ($\mathcal{L}_d$ is the Lebesgue measure), obtained by intersecting A with any d-dimensional lattice scaled by a factor going to zero is asymptotically d-energy minimizing. When $\mathcal{L}_d(\partial A) > 0$, an asymptotically d-energy minimizing sequence is constructed. When A is contained in a d-dimensional C^1 manifold, any asymptotically best-packing sequence of N-point configurations is asymptotically d-energy minimizing.

Sergiy Borodachov
Towson University
Baltimore, USA
sborodachov@towson.edu

Douglas Hardin, Edward Saff
Vanderbilt University
doug.hardin@vanderbilt.edu, edward.b.saff@vanderbilt.edu

MS4

Covering and Separation for Points on the Sphere

The covering radius of an N-point set X_N on the unit sphere in R^{d+1} is the radius of the largest spherical cap that contains no points from X_N whereas the separation distance gives the least distance between two different points in X_N. In the theory of approximation and interpolation, the covering radius arises in the error of approximation whereas good separation of points is generally associated with the stability of an approximation or interpolation method. We present resent results and open questions regarding the covering and separation properties of, in particular, random points on the sphere. This talk is based on joint work with Josef Dick, Ed Saff, Ian Sloan, Yuguang Wang and Rob Womersley.

Johann Brauchart
Technical University at Graz
Graz, Austria
j.brauchart@tugraz.at

MS4

Characterization of Complex Variable Positive Definite Functions

A holomorphic positive definite function f defined on a horizontal strip of the complex plane is characterized as the Fourier-Laplace transform of a unique exponentially finite measure on $\mathbf{R}$. This characterization unifies the integral representations in Bochner's theorem on positive definite functions and Widder's theorem on exponentially convex functions. Special interesting cases include the Gamma function and the Riemann Zeta function. Further characterizations are derived from the usual concepts of moment, moment-generating function and characteristic function.

Jorge C. Buescu
Faculdade de Ciencias
Universidade de Lisboa
jsbuescu@fc.ul.pt

António Paixo
ISEL
apaixao@dem.isel.ipl.pt

Alexandra Symeonides
IST
alexandra.symeonides@gmail.com

MS5

On Perturbations of the Toda Lattice

We present the results of an analytical and numerical study (with Nenciu) of the long-time behavior for certain Fermi-Pasta-Ulam lattices viewed as perturbations of the completely integrable Toda lattice. Our main tools are the direct and inverse scattering transforms for doubly-infinite Jacobi matrices. We also present our work (with Trogdon) where we solve the Cauchy problem for the Toda lattice by computing the Riemann-Hilbert formulation of the inverse scattering transform.

Deniz Bilman, Irina Nenciu
University of Illinois at Chicago
dbilma2@uic.edu, nenciu@uic.edu

Thomas Trogdon
Courant Institute of Mathematical Sciences
NYU
trogdon@cims.nyu.edu

MS5

Transition Asymptotics for the Painlevé II Transcendent

We consider real-valued solutions $u = u(x|s), x \in \mathbb{R}$ of the second Painlevé equation $u_{xx} = xu + 2u^3$ which are parametrized in terms of the monodromy data $s \equiv (s_1, s_2, s_3) \subset \mathbb{C}^3$ of the associated Flaschka-Newell system of rational ODEs. Our analysis describes the transition asymptotics as $x \to -\infty$ between the oscillatory power-like decay asymptotics for $|s_1| < 1$ (Ablowitz-Segur) to the power-like growth behavior for $|s_1| = 1$ (Hastings-McLeod) and from the latter to the singular oscillatory power-like growth for $|s_1| > 1$ (Kapaev). It is shown that the transition asymptotics is of Boutroux type, i.e. it is expressed in terms of Jacobi elliptic functions.

Thomas J. Bothner
Université de Montréal
thomas.bothner@gmail.com

MS5

On Non-Linearizable Boundary Value Problems for the Defocusing Nonlinear Schrödinger Equation on the Half-Line

We study non-linearizable Dirichlet initial/boundary-value problems for the defocusing nonlinear Schrödinger equation on the half-line $x > 0$ using a modification of the unified transform method. We sidestep the global relation by instead introducing an explicit approximation of the

Dirichlet-to-Neumann mapping for the problem suggested by dispersionless theory. Accuracy is proven directly in the semiclassical/dispersionless limit, with the help of steepest descent methods. This is joint work with Zhenyun Qin, Fudan University.

Peter D. Miller
University of Michigan, Ann Arbor
millerpd@umich.edu

Zhenyun Qin
Fudan university
zyqin@fudan.edu.cn

MS5

Numerical Inverse Scattering for the BenaminOno Equation

Numerical inverse scattering has proven a successful tool for calculating the Korteweg–de Vries and nonlinear Schrdinger equations, which consists of calculating spectral data for a differential equation (direct scattering) and solving a Riemann–Hilbert problem (inverse scattering). The Benjamin–Ono equation does not fit immediately in this form, as the direct scattering is a Riemann–Hilbert/differential equation and the inverse scattering is a nonlocal Riemann–Hilbert problem involving an oscillatory integral term. In this talk we investigate numerics for these more difficult problems.

Sheehan Olver
The University of Sydney
Sheehan.Olver@sydney.edu.au

MS6

Multivariate Meixner Polynomials and a Discrete Model of the Two-dimensional Harmonic Oscillator with $su(2)$ Symmetry

A discrete model of the two-dimensional harmonic oscillator based on the two-variable Meixner polynomials is presented. It is shown that the system is superintegrable and has $su(2)$ symmetry. The interpretation of the d-variable Meixner polynomials as matrix elements for $SO(d, 1)$ representations on oscillator states is reviewed.

Julien Gaboriaud
Université de Montréal
julien.gaboriaud@umontreal.ca

Vincent Genest
Centre de recherches mathématiques
Université de Montréal
vincent.genest@umontreal.ca

Jessica Lemieux
University of Ottawa
jlemi049@uottawa.ca

Luc Vinet
Centre de recherches mathematiques
luc.vinet@umontreal.ca

MS6

Recent Developments in Hyperinterpolation

Hyperinterpolation is an approximation technique developed by Ian Sloan in 1995. In this talk, a survey of recent results in the field will be presented.

Jeremy Wade
Pittsburg State University
jwade@pittstate.edu

MS6

A New Identity for Gegenbauer Polynomials and Reproducing Kernels for Multivariable Orthogonal Polynomials

A new integral identity that expresses the Gegenbauer polynomial C_n^λ in terms of an integral of C_n^μ, with $\mu > \lambda$, is established. It is used to derive concise formulas for reproducing kernels of orthogonal polynomials for two families of weight functionsL: the product Gegenbauer weight functions on the unit cube and the generalized Gegenbauer weight functions on the unit ball of R^d.

Yuan Xu
University of Oregon
yuan@uoregon.edu

MS6

Some Topics on Basis Function Approximation on the Sphere

In several cases, kernel-based methods for scattered data nicely frame within the context of approximation on Gelfand pairs. With the Euclidean motion group being the most prominent example, another geometrical setting is given by zonal basis function approximation. The sphere is tied to the pair (SO_d, SO_{d-1}) with Gegenbauer polynomials defining the spherical functions. Aiming at transferring well-established results from radial basis function approximation to the geometric setting of the sphere, certain properties of Gegenbauer polynomials are needed. While some of these properties are already known, others lead to interesting problems dealing with this family of orthogonal polynomials. We will address these issues and explain some of the results. The talk is based on joint work with R. Beatson and Y. Xu.

Wolfgang zu Castell
Helmholtz Zentrum Muenchen, German Research Center for Envir
Ingolstaedter Landstrasse 1, 85764 Neuherberg, Germany
castell@gsf.de

MS7

Algorithms for An Interpolation Problem on the Unit Circle Between Lagrange and Hermite Problems

This piece of work is devoted to obtain formulae for an interpolation problem on the Unit Circle ($\mathbb{T}$). The interpolation conditions gather the values of the polynomial at $2n$ nodes and its first derivative at n nodal points; the nodal system is constituted by $2n$ equispaced points of modulus 1. We give different solutions, between them we point out an explicit solution in terms of the natural basis and two barycentric type formulae. All these formulae are suitable for numerical applications.

Elias Berriochoa, Alicia Cachafeiro
University of Vigo
esnaola@uvigo.es, acachafe@uvigo.es

José GarcíA-Amor

I. E. S. Valle Inclan, Xunta de Galicia
garciaamor@uvigo.es

MS7

Bernoulli Polynomials As a Base in Numerical Methods

In theory, since the best approximation in $L^2[a,b]$ is unique, all polynomials which are in $L^2[a,b]$ can give the same approximation of known functions. However, in application, it does not occur because we have computational error. We know usually Legendre polynomials have the less computational error than other polynomials. We show Bernoulli polynomials create less computational error than Legendre polynomials. For this reason, we introduced the Bernoulli polynomials as trial functions.

Somayeh Mashayekhi
Mississippi state university
sm2395@msstate.edu

MS7

Approximation of Periodic Functions in Terms of Moduli of Continuity

A 2p periodic function in Lp spaces is approximated by the trigonometric polynomials of degree and the error of approximation is given by norm minimozation of the norm $||Tn(x) - f(x)||p.$. The error defined above depends on the various properties of the function f. In this paper, we obtain the error of approximation in terms of the modulus of continuity of the function f. We use some summability techniques to accelerate the rate of convergence. We also derive some corollaries from our results.

Uaday Singh
Indian Institute of Technology Roorkee
Roorkee-247667 (India)
usingh2280@yahoo.co.in

Soshal Saini
Indian Institute of Technology Roorkee
Roorkee-247667
sainisoshal125@gmail.com

MS7

A New Way to Compute Sine Integral Function

Considering its wide application areas in science and engineering, efficient calculation of the sine integral function is essential. We present a new method to calculate the sine integral function which is based on an accurate representation of the sinc function as a sum of scaled cosines. The proposed method is as accurate as MATLAB's sinint and is more efficient than both MATLAB's sinint and truncated spherical Bessel function expansion, especially for large number of points.

Evren Yarman
Schlumberger
cyarman@slb.com

Garret Flagg
Schlumberger, Houston, TX
gflagg@slb.com

MS8

Boundary Integrals and Approximations of Harmonic Functions

Steklov expansions for a harmonic function on a rectangle are analyzed. The value of a harmonic function at the center of a rectangle is shown to be well approximated by the mean value of the function on the boundary plus a very small number (often 3 or fewer) of additional boundary integrals. Similar approximations are found for the central values of solutions of Robin and Neumann boundary value problems. These results are based on finding explicit expressions for the Steklov eigenvalues and eignfunctions. This is joint work with Professor Giles Auchmuty.

Giles Auchmuty
University of Houston, USA
giles@math.uh.edu

Manki Cho
University of Houston
realmann@math.uh.edu

MS8

Universal Lower Bounds for Potential Energy of Spherical Codes

Minimal energy configurations (codes), maximal codes, and spherical designs have wide ranging applications in various fields of science, such as crystallography, nanotechnology, material science, information theory, wireless communications, etc. We derive universal lower bounds for the potential energy of spherical codes, that are optimal in the framework of the standard linear programming approach and provide a characterization of when improvements are possible. Our bounds are universal in the sense of Cohn and Kumar; i.e., they apply to any absolutely monotone potential.

Peter Dragnev
Indiana UniversityPurdue University Fort Wayne
dragnevp@ipfw.edu

Peter Boyvalenkov
Institute for Mathematics and Informatics, Sofia, Bulgaria
peter@math.bas.bg

Douglas Hardin, Edward Saff
Vanderbilt University
doug.hardin@vanderbilt.edu,
edward.b.saff@vanderbilt.edu

Maya Stoyanova
Sofia University, Bulgaria
stoyanova@fmi.uni-sofia.bg

MS8

Asymptotics of Minimal Discrete Periodic Energy Problems

Let L be a d-dimensional lattice in R^d. For a parameter $s > 0$, we consider the asymptotics of N point configurations minimizing the L-periodic Riesz s-energy as the number of points N goes to infinity. In particular, we focus on the case $0 < s < d$ of long-range potentials where we establish that the minimal energy $E_s(L,N)$ is of the form $E_s(L,N) =$

$C_0 N^2 + C_1 N^{1+s/d} + o(N^{1+s/d})$ as $N \to \infty$. The constants C_0 and C_1 depend only on s, d, and covolume of the lattice L.

Douglas Hardin
Vanderbilt University
doug.hardin@vanderbilt.edu

MS8

Asymptotics for Maximal Polarization Configurations

For a compact manifold A in Euclidean space and a lower semi-continuous kernel K on $A \times A$ we consider the problem of determining the asymptotic behavior of N-point configurations $\{x_i\}_{i=1}^N$ that maximize the minimum of the potential $\sum_{i=1}^N K(x, x_i)$ for x in A. We refer to such extremal problems as polarization problems (the terminology Chebyshev problems is also used). We focus especially on Riesz s-kernels $K(x, y) = 1/|x - y|^s$ for $s > 0$ and their connection with best covering problems on A. Results for $s > \dim(A)$ are of particular interest.

Edward Saff
Vanderbilt University
edward.b.saff@vanderbilt.edu

MS9

Saturated Bands in Many-Pole Riemann-Hilbert Problems

Many-pole Riemann-Hilbert problems arise in the analysis of semiclassical limits for differential equations and orthogonal polynomials. In current methods certain parameter ranges corresponding to saturated bands must be excluded for technical reasons. We present progress on obtaining asymptotic results in these regions with applications to the nonlinear Schrodinger equation, the sine-Gordon equation, and discrete orthogonal polynomials.

Robert J. Buckingham
Dept. of Mathematical Sciences
The University of Cincinnati
buckinrt@uc.edu

Peter D. Miller
University of Michigan, Ann Arbor
millerpd@umich.edu

David A. Smith
University of Crete
smith5dd@ucmail.uc.edu

Megan Stone
University of Arizona
mmccormick@math.arizona.edu

MS9

Long Time Asymptotics and Stability of Finite Density Solutions of the Defocusing NLS Equation

We consider the Cauchy problem for the defocusing nonlinear Schrödinger equation (NLS). Using the $\overline{\partial}$ generalization of the nonlinear steepest descent method of Deift and Zhou we derive the leading order approximation to the solution of NLS in the solitonic region of space–time $|x| < 2t$ for large times and provide bounds for the error which decays as $t \nearrow \infty$ for a general class of initial data whose difference from the non vanishing background possess's a fixed number of finite moments and derivatives. Using properties of the scattering map of NLS we derive as a corollary an asymptotic stability result for initial data which are sufficiently close to the N-dark soliton solutions of NLS.

Robert Jenkins
Dept. of Mathematics
University of Arizona
rjenkins@math.arizona.edu

MS9

Long-Time Asymptotics for a Discrete-Discrete Integrable Equation

The lattice potential KdV equation is a fully discrete integrable equation, whose initial value problem was recently studied by Butler and Joshi. We obtain long-time asymptotics for this equation, using a suitable modification of the methods of Teschl's group. The results show a striking similarity with corresponding results for the continuous KdV equation, reinforcing the value of this discretization.

David A. Smith
University of Crete
smith5dd@ucmail.uc.edu

MS9

Direct Scattering and Small Dispersion for the Benjamin-Ono Equation with Rational Initial Data

We propose a construction procedure for the scattering data of the Benjamin-Ono equation with a rational initial condition, under mild restrictions. The construction procedure entails building the Jost solutions of the Lax pair explicitly and use their analyticity properties to recover the reflection coefficient, eigenvalues, and phase constants. We finish by showing that this procedure validates certain well-known formal results obtained in the zero-dispersion limit.

Alfredo N. Wetzel, Peter D. Miller
University of Michigan, Ann Arbor
wreagan@umich.edu, millerpd@umich.edu

MS10

An Historical Approach to Sobolev Orthogonal Polynomials

We present a survey about analytic propeties of polynomials orthogonal with respect to a Sobolev inner product. The distribution of their zeros, asymptotic properties, spectral theory as well as some applications will be considered. Some open problems will be discussed.

Francisco Marcellan
University of Carlos III of Madrid
pacomarc@ing.uc3m.es

MS10

Linearly Related Sequences of Continuous and Discrete Derivatives of Orthogonal Polynomial Sequences. Connections with Sobolev Orthogonal

Polynomial Sequences

In this talk we consider structure relations of the form

$$\sum_{i=0}^{N} r_{i,n}\, D^m Q_{n-i+m}(x) = \sum_{i=0}^{M} s_{i,n} D^k P_{n-i+k}(x)\ ,$$

where $(P_n)_n$ and $(Q_n)_n$ are two sequences of orthogonal polynomials (OPs), continuous or discrete, and, for a nonnegative integer number s, D^s represents the continuous derivative or a discrete difference-derivative operator of order s (the possibilities for D include the usual discrete operators D_ω and D_q). In the above relation, the orders of the derivatives, k and m, are arbitrarily fixed nonnegative integer numbers, and $(r_{i,n})_n$ and $(s_{i,n})_n$ are sequences of numbers fulfilling some appropriate conditions. Such structure relation leads to the notion of $(M,N)-$coherence of order (m,k), being an extension of the concept of coherence of measures as introduced by A. Iserles, P. E. Koch, S. P. Nørsett, and J. M. Sanz-Serna [On polynomials orthogonal with respect to certain Sobolev inner products, J. Aprox. Theory, **65**(2) (1991) 151-175]. We will present a survey of some results obtained in the last years around the concept of $(M,N)-$coherence of order (m,k). In the continuous case, we will be able to present some examples showing how such a concept may be an useful tool in Approximation Theory to explore polynomials orthogonal with respect to a Sobolev inner product of the form

$$\langle f,g\rangle_{\lambda,s} := \int_{-\infty}^{+\infty} fg\,\mathrm{d}\mu + \lambda\int_{-\infty}^{+\infty} f^{(s)}g^{(s)}\,\mathrm{d}\nu\ ,$$

provided $(P_n)_n$ and $(Q_n)_n$ are orthogonal with respect to positive Borel measures μ and ν (resp.). Most results presented in this talk are the result of several collaborations along the last years, specially with F. Marcellán, R. Álvarez-Nodarse, M.N. de Jesus, N.C. Pinzón-Cortés, and R. Sevinik-Adıgüzel.

Jose Carlos S. Petronilho
Departamento de Matematica, Universidade de Coimbra
josep@mat.uc.pt

MS10

(M,N)-Coherent Pairs of Linear Functionals and Jacobi Matrices

We give a matrix interpretation of (M,N)-coherent pairs of linear functionals: An algebraic relation between the corresponding Jacobi matrices associated with such functionals is stated, and the classical case is analyzed; The relation between the Jacobi matrices associated with (M,N)-coherent pairs of order m and the Hessenberg matrix associated with the multiplication operator in terms of the basis of polynomials orthogonal with respect to the Sobolev inner product defined by such coherent pair is deduced.

Natalia C. Pinzón-Cortés
Universidad Nacional de Colombia
ncpinzonco@unal.edu.co

Francisco Marcellan
University of Carlos III of Madrid
pacomarc@ing.uc3m.es

MS10

$W^{(1,p)}$ - Convergence of Fourier-Sobolev Expansions Associated to (M,N) Coherent Pairs of Measures

Let $\{q_{n,\lambda}\}_{n\geq 0}$ be the sequence of polynomials orthonormal with
respect to the Sobolev inner product

$$\langle f,g\rangle_S := \int_{-1}^{1} f(x)g(x)d\mu_0(x) + \lambda\int_{-1}^{1} f'(x)g'(x)d\mu_1(x),$$

where λ is a non-negative constant and $(d\mu_0, d\mu_1)$ is an (M,N)-coherent pair of measures supported on the interval $[-1,1]$ with M and N fixed non-negative integer numbers. In this talk we discuss necessary and/or sufficient conditions for the convergence in the $W^{1,p}([-1,1],(d\mu_0,d\mu_1))$ norm of the Fourier expansion in terms of $\{q_{n,\lambda}\}_{n\geq 0}$, with $1<p<\infty$.

Alejandro J. Urieles
Departamento de Matematicas, Universidad del Atlantico
aurielesg@gmail.com

MS11

Fast Arbitrary-precision Evaluation of Special Functions in the Arb Library

We demonstrate Arb, a library for arbitrary-precision real and complex ball/interval arithmetic with support for many special functions, including erf, gamma, zeta, polylog, Bessel, hypergeometric, theta, elliptic and modular functions. Our implementations are competitive with existing non-interval high-precision software, and often significantly faster. One highlight is a new algorithm for elementary functions, giving up to an order of magnitude speedup at "medium" precision (approximately 200-2000 bits).

Fredrik Johansson
Institut de Mathématiques de Bordeaux
Université de Bordeaux 351, cours de la Libération - F 3340
fredrik.johansson@gmail.com

MS11

Pushing Forward the Dimension of fcc Lattices

The generating function G_d for the return probabilities in a d-dimensional face-centered cubic lattice satisfies an ODE whose order grows quadratically with d. Until recently only the ODEs for $d\leq 7$ were known; using a recursive method for computing the coefficients of G_d, proposed by Zenine, Hassani, Maillard, we are able to go up to $d=11$. These ODEs share many remarkable properties which we shall discuss in this talk. This is joint work with Jean-Marie Maillard.

Christoph Koutschan
RICAM Linz
Austrian Academy of Sciences (AW)
christoph.koutschan@ricam.oeaw.ac.at

MS11

Divisibility Properties of Sporadic Apéry-like Numbers

It was shown by Gessel that the Apéry numbers, introduced by Apéry in his proof of the irrationality of $\zeta(3)$, are periodic modulo 8. We investigate this, and other divisibility results, for Apéry-like numbers. For instance, we prove that the Almkvist–Zudilin numbers are periodic modulo 8 as well. The ingredients of our proof are a multivariate rational function whose diagonals are the Almkvist–Zudilin numbers and a theorem originating with Furstenberg which states that, modulo a fixed prime power, these values have algebraic generating function and, hence, can be generated by a finite state automaton. As demonstrated recently by Rowland and Yassawi, these automata can be computed mechanically. This talk includes joint work with Arian Daneshvar, Pujan Dave, Amita Malik and Zhefan Wang that was done as part of an Illinois Geometry Lab project.

Armin Straub
University of Illinois at Urbana-Champaign
astraub@illinois.edu

MS11

Positive Rational Functions and their Diagonals

An interesting phenomenon about some arithmetically significant sequences, like the sequence of Apéry numbers (`http://oeis.org/A005259`), is that they are the diagonal sequences of the Taylor expansions of "interesting' rational functions, and the positivity of the coefficients in the latter expansions are (conjecturally) determined by the positivity of the sequences. In my talk I will explain and illustrate the phenomenon and also relate it to other classical results in analysis, in particular, to some 60-year old theorems of Hörmander. This is joint work with Armin Straub.

Wadim Zudilin
University of Newcastle
wzudilin@gmail.com

MS12

Mock Modular Forms of Weight 5/2 and Partitions

We study the coefficients of a natural basis for the space of mock modular forms of weight 5/2 on the full modular group. The "shadow' of the first element of this infinite basis encodes the values of the partition function $p(n)$. We show that the coefficients of these forms are given by traces of singular invariants. These are values of modular functions at CM points or their real quadratic analogues: cycle integrals of such functions along geodesics on the modular curve. The real quadratic case relates to recent work of Duke, Imamoglu, and Toth on cycle integrals of the j-function, while the imaginary quadratic case recovers the algebraic formula of Bruinier and Ono for the partition function.

Nickolas Andersen
University of Illinois at Urbana-Champaign
nandrsn4@illinois.edu

MS12

Partition Identities and Mock Theta Functions

This is joint work with Stephen Hill, a Penn State undergraduate. In 1961, Basil Gordon proved a sweeping generalization of the Rogers-Ramanujan identities. His theorem may be broadly characterized as identifying the generating function for partitions having specified difference conditions on the parts with the quotient of two theta functions. We shall provide a new class of partitions (similar to those studied by Gordon) where the generating function is identified with the quotient of a Hecke-type theta series divided by the Dedekind eta function. The simplest case is related to one of the fifth order mock theta functions of Ramanujan. The partitions in question are similar in kind to those described in: G.E Andrews, Partitions with initial repetitions, Acta Math. Sinica, English Series, 25(2009), 1437-1442, and in G.E Andrews, Partitions with early conditions, In Advances in Combinatorics Waterloo Workshop in Computer Algebra, W80 May. 26-29, 2011.

George E. Andrews
Penn State University
Department of Mathematics
andrews@math.psu.edu

MS12

Mock Theta Functions, Partial Theta Functions, and Their Ghosts - Rhoades

There is an association between mock theta functions and partial theta functions; they each carry half of the Fourier coefficients of a non-holomorphic modular form. In this association some mysterious q-hypergeometric series may arise. We call these "ghost terms". This talk explains the association and gives hints of the curious structure in ghost terms which remains without a theory.

Robert Rhoades
Stanford University
rob.rhoades@gmail.com

MS12

Mock Theta Functions and Quantum Modular Forms

In this talk, I will describe several related recent results related to mock theta functions. These functions have very recently been understood in a modern framework thanks to the work of Zwegers and Bruinier-Funke. Here, we will revisit the original writings of Ramanujan and look at his original conception of these functions, which gives rise to a surprising picture connecting important objects such as generating functions in combinatorics and quantum modular forms.

Larry Rolen
University of Cologne
larryrolen@gmail.com

MS13

Painleve Equations - Nonlinear Special Functions

The Painlevé equations, discovered over a hundred years ago, are special amongst nonlinear ordinary differential equations in that they are "integrable" due to their representation as Riemann-Hilbert problems. The Painlevé equations can be thought nonlinear analogues of the classical special functions and have numerous remarkable properties. In this talk I shall give an introduction to the Painlevé equations and discuss some open problems.

Peter Clarkson
University of Kent
Kent UK
p.a.clarkson@kent.ac.uk

MS13

Numerics for Classical Applications of Riemann-Hilbert Problems

We overview several classical problems that can be reduced to RiemannHilbert problems, falling into three categories: integral representations, differential equations and inverse spectral problems. The integral representation of error functions and elliptic integrals can be rewritten in terms of scalar RiemannHilbert problems. The Stokes' phenomenon for Airy's equation and monodromy problems for Fuchsian differential equations lead to matrix RiemannHilbert problems. The inverse spectral problem for Jacobi and Schrdinger operators can be solved via matrix RiemannHilbert problems depending on a parameter, the latter of which encodes the solution to the Kortewegde Vries equation. In all three cases, applying numerics to the RiemannHilbert problem allows for efficient approximation, that is uniformly accurate in the complex plane. Joint work with Thomas Trogdon

Sheehan Olver
The University of Sydney
Sheehan.Olver@sydney.edu.au

MS13

Explorations of the Solution Space of the Fourth Painlev Equation

Solutions to the Painlevé equations (P_I-P_{VI}) that are free of poles over extensive regions of the complex plane are of considerable interest, dating back to the tronquée and tritronquée solutions of P_I (Boutroux 1913). This talk presents parameter choices and initial conditions of P_{IV} that exhibit these pole-free sectors, highlighting the existence of the families of solutions that are free from (or have only a finite number of) poles on the real axis.

Jonah A. Reeger
USAF
jonah.reeger@afit.edu

Bengt Fornberg
University of Colorado
Boulder
fornberg@colorado.edu

MS13

Uniformly Accurate Computation of Painlevé II Transcendents

The Riemann–Hilbert approach for the Painlevé transcendents has proved to be a powerful and rigorous tool for asymptotic analysis. The approach makes use of the Deift–Zhou method of nonlinear steepest descent. Recently, this approach has been adapted for numerical purposes and the resulting computations are seen to be uniformly accurate. In this talk, I will outline the method and describe the rigorous justification for the accuracy that is observed. This is joint work with Sheehan Olver.

Thomas Trogdon
Courant Institute of Mathematical Sciences
NYU
trogdon@cims.nyu.edu

MS14

The Statistical Behaviour of the Low-end Spectra of Several Coupled Matrices and the Meijer-G random Point Field

Universality results in the statistical behaviour of spectra of random matrices typically consider the fluctuations near a macroscopic point of the spectrum. In an ensemble of positive definite matrices, there are two essentially different distinguished points; the largest eigenvalues and the smallest. Since they are positive definite the study of the smallest eigenvalue is the study of the statistics near the origin of the spectral axis. The simplest instance is the Laguerre ensemble; in this case the fluctuations are described by a determinant random point field defined in terms of the celebrated Bessel kernel. This behaviour is also proven in the literature to be ?universal?. We consider a model of several coupled matrices of Laguerre type (with a specific interaction) and address the corresponding study of the origin of the spectrum. We find a natural generalization of the Bessel random point field to a multi-specie analog that involves special functions (Meijer-G). It is then natural to formulate a universality conjecture.

Marco Bertola
Concordia University, Canada
marco.bertola@concordia.ca

Thomas Bothner
CRM, Concordia University
Canada
bothner@crm.umontreal.ca

MS14

The Normal Matrix Model in the Supercritical Regime, and Asymptotics of the Associated Orthogonal Polynomials

The normal matrix model is a random matrix model with eigenvalues in the complex plane. The average characteristic polynomials are orthogonal with respect to weighted area measure on the full plane. However, the integrals that are involved in the orthogonality do not converge in many interesting cases, and then the orthogonality has to be redefined. We follow the approach of [Bleher and Kuijlaars, Adv. Math. (2012)] for the model with a cubic potential, where the orthogonality is redefined as orthogonality on contours in the complex plane. The orthogonal polynomials admit a Riemann-Hilbert characterization of size 3x3, which was analyzed in the subcritical case by means of the Deift-Zhou method of steepest descent. Here we analyze the supercritical case, and we find the limiting behavior of zeros. We also find a new critical behavior, and our results only hold up to this second criticality.

Arno Kuijlaars
Katholieke Universitaet Leuven
arno.kuijlaar@wis.kuleuven.be

Alexander Tovbis
University of Central Florida
Department of Mathematics
alexander.tovbis@ucf.edu

MS14

Discrete Toeplitz Determinants and Their Applications

We will discuss the asymptotics of the Toeplitz determinants with discrete measure. We first show how one can convert this problem to the asymptotics of continuous orthogonal polynomials by using a simple identity. Then we apply this method to the width of non-intersecting processes of several different types.

Zhipeng Liu
Courant Institute of Mathematical Sciences
NYU

zhipeng@cims.nyu.edu

MS14

The Condition Number of the Critically–Scaled Laguerre Unitary Ensemble

In recent computations with S. Olver, we demonstrated universal fluctuations in the iteration count, called the halting time, of classical numerical algorithms with random initial data. In particular, we noticed this phenomenon in the conjugate gradient algorithm for solving $Ax = b$ where $A > 0$. The random data is formed by letting $A = XX^*$ where X is an $N \times n$ matrix with iid entries distributed according to some random variable $\mathcal{D}$ and n is scaled in a critical manner. In this talk, I will present a limit theorem for the condition number of the Laguerre Unitary Ensemble ($\mathcal{D}$ is a complex Gaussian) with this critical scaling and relate it to the performance of the conjugate gradient algorithm. This is joint work with P. Deift and G. Menon.

Thomas Trogdon
Courant Institute of Mathematical Sciences
NYU
trogdon@cims.nyu.edu

MS15

Differential Equations for Discrete Sobolev Orthogonal Polynomials

The aim of this talk is to study differential properties of orthogonal polynomials with respect to a discrete Sobolev bilinear form with mass points. We will focus on the Laguerre case, where the mass point is located at zero, and the Jacobi case, where the mass point can be located either at -1 or 1, or at the two points at the same time. We construct the orthogonal polynomials using certain Casorati determinants. Using this construction, we prove that they are eigenfunctions of a differential operator. Moreover, the order of this differential operator is explicitly computed in terms of the matrices which defines the discrete Sobolev bilinear form.

Manuel Dominguez de la Iglesia
Instituto de Matemáticas, Universidad Nacional
Autonoma Mexi
mdi29@us.es

Antonio Duran
Universidad de Sevilla
duran@us.es

MS15

Laguerre Polynomials and Sobolev Orthogonality

When $\alpha > -1$, the classical Laguerre polynomials $\{L_m^\alpha\}_{m=0}^\infty$ form a complete orthogonal set in the Hilbert space $L^2((0,\infty); x^\alpha e^{-x})$. More generally, from classical properties of these polynomials, it is the case that, for each non-negative integer n, these Laguerre polynomials form a complete orthogonal set in a certain Sobolev space W_n with inner product

$$(f,g)_n = \sum_{j=0}^{n} \int_0^\infty S_n^{(j)} f^{(j)}(x)\overline{g}^{(j)}(x) x^{\alpha+j} e^{-x} dx \quad (f,g \in W_n),$$

where $\{S_n^{(j)}\}$ are the Stirling numbers of the second kind. When $\alpha \le -1$ but $-\alpha$ is not a positive integer, the Laguerre polynomials are orthogonal with respect to a signed measure.
A natural question to ask is what happens in the remaining case when α is a real number: namely when $-\alpha := k$ is a positive integer? In this case, it is not difficult to see that the Laguerre polynomials $\{L_m^{-k} \mid m \ge k\}$ form a complete orthogonal set in the Hilbert space $L^2((0,\infty); x^{-k}e^{-x})$. A not-so-obvious question to ask is whether there is a 'natural' inner product in which the entire sequence of Laguerre polynomials $\{L_m^{-k}\}_{m=0}^\infty$ are orthogonal? The answer, surprisingly, is yes. We will show that the Laguerre polynomials $\{L_m^{-k}\}_{m=0}^\infty$ form a complete orthogonal set in a Sobolev space S_k with an inner product of the form

$$(f,g)_{S_k} = \sum_{j=0}^{k} \int_0^\infty f^{(j)}(x)\overline{g}^{(j)}(x) d\mu_j.$$

We will also discuss a self-adjoint operator T_k, generated by the classical second-order Laguerre differential expression, in the space S_k having the Laguerre polynomials $\{L_m^{-k}\}_{m=0}^\infty$ as a complete set of eigenfunctions.

Lance L. Littlejohn
Department of Mathematics, Baylor University
lance_littlejohn@baylor.edu

MS15

A Study of the Exceptional X_m-Jacobi Expression for Extreme Parameter Choices

In 2009, Gómez-Ullate, Kamran, and Milson extended the well-established Bochner Classification (1929) by showing that the only polynomial sequences $\{p_n\}_{n=1}^\infty$ which simultaneously form a complete set of eigenstates for a second-order differential operator and are orthogonal with respect to a positive Borel measure having positive moments are the exceptional X_1-Laguerre and X_1-Jacobi polynomials. We will focus on the exceptional X_1-Jacobi polynomials. The second-order differential expression has rational coefficients and as a result, there is no solution of degree zero. The X_1-Jacobi polynomials $\left\{\hat{P}_n^{(\alpha,\beta)}\right\}_{n=1}^\infty$ form a complete orthogonal set in the weighted Hilbert space $L^2((-1,1); \hat{w}_{\alpha,\beta})$, where $\hat{w}_{\alpha,\beta}$ is a positive rational weight function. Restrictions are placed on parameters α and β; in particular, $\alpha, \beta > 0$. We are particularly interested in the 'extreme' parameter choice when $\alpha = 0$. In this situation, $\left\{\hat{P}_n^{(\alpha,\beta)}\right\}_{n=2}^\infty$ are in the associated L^2 space, but we may actually study the *full* sequence of solutions $\left\{\hat{P}_n^{(0,\beta)}\right\}_{n=0}^\infty$ in a certain Sobolev space S.

Jessica Stewart
Department of Mathematics and Computer Science,
Goucher Coll
jessica.stewart@goucher.edu

MS15

Orthogonal Polynomials for Learning

Within the burgeoning field of machine learning kernel based learning has produced not only state of the art algorithms but stands out for the depth of its mathematical sophistication and elegance. The promise of kernel learning for applications lies in the abundance of reproducing kernels. In practice however there is a dearth of computable kernels. Orthogonal polynomials provide a compelling way for producing useful learning kernels. Associated spectral theory gives key insight into the nature of the regularization properties for learning machines with Sobolev orthogonal polynomial kernels. Well discuss general theory as well as several specific examples used in commercial applications.

Richard Wellman
Department of Mathematics, Westminster College
rwellman@westminstercollege.edu

MS16
Transformations of Hypergeometric Functions

It is well known that the generalized hypergeometric functions ${}_pF_q$ satisfy many useful functional identities including the Kummer, Pfaff or Euler transform. Less is known in higher dimensions. Some results of a similar kind are known for example for Appell's functions and other functions. We are going to provide a generalization of Euler, Kummer and "Analytic continuation' transform for n dimensional hypergeometric function for arbitrary n.

Petr Blaschke
Mathematical Institut in Opava, Silesian university in Opava
Czech Republic
Petr.Blaschke@math.slu.cz

MS16
Computing the Probability of Collision for Short-term Space Encounters: a Symbolic-Numeric Approach

The increasing number of space debris in Low Earth Orbits constitute a serious hazard for operational satellites. In order to provide adequate collision avoidance strategies, it is important to determine the collision probability between two orbiting objects. Three-dimensional Gaussian probability densities represent the position uncertainties of the objects. With some simplifying assumptions, the problem of computing the collision probability, for short-term encounters between space-borne objects, is, in practice, reduced to a two-dimensional integral of a Gaussian function over a bounded region in a plane normal to the relative velocity vector (encounter frame). The method presented here is based on an analytical expression for the integral, derived by use of Laplace transform and properties of D-finite functions. The formula has the form of a product between an exponential term and a convergent power series with positive P-recursive coefficients. Analytic bounds on the truncation error are also derived. This allows for an efficient and reliable numerical evaluation of the risk. This talk is based on R. Serra, D. Arzelier, M. Joldes, J.-B. Lasserre, A. Rondepierre and B. Salvy, *A New Method to Compute the Probability of Collision for Short-term Space Encounters, AIAA/AAS astrodynamics specialist conference, American Institute of Aeronautics and Astronautics, pages 1-7, 2014.*

Mioara Joldes
LAAS-CNRS
7 Avenue du Colonel Roche, 31077 Toulouse, Cedex 4
France
joldes@laas.fr

MS16
The Positive Part of Multivariate Infinite Series

The positive part of an infinite series is defined as the formal power series which is obtained from it by discarding all the terms involving negative exponents. In the univariate case, it is easy to see that the positive part of a rational series is again a rational series. While this is no longer true in the multivariate case, it is still true there that the positive part of a D-finite series is D-finite. This can be used in combinatorics to show that certain generating functions are D-finite. In the talk, we will show how to compute the positive part of a D-finite multivariate Laurent series using creative telescoping. This is joint work with Alin Bostan, Frederic Chyzak, Lucien Pech and Mark van Hoeij.

Manuel Kauers
Johannes Kepler University
mkauers@risc.jku.at

MS16
About Some Identities by Bayad and Beck Involving Bernoulli-Barnes Numbers, Barnes Zeta Functions and Fourier Dedekind Sums

A. Bayad and M. Beck have recently derived new identities involving the Bernoulli-Barnes numbers, the Barnes zeta functions and the Fourier Dedekind sums. Using symbolic calculus methods, we will provide some extensions and interpretations of these identities. We will also show a link between the Bernoulli-Barnes numbers and the p-Bernoulli numbers as recently introduced by Rahmani, which are related to the generating function of some special zeta values.

Christophe Vignat
Dpt. of Physics, Universite d'Orsay
Dpt. of Mathematics, Tulane University
cvignat@tulane.edu

MS17
A Generalized Lebesgue Identity in Ramanujan's Lost Notebook

The Lebesgue identity is one of the important identities in the theory of partitions and q-hypergeometric series (q-series). The identity has a free parameter. By dilations of the base q, and choices of the free parameter, several fundamental partition identities follow. In Ramanujan's Lost Notebook there is an identity in two free parameters. By viewing this as an extension of the Lebesgue identity, we obtain new weighted partition theorems. Our approach yields new combinatorial information about certain Hecke-Rogers type series, and also leads to new companions to Euler's celebrated Pentagonal Numbers Theorem.

Krishnaswami Alladi
University of Florida
alladik@ufl.edu

MS17
Exotic Bailey-Slater Spt-Functions and Hecke-Rogers Double Series

We study SPT-crank type functions that arise from Bailey pairs and that have interesting arithmetic properties. We find representations of these functions in terms of infinite products or two-variable Hecke-Rogers double series. The method uses Bailey's Lemma and conjugate Bailey pairs.

Frank Garvan, Chris Jennings-Shaffer
University of Florida

fgarvan@ufl.edu, cjenningsshaffer@ufl.edu

MS17

Rogers-Ramanujan Type Identities

The Rogers–Ramanujan identities and a number of closely related identities first appeared just over 120 years ago. Early contributors to the theory of Rogers–Ramanujan-type identities included Rogers, Ramanujan, F. H. Jackson, W. N. Bailey, F. J. Dyson, and L. J. Slater. Since their introduction, identities of Rogers–Ramanujan type have found applications in a variety of areas including the theory of partitions, statistical mechanics, and vertex operator algebras. I will review some of the history of Rogers–Ramanujan type identities, including some of the more recent contributions which relied rather heavily on computer algebra.

Andrew V. Sills
Georgia Southern University
asills@georgiasouthern.edu

MS17

Partitions Associated with the Ramanujan/Watson Mock Theta Functions omega(q) and nu(q)

Recently, George Andrews, Atul Dixit, and I have discovered very interesting partition theorems that are related to the mock theta functions $\omega(q)$ and $\nu(q)$. For instance, the generating function for partitions where each part is less than twice the smallest part equals $q\ \omega(q)$. In this talk, I will present those discoveries and some related arithmetic properties.

Ae Ja Yee
Department of Mathematics
Pennsylvania State University
yee@math.psu.edu

George E. Andrews
Penn State University
gea1@psu.edu

Atul Dixit
Tulane University
adixit@tulane.edu

MS18

Zeros of Large Degree Vorob'ev-Yablonski Polynomials Via a Hankel Determinant Identity

It is well known that all rational solutions of the second Painlevé equation and its associated hierarchy can be constructed with the help of Vorob'ev-Yablonski polynomials and generalizations thereof. The zero distribution of the aforementioned polynomials has been analyzed numerically by Clarkson and Mansfield and the authors observed a highly regular and symmetric pattern: for the Vorob'ev polynomials itself the roots form approximately equilateral triangles whereas they take the shape of higher order polygons for the generalizations. Very recently Buckingham and Miller completely analyzed the zero distribution of large degree Vorob'ev-Yablonski polynomials using a Riemann-Hilbert/nonlinear steepest descent approach to the Jimbo-Miwa Lax representation of PII equation. In our work, joint with Marco Bertola, we rephrase the same problem in the context of orthogonal polynomials on a contour in the complex plane. The polynomials are then analyzed asymptotically and the zeros localized through the vanishing of a theta divisor on an appropriate hyperelliptic curve. Our approach starts from a new Hankel determinant representation for the square of the Vorob'ev-Yablonski polynomial. This identity is derived using the representation of Vorob'ev polynomials as Schur functions.

Thomas Bothner
CRM, Concordia University
Canada
bothner@crm.umontreal.ca

MS18

Asymptotics of Large-Degree Rational Painleve-IV Functions

The Painlevé-IV equation admits a family of rational solutions, indexed by two integers, that can be expressed in terms of generalized Hermite polynomials. In the large-degree limit the zeros of these polynomials form remarkable patterns in the complex plane resembling rectangles with arbitrary aspect ratios depending on how the indexing integers grow. Using Riemann-Hilbert analysis we analytically determine the boundary of the zero/pole region for these rational functions.

Robert Buckingham
University of Cincinnati
buckinrt@ucmail.uc.edu

MS18

Painleve I in the Cubic Random Matrix Model

In his talk we analyze the double scaling regime in the asymptotics of the partition function in the cubic random matrix model. In particular, we show the appearance of solutions of the Painlev I differential equation in this setting. We also provide a uniform asymptotic expansion of the Painlev I function $\Psi(\zeta, \lambda, \alpha)$ introduced by Kapaev to describe the Riemann-Hilbert problem corresponding to Painlev I.

Alfredo Deaño
Departamento de Matemáticas
Universidad Carlos III de Madrid
alfredo.deanho@uc3m.es

Pavel Bleher
Department of Mathematics, IUPUI
bleher@math.iupui.edu

MS18

On Large Degree Rational Solutions of Painleve II

The inhomogeneous Painlevé-II equation with parameter m has a (unique) rational solution exactly when m is an integer. Motivated in part by the universal appearance of these functions in a certain double-scaling limit involving the semiclassical sine-Gordon equation, we study their asymptotic behavior in the limit of large m by means of steepest descent methods applied to a Riemann-Hilbert problem encoding them that naturally arises in the sine-Gordon theory. Joint work with Robert Buckingham (Cincinnati).

Peter D. Miller
University of Michigan, Ann Arbor
millerpd@umich.edu

Robert Buckingham
University of Cincinnati
buckinrt@ucmail.uc.edu

MS19

The Riemann-Hilbert Approach to Polynomials Orthogonal with Respect to Complex Weight Functions

In this contribution we illustrate how the Riemann-Hilbert approach and the Deift-Zhou steepest descent method can be used to analyze the asymptotic behavior and zero distribution of polynomials that are orthogonal with respect to complex weight functions. Some examples studied recently include a Fourier-type weight function, $w(x) = e^{i\omega x}$ on $[-1, 1]$, an exponential weight with a cubic potential on suitable contours in the complex plane, and Bessel functions on the semiaxis $[0, \infty)$.

Alfredo Deaño
Departamento de Matemáticas
Universidad Carlos III de Madrid
alfredo.deanho@uc3m.es

MS19

The Riemann-Hilbert Approach to Critical Phenomena in the Two Matrix Model

In this talk, an overview will be presented on various results for the asymptotic analysis of the two matrix model, based on the Riemann-Hilbert approach for the biorthogonal polynomials that integrate this model. Particular emphasis will be on recent results on the classification of the critical phenomena that can occur in the even quadratic/quartic case and the even quartic/quartic case.

Maurice Duits
Mathematics Department
Stockholm University
duits@kth.se

MS19

Asymptotics of Orthogonal Polynomials with Complex Varying Weight: Critical Point Behaviour and the Painleve Equations

We study the asymptotics of recurrence coefficients for complex monic orthogonal polynomials $\pi_n(z)$ with the quartic exponential weight

$$\exp\left[-n\left(\frac{z^2}{2}+\frac{tz^4}{4}\right)\right],$$

where $t \in \mathbb{C}$ and $N \to \infty$. We consider neighborhoods of the critical points $t_0 = -\frac{1}{12}$, $t_1 = \frac{1}{15}$ and $t_2 = \frac{1}{4}$, where the subleading terms can be expressed via Painlevé transcendents. These subleading terms can become dominant near the poles of the corresponding Painlevé transcendents. We use the nonlinear steepest descent analysis for Riemann-Hilbert Problems to describe the recurrence coefficients in full neighborhoods of t_j, $j = 0, 1, 2$, including the location of the poles. We also provide the global (in the t-plane) "phase diagrams, where the recurrence coefficients exhibit different asymptotic behaviors (Stokes' phenomenon).

Alex Tobvis
Department of Mathematics
University of Central Florida
alexander.tovbis@ucf.edu

Marco Bertola
Concordia University, Canada
marco.bertola@concordia.ca

MS19

Asymptotics in the Complex Plane for Multiple Laguerre Polynomials

There are two kinds of multiple Laguerre polynomials. The multiple Laguerre polynomials of the first kind have orthogonality conditions with respect to weights $x^{\alpha_j} e^{-x}$ on $[0, \infty)$ for $1 \le j \le r$, where $\alpha_j > -1$ are such that $\alpha_i - \alpha_j \notin \mathbb{Z}$. Multiple Laguerre polynomials of the second kind have orthogonality properties with respect to $x^{\alpha} e^{-c_j x}$ on $[0, \infty)$ for $1 \le j \le r$, where $c_j > 0$ are such that $c_i \neq c_j$ whenever $i \neq j$. Wielonsky and Lysov (Constr. Approx. 28 (2008), 61–111) have obtained strong asymptotics for the multiple Laguerre polynomials of the second kind using the Riemann-Hilbert problem and the Deift-Zhou steepest descent method. We will obtain strong asymptotics of the multiple Laguerre polynomials of the first kind using the same techniques. This is joint work with Thorsten Neuschel.

Walter van Assche
Katholieke Universitaet Leuven
walter.vanassche@wis.kuleuven.be

MS20

A Particular Case of a Higher Order Sobolev-Type Inner Product of Orthogonal Polynomials in Several Variables

We consider sequences of polynomials of several variables, orthogonal with respect to the Sobolev-type inner product given in, which is obtained by adding to an inner standard product, the gradient operator of order j evaluated in a particular point. We present an expression for the perturbed orthogonal polynomials in terms of the original polynomials and a particular example on the unit ball in which we analize the asymptotic behaviour of the Kernel associated to the Sobolev-type polynomials.

Herbert Dueñas
Departamento Matematicas, Universidad Nacional Colombia
haduenasr@unal.edu.co

Wilmer M. Gómez
Universidad Pedagógica y Tecnológica de Colombia
wilmer.gomez@uptc.edu.co

MS20

Sobolev Orthogonal Polynomials on the Unit Ball via Outward Normal Derivatives

The purpose of this work is to analyze a family of mutually orthogonal polynomials on the unit ball with respect to an inner product which involves the outward normal derivatives on the sphere. Using the representation of these polynomials in terms of spherical harmonics, algebraic and analytic properties will be deduced. First, we will get connection formulas relating classical multivariate orthogonal polynomials on the ball with our family of Sobolev orthogonal polynomials. Then explicit expressions for the norms will be obtained, among other properties.

Lidia Fernandez

Departamento Matematica Aplicada, Universidad Granada
lidiafr@ugr.es

Antonia Delgado
Universidad de Granada
amdelgado@ugr.es

Teresa E. Pérez
Universidad de Granada
Departamento de Matemática Aplicada
tperez@ugr.es

Miguel Pinar
University of Granada
mpinar@ugr.es

MS20

On Asymptotic Properties of Sobolev Orthogonal Polynomials on the Unit Circle

In this contribution, we study the sequence of polynomials orthogonal with respect to the Sobolev inner product

$$\langle f,g\rangle_S := \int_{\mathbb{T}} f(z)\overline{g(z)}d\mu(z) + \lambda f^{(j)}(\alpha)\overline{g^{(j)}(\alpha)},$$

where μ is a nontrivial probability measure supported on the unit circle, α is a complex number, λ is a positive real number, and j is a positive integer. In particular, we analyze some asymptotic properties of such polynomials and the behavior of their zeros when n and λ tend to infinity, respectively. We also provide some numerical examples to illustrate the behavior of these zeros with respect to α.

Luis E. Garza
Departamento Matematicas,Universidad Nacional Colombia
garzaleg@gmail.com

Francisco Marcellan
University of Carlos III of Madrid
pacomarc@ing.uc3m.es

Kenier Castillo
University Carlos III de Madrid
kcastill@math.uc3m.es

MS20

Orthogonal Polynomials with Respect to a Sobolev Inner Product on the Unit Circle

We look at some information concerning the orthogonal polynomials with respect to the Sobolev inner product

$$\langle f,g\rangle_S = (1-t)\,\langle f,g\rangle_b + t\,f(1)\,\overline{g(1)} + \kappa\langle f',g'\rangle_{b+1},$$

where $0 \le t < 1$, $\kappa \ge 0$, $\mathcal{R}e(b) > -1/2$ and

$$\langle f,g\rangle_b = \frac{\tau(b)}{2\pi}\int_0^{2\pi} f(e^{i\theta})\,\overline{g(e^{i\theta})}\,(e^{\pi-\theta})^{\mathcal{I}m(b)}(\sin^2(\theta/2))^{\mathcal{R}e(b)}d\theta.$$

Here, $\tau(b) = \frac{2^{b+\bar{b}}|\Gamma(b+1)|^2}{\Gamma(b+\bar{b}+1)}$ is such that $\langle 1,1\rangle_b = 1$ and hence also $\langle 1,1\rangle_S = 1$. For example, the monic orthogonal polynomials S_n with respect to the inner product $\langle f,g\rangle_S$ satisfy

$$S_n(z) + a_n S_{n-1}(z) = \Phi_n(z), \quad n \ge 1,$$

where $\Phi_n(z)$ are the monic orthogonal polynomials with respect to the inner product $\langle f,g\rangle_{b,t} = (1-t)\,\langle f,g\rangle_b + t\,f(1)\,\overline{g(1)}$.

Alagacone S. Ranga
Departamento de Matematica Aplicada
Computacao,IBILCE-UNESP
ranga@ibilce.unesp.br

MS21

A Human Proof of Gessel's Lattice Path Conjecture

Gessel walks are lattice paths confined to the quarter plane that start at the origin and consist of unit steps going either West, East, South-West or North-East. In 2001, Ira Gessel conjectured a nice closed-form expression for the number of Gessel walks ending at the origin. In 2008, Kauers, Koutschan and Zeilberger gave a computer-aided proof of this conjecture. The same year, Bostan and Kauers showed, again using computer algebra tools, that the complete generating function of Gessel walks is algebraic. In this talk, I will report on the first human proofs of these two results, obtained in collaboration with Irina Kurkova and Kilian Raschel. The proofs are derived from a new expression for the generating function of Gessel walks in terms of Weierstrass zeta-functions.

Alin Bostan
INRIA Saclay Ile-de-France
alin.bostan@inria.fr

MS21

Numerical Evaluation of Contour Integrals for Computation of Stirling Numbers

Flajolet and Prodinger (SIAM J. Discrete Math, 12:155–159, 1999) proposed an interpolation of Stirling partition numbers by the integral definition

$$\left\{ \begin{array}{c} x \\ y \end{array} \right\} = \frac{(x-1)!}{(y-1)!}\frac{1}{2i\pi}\int_H \exp(z)(\exp(z)-1)^{y-1}\frac{dz}{z^x}.$$

Because the Γ functions are singular at negative integers this is not entirely satisfactory and some authors have proposed instead a related integral. This talk explores these issues and discusses an explicit evaluation by numerical means, using a parametrization of the Hankel contour that is related to the Tree T function (a cognate of Lambert W more suited to combinatorial applications, as is well known).

Rob Corless
University of Western Ontario in London
Rob.Corless@uwo.ca

David Jeffrey
Department of Applied Mathematics
University of Western Ontario
djeffrey@uwo.ca

Yang Wang
University of Western Ontario
ywan342@gmail.com

MS21

Utility Maximization and Symbolic Computation

We give a brief introduction to utility maximization, which

is a central topic of mathematical finance. Recently, there has been a lot of work on extending the classical framework by modelling the bid-ask spread. For small spread, asymptotic expansions of the economically relevant quantities can be obtained. The special functions in our talk are the value functions of the optimization problems. They are characterized by Riccati ODEs with free boundary, and have explicit expressions in some cases. Computer algebra, in particular symbolic manipulation of polynomials, is very useful to obtain asymptotic expansions of optimal strategies, optimal utility, and trading volume.

Stefan Gerhold
Financial and Actuarial Mathematics
Vienna University of Technology
sgerhold@fam.tuwien.ac.at

MS21

On $_2F_1$-type Solutions of Second Order Linear Differential Equations

Differential equations with $_2F_1$-type solutions are very common in Mathematics and they also occur in several areas such as Physics and Combinatorics. Given a second order linear differential operator $L \in C(x)[\partial]$, we want to find a $_2F_1$-type solution of the form

$$\exp(\int r\, dx) \cdot {}_2F_1(a, b; c; f)$$

where $r, f \in \overline{Q(x)}$, and $a, b, c \in Q$. This form is both more and less general than in prior work. In prior work, solutions involving a sum of two $_2F_1$'s were also considered, however, f was restricted to rational functions, while our method allows algebraic functions.

Erdal Imamoglu
Department of Mathematics
Florida State University
eimamogl@math.fsu.edu

Mark van Hoeij
Florida State University
Tallahassee, FL
hoeij@math.fsu.edu

MS22

Ramanujan, Voronoi Summation Formula, Circle and Divisor Problems and Some Modular Transformations

On page 336 in his Lost Notebook, Srinivasa Ramanujan proposed an identity that may have been devised to attack a divisor problem. Unfortunately, the identity is vitiated by a divergent series appearing in it. We present here a corrected version of Ramanujan's identity. This study has a natural connection with the Voronoi summation formula. We also obtain a one-variable generalization of two double Bessel series identities of Ramanujan on page 335 of the Lost Notebook which are intimately connected with the circle and divisor problems. Finally, we also obtain a new modular-type transformation involving infinite series of Lommel functions. Such a transformation is extremely rare, and is the only known example of its kind.

Atul Dixit
Tulane University
adixit@tulane.edu

Bruce Berndt, Arindam Roy, Alexandru Zaharescu
University of Illinois at Urbana-Champaign
berndt@math.uiuc.edu, roy22@illinois.edu, zaharesc@illinois.edu

MS22

On Theta Quotients Generating Graded Algebras of Modular Forms

We construct theta quotients that generate graded algebras of modular forms on principal congruence subgroups of prime level. A variety of consequences ensue, including coupled systems of differential equations for sums of twisted Eisenstein series analogous to those on the full modular group.

Tim Huber
Department of Mathematics
University of Texas-Pan American
hubertj@utpa.edu

MS22

Cubic Modular Equations in Two Variables

By adding certain equianharmonic elliptic sigma functions to the coefficients of the Borwein cubic theta functions, an interesting set of six two-variable theta functions may be derived. These theta functions invert the $F_1\left(\frac{1}{3}; \frac{1}{3}; \frac{1}{3}; 1|x, y\right)$ case of Appell's hypergeometric function and satisfy several identities akin to those satisfied by the Borwein cubic theta functions. Previous work on these functions is extended and put into the context of Ramanujan's modular equations, resulting in a simpler derivations as well as several new modular equations for Picard modular forms.

Dan Schultz
Pennsylvania State University
schult25@illinois.edu

MS22

Special Values of Trigonometric Dirichlet Series

In his first letter to Hardy and in several entries of his notebooks, Ramanujan recorded many evaluations of trigonometric Dirichlet series such as

$$\sum_{n=1}^{\infty} \frac{\coth(\pi n)}{n^{2r-1}} = \frac{1}{2}(2\pi)^{2r-1} \sum_{m=0}^{r} (-1)^{m+1} \frac{B_{2m}}{(2m)!} \frac{B_{2(r-m)}}{(2(r-m))!},$$

which, in fact, goes back to Cauchy and Lerch. A recent example is the secant Dirichlet series $\psi_s(\tau) = \sum_{n=1}^{\infty} \frac{\sec(\pi n \tau)}{n^s}$, for which Lalín, Rodrigue and Rogers conjecture, and partially prove, that its values $\psi_{2m}(\sqrt{r})$, with $r > 0$ rational, are rational multiples of π^{2m}. We give an overview of special values of such Dirichlet series and their connection with the theory of modular forms. This talk includes joint work with Bruce C. Berndt.

Armin Straub
University of Illinois at Urbana-Champaign
astraub@illinois.edu

MS23

Painleve Equations and Orthogonal Polynomials

In this talk I shall discuss semi-classical orthogonal polynomials arising from perturbations of classical weights. It is shown that the coefficients of the three-term recurrence relation satisfied by the polynomials can be expressed in terms of Wronskians which involve special functions. These Wronskians are related to special function solutions of the Painlevé equations. Using this relationship recurrence relation coefficients can be explicitly written in terms of exact solutions of Painlevé equations.

Peter Clarkson
University of Kent
Kent UK
p.a.clarkson@kent.ac.uk

MS23

Determinantal Representations of Exceptional Orthogonal Polynomials

Exceptional orthogonal polynomials represent an escape to the Bochner Classification Theorem by not allowing polynomials of certain degrees as eigenfunctions to the corresponding differential expressions. Much of their fascinating rich structure had been exposed over the past 6 (or so) years, and still more interesting research is being conducted. We focus on the exceptional X_1−Laguerre polynomials, which do not contain the constant polynomial. In particular, we find determinantal representations in terms of moments where the first row of the determinant is adjusted to implement the 'exceptional condition'. A slight alteration of the moments greatly simplifies the representation.

Constanze Liaw
Baylor University
USA
constanze_liaw@baylor.edu

MS23

On the Alternative Discrete Painleve I

During this talk I will discuss special solutions of the alternative discrete Painlevé-I equation. At the centre will be the uniqueness of the positive solution, which somehow links to the recurrence coefficients of exponential cubic weights.

Ana Loureiro
University of Kent
a.loureiro@kent.ac.uk

Peter Clarkson
University of Kent
Kent UK
p.a.clarkson@kent.ac.uk

Walter van Assche
Katholieke Universitaet Leuven
walter.vanassche@wis.kuleuven.be

MS24

Orthogonal Polynomials for a Class of Measures with Discrete Rotational Symmetries in the Complex Plane

Normal matrix models for external potentials of the form $|z|^{2n} + tz^d + \overline{tz^d}$ are considered with integers $0 \leq d \leq 2n$. A symmetry reduction procedure is used to find the equilibrium measure for all values of n, d and t. For fixed n and d, there is a critical value $|t| = t_{cr}$ such that the support is simply connected for $|t| < t_{cr}$ and has d connected components for $|t| > t_{cr}$. Moreover, the strong asymptotics of orthogonal polynomials with respect to measures of the form $e^{-|z|^{2d}+tz^d+\overline{tz}^d}dA(z)$ are obtained, where t is complex and dA is the area measure on the plane. Since the orthogonality can be written in terms of contour integrals, the asymptotics are found via a Riemann–Hilbert problem. In particular, the Cauchy transform of the normalized counting measure of the zeroes converges to that of the equilibrium measure in the exterior of the support (based on joint works with T. Grava and D. Merzi).

Ferenc Balogh
SISSA Trieste
ferenc.balogh@concordia.ca

MS24

Asymptotics for the Partition Function in Two-cut Random Matrix Models

I will present a new method to obtain asymptotics for the partition function in two-cut random matrix models, based on Riemann-Hilbert problems. The method enables us to re-derive potential-dependent terms which were known in the physics literature, but also to evaluate rigorously potential-independent terms. The talk will be based on joint work with T. Grava and K. McLaughlin.

Tom Claeys
Department of Mathematics
tom.claeys@uclouvain.be

Tom Claeys
Université catholique de Louvain
tom.claeys@uclouvain.be

MS24

Recent Developments in the Large-N Analysis of Correlation Functions in the Quantum Separation of Variables Method

The scalar products and certain correlation functions of models solvable by the quantum separation of variables can be expressed in terms of N-fold multiple integrals which can be thought of as the partition function of a one dimensional gas of particles evolving on a curve C, trapped in an external potential V and interacting through repulsive two-body interactions of the type $\ln\Big[\sinh[\pi\omega_1(\lambda-\mu)]\cdot \sinh[\pi\omega_2(\lambda-\mu)]\Big]$. The choice of the curve $\mathcal{C}$ and of the confining potential V determines a given model. The analysis of the large-N asymptotic behaviour of these integrals is of interest to the description of the continuum limit of the integrable model. In this talk, I shall report on recent developments in the large-N analysis of such integrals and discuss, on some specific examples, the form taken by the asymptotics. Part of the results that I will present issue from a joint work with G. Borot and A. Guionnet.

Karol K. Kozlowski
Institut de Mathématiques de Bourgogne
karol.kozlowski@u-bourgogne.fr

MS25

Deformed Semi-classical Discrete Orthogonal Polynomials

In this talk a study families of discrete orthogonal polynomials on the real line whose Stieltjes functions satisfy a linear type difference equation with polynomial coefficients is presented. The discrete dynamical systems, obtained as a result of deformations of the recurrence relation coefficients of the orthogonal polynomials related to the above referred Stieltjes functions is derived.

Amílcar Branquinho
UNIVERSIDADE DE COIMBRA
ajplb@mat.uc.pt

MS25

Integral Transforms of d-orthogonal Polynomial Sequences

This talk is mainly focused on index type integral transforms that map certain d-orthogonal polynomial sequences into another d-orthogonal polynomial sequences. It turns out that in several cases the weights are semiclassical.

Ana Loureiro
University of Kent
a.loureiro@kent.ac.uk

MS25

The Correspondence between the Askey Table of Orthogonal Polynomial Systems and the Sakai Scheme of Discrete Painlevé Equations

The Askey Table is a classification of the hypergeometric and basic hypergeometric orthogonal polynomial systems in a single variable, whose members possess a number of characteristic and defining properties. On the other hand the Sakai Scheme is a classification of the non-linear, integrable Painlevé equations and their difference, q-difference and elliptic analogs, based upon algebraic-geometric ideas. It is possible to bring these two into correspondence whereby the Askey Table represents a base, trivial level and the Sakai Scheme is its first deformation, the first of the multi-variable extensions. In the process of doing so we solve a number of related problems: an algorithmic construction of Lax pairs for the discrete Painlevé equations, the construction of an explicit sequence of classical solutions to the discrete Painlevé equations with useful applications to probabilistic models such as found in random matrix theory or random tiling models. In addition the correspondence provides insight into a geometrical reformulation of the Askey Table itself akin to the root system symmetries of the hypergeometric functions.

Nicholas Witte
University of Melbourne
nsw@ms.unimelb.edu.au

MS25

"Abstract" Classical Polynomials: Open Problems

We discuss open problems arising in classification of "abstract" classical orthogonal polynomials. These polynomials satisfy some general "umbral" eigenvalue problems with unknown symbols of operators.

Alexei Zhedanov
Donetsk Institute for Physics and Engineering, Ukraine
zhedanov@yahoo.com

MS26

Interlacing and Bounds for Zeros of Quasi-Orthogonal Laguerre Polynomials

We discuss interlacing properties of zeros of polynomials of consecutive degree in sequences of Laguerre polynomials $\{L_n^{(\alpha)}\}_{n=0}^{\infty}$ characterized by $-2 < \alpha < -1$. Stieltjes interlacing between the zeros of q_m and p_{n+1}, $m \leq n$, where $\{q_n\}_{n=0}^{\infty}$ is a sequence of quasi-othogonal Laguerre polynomials and $\{p_n\}_{n=0}^{\infty}$ is an orthogonal Laguerre sequence, is also investigated. Upper and lower bounds for the negative zero of $L_n^{(\alpha)}$, $-2 < \alpha < -1$, are derived.

Kathy A. Driver
University of Cape Town
South Africa
kathy.driver@uct.ac.za

Martin E. Muldoon
Department of Mathematics & Statistics
York University
muldoon@yorku.ca

MS26

Weighted Norm Inequalities for Some Special Functions

A method of obtaining new weighted norm inequalities for generalized hypergeometric functions and special functions of hypergeometric type will be presented. It is based on the limit version of the sequence of weighted convolution inequalities generated by a seminorm problem for formal power series and its binomial solution. It will be shown that the limit inequality involves some deep properties of the Eulerian integrals and Bernstein polynomial relation. Several related examples and applications of our method will be discussed.

Arcadii Grinshpan
University of South Florida
Tampa, FL, USA
agrinshp@usf.edu

MS26

Meijer's G Function and Fox's H Function Near a Regular Singularity

In the talk, we will discuss some new properties of Meijer's G function $G^{p,0}_{p,p}$ and Fox's H-function $H^{p,0}_{q,p}$. The first function has been studied by various authors under different names and our first goal is to reveal the connections between those investigations. In particular, we will present new formulas for the expansion coefficients in the vicinity of the singular point at 1. The Fox H-function does not satisfy any known differential equation but it has a similar type of singularity at a finite real point (which need not be unity). We will discuss its behavior in the neighborhood of this point. Further, we present new integral and functional equation for both functions and inequalities for some other cases of Meijer's G-function. The talk will reflect the joint work with Elena Prilepkina.

Dmitry Karp
Russian Academy of Sciences
Vladivostok, Russia
dimkrp@gmail.com

MS26

Interlacing of Zeros of General Laguerre Polynomials

We consider interlacing properties of the real zeros of the Laguerre polynomial $L^{(\alpha)}_n(x)$, in the case $\alpha < -1$. The main tool used is the Sturm comparison theorem.

Martin E. Muldoon
Department of Mathematics & Statistics
York University
muldoon@yorku.ca

Kathy A. Driver
University of Cape Town
South Africa
kathy.driver@uct.ac.za

MS27

The Quantum Superalgebra $osp_q(1|2)$ and a q-generalization of the Bannai-Ito Polynomials

The Racah problem for the quantum superalgebra $osp_q(1|2)$ is considered. A quantum deformation of the Bannai-Ito algebra is realized by the intermediate Casimir operators entering the Racah problem. A q-generalization of the Bannai-Ito polynomials is presented. The relation between these basic polynomials and the q-Racah/Askey-Wilson polynomials is discussed.

Vincent Genest
Centre de recherches mathématiques
Université de Montréal
vincent.genest@umontreal.ca

Luc Vinet
Centre de recherches mathematiques
luc.vinet@umontreal.ca

Alexei Zhedanov
Donetsk Institute for Physics and Engineering, Ukraine
zhedanov@yahoo.com

MS27

Separation of Variables, Superintegrability and Bôcher Contractions

Two-dimensional quadratic algebras are generalizations of Lie algebras that include the symmetry algebras of 2nd order superintegrable systems in 2 dimensions as special cases. Distinct superintegrable systems and their quadratic algebras are related by geometric contractions, induced by generalized Inönü-Wigner Lie algebra contractions and these contractions have important physical and geometric consequences, such as the Askey scheme for hypergeometric orthogonal polynomials. This approach can be unified by ideas first introduced in the 1894 thesis of Bôcher to study separable solutions of the wave equation.

Willard Miller Jr
University of Minnesota
School of Mathematics
miller@ima.umn.edu

MS27

Exceptional Orthogonal Polynomials, Wronskians, and the Darboux Transformation

Exceptional orthogonal polynomials (so named because they span a non-standard polynomial flag) are defined as polynomial eigenfunctions of Sturm-Liouville problems. By allowing for the possibility that the resulting sequence of polynomial degrees admits a number of gaps, we extend the classical families of Hermite, Laguerre and Jacobi. In recent years the role of the Darboux (or the factorization) transformation has been recognized as essential in the theory of orthogonal polynomials spanning a non-standard flag. In this talk we will focus on exceptional Hermite polynomials: their regularity properties, asymptotics of zeros and their relation to the recent conjecture that ALL exceptional orthogonal polynomials are related via factorization transformations to classical orthogonal polynomials.

Robert Milson
Dept. Mathematics & Statistics
Dalhousie University
rmilson@dal.ca

David Gomez-Ullate
Complutense University
dgomezu@ucm.es

MS27

Doubling-up Hahn Polynomials: Classification and Applications

Joint work with J. Van der Jeugt. We examine how two sets of Hahn polynomials $Q_n(x;\alpha,\beta,N)$ and $Q_n(x;\hat{\alpha},\hat{\beta},\hat{N})$ can be combined into a single set of (discrete) orthogonal polynomials $P_n(x)$ ($n = 0,1,\ldots,N+\hat{N}+1$). Our analysis uses (new and old) shift operator relations for Hahn polynomials, coming from contiguous relations. This investigation gives rise to new classes of (rather pretty) tridiagonal matrices with a closed form spectrum, and has applications in finite quantum oscillators or in linear spin chains.

Roy Oste, Joris Van Der Jeugt
Ghent University
Roy.Oste@UGent.be, joris.vanderjeugt@ugent.be

MS28

Overview of Digital Mathematics Libraries (DML) and the NIST Digital Repository of Mathematical Formulae (DRMF)

The DRMF is designed for a mathematically literate audience and should (1) facilitate interaction among mathematicians and scientists interested in compendia OPSF formulae data; (2) be expandable, allowing for input of new formulae from the literature; (3) use context-free semantic markup; (4) have a user friendly, consistent, and hyperlinkable viewpoint and authoring perspective; (5) perform math-aware search; and (6) use MathML for easily read, scalably rendered, content driven mathematics. A DML overview will be given.

Howard Cohl
Applied and Computational Mathematics Division
National Institute of Standards and Technology
howard.cohl@nist.gov

MS28

Steps Toward Realizing a World Information System for Digitally Organized Mathematics

The research mathematics available online has greatly increased over the last decade and a half; the methods for describing, linking and discovering these resources has evolved much more slowly. A Global Digital Mathematical Library Working Group was formed at the 2014 International Congress of Mathematicians. Concrete ideas for creating a community-owned mathematical information system for the digital age of science better supporting advanced research in mathematics, and steps already taken will be discussed.

Patrick Ion
American Mathematical Society
ion@ams.org

MS28

Building DLMF

I will describe the technical processes and issues involved in developing and building the Digital Library of Mathematical Functions (DLMF). These include: use of LaTeX for authoring for print; LaTeXML to convert to web formats with Presentation MathML; semantic markup for math-aware search and with an eye to future Content MathML. The applicability of these techniques to Digital Mathematical Libraries and the web publishing of mathematics will be addressed, as well as future work.

Bruce Miller
Applied and Computational Mathematics Division
National Institute of Standards and Technology
bruce.miller@nist.gov

MS28

The Mathematica Computable Library of Special Function Identities

We report on the enhancement and extension of the substantial body of special function identities originally collected on the Wolfram Functions Site. A greatly augmented set of identities including a number of new functions and mathematical constants has now been integrated into Wolfram|Alpha, making finding and working with these identities easier than ever before. In addition, using the computable data framework pioneered for Mathematica 10, this collection will soon be available in Mathematica itself.

Eric Weisstein
Wolfram Research, Inc.
eww@wolfram.com

MS29

Resurgence and Special Functions

The talk aims at presenting advances on some special functions from the viewpoint of resurgence theory. Among other examples, the Painlevé transcendents will be particularly adressed.

Eric Delabaere
Université dAngers
eric.delabaere@univ-angers.fr

MS29

Accelerating the Computation of Special Functions Using Hybrid Hyper-Borel-Padé Approximations

In this talk we will unify the two current techniques of hyperasymptotic expansions and Borel-Padé summation to accelerate the high accuracy computation of special functions and their associated Stokes constants. Examples will be given demonstrating the advantages, and disadvantages, of the techniques.

Chris Howls
Southampton University

c.j.howls@maths.soton.ac.uk

MS29

The Resurgence Properties of the Bessel and Hankel Functions of Large Order and Argument

We reconsider the classical large order and argument asymptotic series of the Bessel and Hankel functions due to Debye. Employing the reformulation of the method of steepest descents by Berry and Howls, we derive resurgence-type formulas for the error terms of these expansions. Using these new representations of the remainder terms, we obtain numerically computable bounds and exponentially improved versions of Debye's series, together with asymptotic expansions for their late coefficients. Our analysis also provides a rigorous treatment of the formal results derived earlier by Dingle.

Gergo Nemes
Central European University
nemesgery@gmail.com

MS29

Exponentially Small Difference Between the Eigenvalues a_m and b_{m+1} of Mathieu's Equation

We consider Mathieu's equation $\frac{d^2w(z)}{dz^2} + \left(\lambda - 2h^2\cos 2z\right)w(z) = 0$, which has special solutions called Mathieu functions of integral order when $\lambda = a_m$ or $\lambda = b_{m+1}$, for m a positive integer. When $h \to \infty$, these special eigenvalues have the same asymptotic expansions to all orders in descending powers of h. I will present rigorous asymptotics to obtain the exponentially small difference between a_m and b_{m+1} as $h \to \infty$.

Karen Ogilvie
The University of Edinburgh
k.ogilvie@sms.ed.ac.uk

MS30

Recurrence Relations and Hamburger and Stieltjes Moment Problems

Consider an orthogonal polynomial sequence (OPS) $\{P_n(x)\}_{n=0}^{\infty}$ which satisfies the three term recurrence relation

$$P_n(x) = (x - c_n)P_{n-1}(x) - \lambda_n P_{n-2}(x), \quad c_n \text{ real}, \quad \lambda_{n+1} > 0, n = 1, 2, \ldots,$$
$$P_0(x) = 1, P_{-1}(x) = 0.$$

We are concerned with finding criteria which permit us to decide the determinacy or indeterminacy of the Hamburger and/or Stieltjes moment problems associated with the recurrence relation above on the basis of the behavior of the two coefficient sequences in the recurrence relation. The prototype for this type of result is the classical theorem of Carleman [4]: The Hamburger moment problem associated with the recurrence relation above is *determined* if

$$\sum_{n=2}^{\infty} \lambda_n^{-1/2} = \infty.$$

We will survey what has been done along these lines and present new results as well. We also recall a comparison theorem due to Carleman which compares the coefficients in a second three term recurrence relation with those in the recurrence relation above and concludes the determinacy or indeterminacy of the OPS determined by the second recurrence relation from the determinacy or indeterminacy of the original OPS. We will also give new results of this sort.

Theodore Chihara
Purdue University
chihara@Purduecal.edu

MS30

From Indeterminate to Determinate

In the talk, I'll discuss various solutions to indeterminate moment problems and explain how much (or little) one needs to modify the measure in order to change the nature of the problem. I'll also discuss a number of conditions that usually won't result in determinacy. This is mainly done through examples and counterexamples.

Jacob S. Christiansen
University of Lund
stordal@maths.lth.se

MS30

Spectral Properties of Unbounded Jacobi Matrices and Chihara's Problem

Let μ be a determinate measure on the real line. Assume the orthonormal polynomials $\{p_n\}_{n=0}^{\infty}$ with respect to μ satisfy

$$p_{-1}(\lambda) \equiv 0, \quad p_0(\lambda) \equiv 1,$$
$$\lambda p_n(\lambda) = a_{n-1}p_{n-1}(\lambda) + b_n p_n(\lambda) + a_n p_{n+1}(\lambda) \quad (n \geq 0)$$

for sequences $\{a_n\}$ and $\{b_n\}$. The Chihara problem is the following: assume

$$\lim_{n\to\infty} b_n = \infty, \quad \lim_{n\to\infty} \frac{a_n^2}{b_n b_{n+1}} = \frac{1}{4}$$

and the smallest accumulation point ρ of $\mathrm{supp}(\mu)$ is finite. Find additional conditions which imply $\mathrm{supp}(\mu) \supseteq [\rho, +\infty)$. We are going to present a solution to this problem.

Grzegorz Swiderski
Instytut Matematyczny, Uniwersytet Wroclawski
gswider@math.uni.wroc.pl

MS31

Computer Algebra and Special Functions Inequalities

Proving special functions inequalities can be a tedious task requiring (a combination of) several different techniques. Among these techniques, algorithms for special functions deserve increasing attention. We will illustrate the scope and limitations of existing computer algebra methods for proving special functions inequalities for some specific examples that arose in some recent collaborations with Geno Nikolov (et al.).

Veronika Pillwein
Johannes Kepler University
LInz, Austria
vpillwei@risc.jku.at

MS31

Inequalities and Bounds for Some Cumulative Dis-

tribution Functions

Bounds on some cumulative distributions functions and their inverses are discussed. In particular, we present recent bounds for incomplete gamma functions and Marcum functions (also called central and non-central cumulative gamma distributions) and we extend this type of results to other probability distributions like the beta distribution (central and noncentral). The inverse of the cumulative distribution functions (quantile functions) are also important functions in statistics; we discuss how monotonically convergent inversion methods can be used both for the numerical inversion and for obtaining analytical bounds for these functions.

Javier Segura
University of Cantabria
Santander, Spain
segurajj@unican.es

MS31

Turán Type Inequalities for Struve Functions

We deduce some Turán type inequalities for Struve functions of the first kind by using various methods developed in the case of Bessel functions of the first and second kind. New formulas, like Mittag-Leffler expansion, infinite product representation for Struve functions of the first kind, are obtained, which may be of independent interest. Moreover, some complete monotonicity results and functional inequalities are deduced for Struve functions of the second kind.

Sanjeev Singh
Indian Institute of Technology Madras
sanjeevsinghiitm@gmail.com

Árpád Baricz
Department of Economics, Babes-Bolyai University,
Cluj-Napoca 400591, Romania.
bariczocsi@yahoo.com

Saminathan Ponnusamy
Indian Statistical Institute, Chennai Centre,
CIT Campus, Taramani, Chennai 600113, India.
samy@iitm.ac.in

MS32

Coupling Coefficients for Quantum SU(2) Representations

We study tensor products of infinite dimensional irreducible $*$-representations (not corepresentations) of the SU(2) quantum group. Eigenvectors of certain 'almost-central' self-adjoint elements can be given explicitly in terms of q-hypergeometric orthogonal polynomials. We compute coupling coefficients between different eigenvectors corresponding to the same eigenvalue; they turn out to be q-analogs of Bessel functions. As a result we obtain several q-integral identities involving q-hypergeometric orthogonal polynomials and q-Bessel-type functions.

Wolter Groenevelt
Delft Institute of Applied Mathematics
Technische Universiteit Delft
w.g.m.groenevelt@tudelft.nl

MS32

Ladder Operators for Rationally-Extended Potentials Connected with Exceptional Orthogonal Polynomials and Superintegrability

I will review results concerning k-step extension of the harmonic oscillator and the radial oscillator. These 1D exactly solvable systems are related to Hermite and Jacobi exceptional orthogonal polynomials of type III and allow different types of ladder operators. I will show how ladder operators involving no isolated multiplets exist and can be constructed via combinations of Darboux-Crum and Krein-Adler SUSYQM approaches. I will also discuss the application to 2D superintegrable systems and derivation of their energy spectrum using polynomial algebras and their finite dimensional unitary representations. I will also discuss how 1-step and 2-step extension of the harmonic oscillator are connected with a quantum Hamiltonian involving the fourth Painleve transcendent with third order ladder operators.

Ian Marquette
School of Mathematics and Physics
The University of Queensland
i.marquette@uq.edu.au

MS32

BC_1 Lamé Polynomials

The potential of the BC_1 quantum elliptic model is a superposition of two Weierstrass functions with doubling of both periods (two coupling constants). The BC_1 elliptic model degenerates to A_1 elliptic model characterized by the Lamé Hamiltonian. It is shown that in the space of BC_1 elliptic invariant, the potential becomes a rational function, while the flat space metric becomes a polynomial. The model possesses the hidden $sl(2)$ algebra for arbitrary coupling constants: it is equivalent to $sl(2)$-quantum top in three different magnetic fields. It is shown that there exist three one-parametric families of coupling constants for which a finite number of polynomial eigenfunctions (up to a factor) occur. They can be called BC_1 Lamé polynomials, being a generalization of the Lamé polynomials.

Alexander Turbiner
Instituto de Ciencias Nucleares, UNAM
Departamento de Gravitación y Teora de Campos
turbiner@nucleares.unam.mx

MS32

A Dirac-Dunkl Equation on S^2 and the Bannai-Ito Algebra

The Dirac-Dunkl operator on the 2-sphere associated to the $\mathbb{Z}_2^3$ reflection group is considered. Its symmetries are found and are shown to generate the Bannai-Ito algebra. Representations of the Bannai-Ito algebra are constructed using ladder operators. Eigenfunctions of the spherical Dirac-Dunkl operator are obtained using a Cauchy-Kovalevskaia extension theorem. These eigenfunctions, which correspond to Dunkl monogenics, are seen to support finite dimensional irreducible representations of the BannaiIto algebra.

Hendrik De Bie
Ghent University
hendrik.debie@ugent.be

Vincent Genest
Centre de recherches mathématiques
Université de Montréal
vincent.genest@umontreal.ca

Luc Vinet
Centre de recherches mathematiques
luc.vinet@umontreal.ca

MS33

An Overview of the Dynamic Dictionary of Mathematical Functions (http://ddmf.msr-inria.inria.fr)

DDMF is a generated, online, interactive dictionary of special functions. For each function, it displays:local and asymptotic expansions; recurrences on and closed forms of the coefficients in those expansions; guaranteed arbitrary-precision numerical approximations; plots; etc. More terms in expansions or more digits in approximations can be obtained upon request. When relevant, human-readable proofs are also automatically generated and displayed. In the talk, I will demonstrate the website and present the algorithms used.

Frédéric Chyzak
INRIA and École Normale Supérieure de Lyon
frederic.chyzak@inria.fr

MS33

MathSciNet: Digital Guide to the Mathematical Literature

MathSciNet is the database based on Mathematical Reviews, a 75-year-old reviewing service covering the research mathematics literature. With the exponential growth in scholarly publications, guides to the literature are increasingly important. There are several options available, ranging from simple internet searches to advanced databases. MathSciNet provides authoritative information about authors and publications, as well as abstracts and reviews of most of the literature. The talk includes a brief history and discussions of scope and functionality.

Edward Dunne
American Mathematical Society
egd@ams.org

MS33

An Introduction to Recent Algorithms Behind the DDMF

The DDMF is built on the observation that many special functions can be completely specified by a linear differential equation and initial conditions. We will describe how this data structure can be used to produce numerical values efficiently, expansions in Chebyshev series or other generalized Fourier expansions and also in some cases, continued fraction expansions.

Bruno Salvy
INRIA, Paris-Rocquencourt
bruno.salvy@inria.fr

MS33

Semantics, Formula Search, Mathematical Software - How the zbMATH Database extends Beyond Publications

Condensing the corpus of mathematical research into quickly accessible structured content has been the task of review services in mathematics for a long time. Traditionally done by humans (reviews, classification,...), the growth of research - increasingly dispersed in nonstandard form like software - requires adapted scalable tools for content analysis. We introduce new automated tools, like math-aware taggers and semantic formula enrichment, currently employed at zbMATH.

Olaf Teschke
Zentralblatt MATH
teschke@zentralblatt-math.org

MS34

Rigorous Borel Summability Methods and Applications to Integrable Models

I will present some key results in rigorous Borel summability and their applications, with a special emphasis on two problems: the Dubrovin conjecture for Painevé P1 and a methods of calculating connection formulae in closed form in integrable models, without resorting to Riemann-Hilbert or similar reformulations, solely from the Painlevé property.

Ovidiu Costin
Department of Mathematics
Rutgers University
costin@math.ohio-state.edu

MS34

Uniform Asymptotics of Orthogonal Polynomials Arising from Coherent States

We study a family of orthogonal polynomials $\{\phi_n(z)\}$ arising from nonlinear coherent states in quantum optics. Based on the three-term recurrence relation only, we obtain a uniform asymptotic expansion of $\phi_n(z)$ as the polynomial degree n tends to infinity. Our asymptotic results suggest that the weight function associated with the polynomials has an unusual singularity, which has never appeared for orthogonal polynomials in the Askey scheme. Our main technique is the Wang and Wong's difference equation method. In addition, the limiting zero distribution of the polynomials $\phi_n(z)$ is provided.

Dan Dai
City University, Hong Kong
dandai@cityu.edu.hk

Weiying Hu
City University of Hong Kong
weiyinhu@cityu.edu.hk

Xiang-Sheng Wang
Southeast Missouri State University
xswang@semo.edu

MS34

Some Recursive Techniques in the Approximation of Special Functions

Many special functions are solutions of a differential or difference equation. An appropriate use of the Green function of a certain part of the equation permits the transformation of the differential or difference equation intro an integral or series equation respectively. Then, from the fixed point theorem of Banach we obtain a sequence of functions that converges to the given special function. As an illustration, we derive a new convergent expansion of the Bessel functions.

Jose L. Lopez
State University of Navarra
jl.lopez@unavarra.es

Chelo Ferreira, Ester Perez Sinusia
Universidad de Zaragoza
cferrei@unizar.es, ester.perez@unizar.es

MS34

Uniform Asymptotic Approximations for Linear Differential Equations with a Bounded Uniformity Parameter

Typically when one studies uniform asymptotic approximations for differential equations, the asymptotics is for a large free parameter, say λ, and the approximations are valid for the differentiation variable, say z, near a critical point. Here we will discuss the opposite case. The differentiation variable is large, and the approximations are supposed to hold for the free parameter near a critical value. Note that in difference equations this is the typical situation, since we normally study the large n asymptotics.

Adri B. Olde Daalhuis
Maxwell Institute and School of Mathematics
The University of Edinburgh
A.OldeDaalhuis@ed.ac.uk

MS35

Generalizations of Generating Functions for Meixner and Krawtchouk Polynomials

In this talk we explain how connection and connection-type relations for Meixner and Krawtchouk polynomials may be derived. Using these relations, we obtain generalizations of generating functions for Meixner and Krawtchouk polynomials. From these generalized generating functions, we develop infinite series expressions using the discrete orthogonality relations for Meixner and Krawtchouk polynomials. We will also explain how one may obtain orthogonality relations for these polynomials in the complex plane using Ramanujan's master theorem.

Howard Cohl
National Institute of Standards and Technology
Applied and Computational Mathematics Division
hcohl@nist.gov

Roberto S. Costas-Santos
Dept. Physics and Mathematics Alcalá de Henares, Madrid, Spa
roberto.costas@uah.es

Wenqing Xu
Montgomery Blair High School
williamxuxu@yahoo.com

MS35

Discrete Orthogonality of Four Types of Orthogonal Polynomials of Weyl Groups

The link between the discrete Fourier calculus of the four families of special functions, $C-$, $S-$, S^s- and S^l- functions, and the four families of the induced orthogonal polynomials is discussed. The affine Weyl groups corresponding to the root systems of simple Lie algebras are recalled and sign homomorphisms, which allow general explicit description of the orbit functions, are described. The discrete Fourier calculus of the four types of orbit functions is performed for each type on a different set of points with the weights, labeling the orthogonal functions, chosen for each type separately. The four types of orthogonal polynomials, induced by the four types of orbit functions, inherit the discrete orthogonality from the orbit functions. The discrete orthogonality of the polynomials is explicitly formulated and its application for the development of numerical integration formulas and polynomial interpolation methods is discussed.

Jiri Hrivnak
Department of physics
Czech Technical University in Prague
jiri.hrivnak@fjfi.cvut.cz

MS35

Lattices of Any Dimension and Their Refinement to Any Density

A simple definition of a lattice in a real Euclidean space R^n of dimension n, can be stated as follows: the infinite set of discrete points $\Lambda \in R^n$ is a lattice provided one has $\Lambda+\Lambda=\Lambda$, where $\Lambda+\Lambda$ stands for every sum of two points of Λ.
Lattices in the two dimensional Euclidean plane are known to be of two kinds, the lattice of squares and the lattice of equilateral triangles. The lattices, characterized by their symmetries, come in two forms each. We identify the four cases by the symbols most often used for the corresponding complex semisimple Lie algebras of rank 2: For square lattices we use $A_1 \times A_1$ and C_2 , for triangular lattice we use A_2 and G_2.

Marzena Szajewska
Institute of Mathematics, University of Bialystok
m.szajewska@math.uwb.edu.pl

MS35

Orthogonality of Macdonald Polynomials with Unitary Parameters

Generalizing previous work with Luc Vinet for the symmetric group case, we show that for parameters q and t on the unit circle and subject to a suitable truncation relation, the Macdonald polynomials associated with crystallographic root systems satisfy a finite-dimensional system of discrete orthogonality relations on the Weyl alcove. The discrete orthogonality weights are positive and their total mass is expressed in product form by means of a finitely truncated Aomoto-Ito type q-Selberg sum. This gives rise to a unitary q-deformed discrete Fourier involution on the Weyl alcove. For $q=t$ our q-deformed Fourier transform amounts to the discrete Fourier transform associated with the discrete orthogonality relations for the Weyl characters studied by Kirillov Jr., Korff and Stroppel, and also in a more general form by Patera et al. This is joint work with Erdal Emsiz.

Jan Felipe van Diejen
Instituto de Matemática y Fsica

Universidad de Talca
diejen@inst-mat.utalca.cl

MS36

Precise and Fast Computation of Elliptic Functions and Elliptic Integrals

New methods are developed to compute three Jacobian elliptic functions, complete and incomplete elliptic integrals of all three kinds, and their derivatives and inversions by the half and double argument formulas. The new methods are of at least 50 bit accuracy and run 1.1-3.5 times faster than the existing methods: Cody's Chebyshev approximations, Bulirsch's `cel` and `el1`, and Carlson's R_F, R_D, and R_J. All the published articles and accompanied Fortran programs are available from `https://www.researchgate.net/profile/Toshio_Fukushima/`

Toshio Fukushima
National Astronomical Observatory of Japan
Toshio.Fukushima@nao.ac.jp

MS36

Computation and Inversion of Certain Cumulative Distribution Functions

Both the direct computation and the inversion of the cumulative central beta and gamma distributions (central and noncentral) are used in many problems in statistics, applied probability and engineering. Reliable and fast algorithms for the inversion and computation of these distribution functions will be presented. Also, we will show comparisons with other existing algorithms implemented in software platforms like Matlab, Mathematica and R. In our algorithms, the computation of the cumulative distribution functions is based on the use of different methods of approximation such as Taylor expansions, continued fractions, uniform asymptotic expansions, numerical quadrature, etc depending on the parameter values. For the inversion, asymptotic expansions in combination with high-order Newton or secant methods are used.

Amparo Gil
Depto. de Matematicas, Est. y Comput.
Universidad de Cantabria
amparo.gil@unican.es

Javier Segura
Departamento de Matematicas, Estadistica y Computacion
Universidad de Cantabria, Spain
javier.segura@unican.es

Nico M. Temme
CWI, Amsterdam
nico.temme@cwi.nl

MS36

On the Evaluation of Prolate Spheroidal Wave Functions and Some Associated Quantities

Prolate spheroidal wave functions (PSWFs) corresponding to band limit $c > 0$ are the eigenfunctions of the truncated Fourier transform $F_c : L^2[-1,1] \to L^2[-1,1]$ defined via the formula

$$F_c [\sigma] (x) = \int_{-1}^{1} \sigma(t) \cdot e^{icxt} \, dt. \quad (2)$$

In this capacity, PSWFs provide a natural tool for dealing with bandlimited functions defined on an interval, as demonstrated by Slepian et. al. in a sequence of classical papers. (A function $f : R \to R$ is called bandlimited with band limit $c > 0$ if its Fourier transform is supported on the interval $[-c, c]$.) Starting with the papers by Slepian et. al., PSWFs have been used as a tool in electrical engineering (design of antenna patterns), digital signal processing (design of digital filters, such as upsampling/downsampling algorithms in acoustics), physics (various wave phenomena, fluid dynamics, uncertainty principles in quantum mechanics), etc. However, the use of PSWFs has been somewhat crippled by their slightly mysterious reputation as being "difficult to compute". This seems to be related to the fact that the classical ("Bouwkamp") algorithm for their evaluation encounters numerical difficulties for $c > 40$ or so. Moreover, the attempt to diagonalize the operator F_c numerically via straightforward discretization meets with numerical difficulties as well. In this talk, we describe several numerical algorithms for the evaluation of PSWFs, some associated quantities, and PSWFs-based quadrature rules for the integration of bandlimited functions. While the underlying analysis is somewhat involved, the resulting numerical schemes are quite simple and efficient in practical computations, even for large values of band limit (e.g. $c = 10^6$).

Andrei Osipov
Mathematics Department
Yale University
andreiosipov2@gmail.com

MS36

A Fast Chebyshev-Legendre Transform Using An Asymptotic Formula

Legendre expansions have applications throughout scientific computing because of their L^2-orthogonality, rapidly decaying Cauchy transform, and association with spherical harmonics. However, fast algorithms for computing with Legendre expansions are not readily available. In this talk we describe a fast and numerically stable $\mathcal{O}(N(\log N)^2/\log\log N)$ algorithm for converting between Legendre and Chebyshev expansions based on carefully exploiting an asymptotic formula. Applications are: fast L^2-projection, mollification, and the effective computation of Painlevé transcendents.

Alex Townsend
Department of Mathematics
MIT
ajt@mit.edu

Nick Hale
Stellenbosch University
nick.p.hale@gmail.com

MS37

The First Szego's Limit Theorem with Varying Coefficients

In this talk, I will present some extensions of Szegö's First Limit Theorem (SFLT) to a class of non Toeplitz matrices. Our results extend those of Kuijlaars and Van Assche for Jacobi matrices, as well as those of Tilli on locally Toeplitz matrices.

Alain Bourget
Cal State University, Fullerton
Department of Mathematics
abourget@fullerton.edu

MS37

The Second Szego's Limit Theorem for a Class of non-Toeplitz Matrices

Let $a(t)$ be an integrable function on the unit circle with Fourier coefficients

$$a_k = \frac{1}{2\pi}\int_{-\pi}^{\pi} a(t)\, e^{-ikt}\, dt, \qquad k \in \mathcal{Z}.$$

The Toeplitz matrices $T_n(a)$ associated with a are

$$T_n(a) = \{a_{j-k}\}_{0\le j,k\le n-1}\,.$$

Szegő's First Limit Theorem (SFLT) states that for any self-adjoint Toeplitz operator $T(a)$ and any continuous φ,

$$\mathrm{Tr}[\varphi(T_n(a))] = \frac{n}{2\pi}\int_{-\pi}^{\pi} \varphi(a(t))\, dt + o(n).$$

Almost 40 years after establishing the FSLT, Szegő established the second Szegő's limit theorem which gives a formula for the $o(n)$ error term above up to exponentially small terms. The SFLT has recently been extended to a class of matrices whose diagonal entries satisfy a small deviation condition. In this talk I will present results toward extending the second limit theorem to this class of non-Toeplitz matrices.

Tyler McMillen
California State University, Fullerton
Department of Mathematics
tmcmillen@fullerton.edu

MS37

Ratio Asymptotics and Weak Asymptotic Measures

We will consider the asymptotics of general orthogonal polynomials in the complex plane. Our focus will be on ratio asymptotics and weak asymptotic measures and how they relate to the right limits of the Bergman Shift matrix. We will also discuss relative forms of these asymptotics and generalize some results from the setting of OPUC to measures with more general support.

Brian Simanek
Vanderbilt University
brian.z.simanek@vanderbilt.edu

MS37

Newman-Rivlin Asymptotics for Partial Sums of Power Series

We discuss analogues of Newman and Rivlin's formula concerning the ratio of a partial sum of a power series to its limit function and present a new general result of this type for entire functions with a certain asymptotic character. The main tool used in the proof is a Riemann-Hilbert formulation for the partial sums introduced by Kriecherbauer et al. This new result makes some progress on verifying a part of the Saff-Varga Width Conjecture concerning the zero-free regions of these partial sums.

Antonio R. Vargas
Dalhousie University
Department of Mathematics and Statistics
antoniov@mathstat.dal.ca

MS38

Universality of Mesoscopic Fluctuations in Orthogonal Polynomial Ensembles

We shall discuss fluctuations on the mesoscopic scale for orthogonal polynomial ensembles and show that these are universal in the sense that two measures with asymptotic recurrence coefficients have the same asymptotic mesoscopic fluctuations (under an additional assumption on the local regularity of one of the measures). The convergence rate of the recurrence coefficients determines the range of scales on which the limiting fluctuations are identical. A particular consequence of our results is a Central Limit Theorem for the modified Jacobi Unitary Ensembles on all mesoscopic scales. This is joint work with Maurice Duits.

Jonathan Breuer
Hebrew University of Jerusalem, Israel
jbreuer@math.huji.ac.il

MS38

Orthogonal Dirichlet Polynomials

We discuss orthogonal "polynomials" formed from linear combinations of Dirichlet functions, as used in L series in number theory. Although formed from the same basis functions as used in the continuous orthogonality of Krein systems, they are distinct from Krein systems. We motivate their study, and present some recent results.

Doron S. Lubinsky
Georgia Institute of Technology
lubinsky@math.gatech.edu

MS38

Analytic Continuation of S-property for Multiple Orthogonal Polynomials

We are interested in the S-property for multiple orthogonal polynomials. As a case study, we analyze a family of multiple orthogonal polynomials arising in the normal matrix model with cubic + linear potential. In order to perform analytic continuation of the S-property, we interpret it in terms of a quadratic differential on the associated spectral curve, and develop a deformation argument on its critical graph.

Guilherme Silva
Katholieke Universiteit Leuven, Belgium
guilherme.silva@wis.kuleuven.be

Pavel Bleher
Department of Mathematics, IUPUI
bleher@math.iupui.edu

MS38

Nuttall's Theorem on Algebraic S-contours

Given a function f holomorphic at infinity, the n-th diagonal Padé approximant to f, say $[n/n]_f$, is a rational function of type (n, n) that has the highest order of contact with f at infinity. Nuttall's theorem provides an asymptotic formula for the error of approximation $f - [n/n]_f$ in the case where f is the Cauchy integral of a smooth density with respect to the arcsine distribution on $[-1, 1]$. I will present an extension of Nuttall's theorem to Cauchy integrals on the so-called algebraic S-contours.

Maxim Yattselev
Indiana University-Purdue University Indianapolis
maxyatts@iupui.edu

MS39

Geometry of Hermite-Padé Approximants for a Pair of Cauchy Transforms with Interlacing Symmetric Supports

We consider the multiple orthogonal polynomials with respect to smooth complex measures supported on the real line. In this talk we are interested in the case where the supports form two interlacing symmetric intervals and the ratio of the measures extends to a holomorphic function in a region that depends on the size of interlacing. This problem was posed and studied by Herbert Stahl in the 80's. We shall speak about algebraic functions (of genus 1 and 2) and their abelian integrals (with purely imaginary periods) which define the main term of the asymptotics for this problem. This is joint work with Walter Van Assche

and Maxim L. Yattselev.

Alexander I. Aptekarev
Keldysh Institute of Applied Mathematics
Russian Academy of Science
aptekaa@keldysh.ru

MS39

Difference Operators on Lattices and Multiple Orthogonal Polynomials

We construct difference operators on Z^2 using multiple orthogonal polynomials. Let us stress that it is not clear whether the eigenvalue problem for a difference equation on Z^2 has a solution and, especially, whether the entries of an eigenvector can be chosen to be polynomials in the spectral variable. However, for the difference operators in question, the existence of a polynomial solution to the eigenvalue problem is guaranteed if the coefficients of the difference operators satisfy a certain discrete zero curvature condition. In turn, this means that there is a discrete integrable system behind the scene and the discrete integrable system can be thought of as a generalization of what is known as the discrete time Toda equation, which appeared for the first time as the Frobenius identity for the elements of the Pade table.

Maxim Derevyagin
University of Mississippi
USA
mderevya@olemiss.edu

MS39

Weak and Strong Asymptotics for the Pollaczek Multiple Orthogonal Polynomials

Pollaczek multiple orthogonal polynomials are type II Hermite–Padé polynomials orthogonal with respect to two simple measures supported on the positive semi–axis. These measures form a socalled Nikishin pair, with the feature that one of its generators is purely discrete. It is known that the largedegree asymptotics of such polynomials is governed by the solution of a vector equilibrium problem, which was previously computed by V. Sorokin. For the strong asymptotics we use the Riemann–Hilbert characterization of the Hermite–Padé polynomials and the corresponding nonlinear steepest descent method. We discuss some of the main ingredients of this analysis and the asymptotic results obtained by this method. This is a joint work with A. Aptekarev and G. López-Lagomasino

Andrei Martinez-Finkelshtein
University of Almeria
andrei@ual.es

MS39

On the Convergence of Mixed Type Hermite-Padé Approximants

The convergence of diagonal sequences of type II Hermite-Padé approximants of Nikishin systems have been known for some time and recently similar results have been obtained for type I Hermite-Padé approximants. In this talk we present new results on the convergence of diagonal sequences of a certain mixed type Hermite-Padé approximation problem of a Nikishin system, which is motivated in finding approximating solutions of a Degasperis-Procesi peakons problem and in the study of the inverse spectral problem for the discrete cubic string.

Sergio M. Medina Peralta
Universidad Carlos III de Madrid
smedinaperalta@gmail.com

Guillermo López Lagomasino
UC3M
lago@math.uc3m.es

MS40

Singular Linear Statistics of the Laguerre Unitary Ensemble and Painleve III: Double Scaling Analysis

We compute the Hankel determinant generated from a singularly deformed weight, $w(x; t, \alpha) := x^\alpha \exp(-x - t/x), \quad x > 0, t > 0$, in a double scaling scheme, namely, $n \to \infty$ and $t \to 0$, such that $s = (2n + 1 + \alpha)t$ is finite. Asymptotic expansions of the double-scaled determinant are obtained for large and small s. These are found through solutions of a particular Painleve III obtained from the original finite n PIII (with a larger number of parameters.)

Yang Chen
University of Macau
yangbrookchen@yahoo.co.uk

MS40

Christoffel-Darboux-type Formulae for Orthonormal Rational Functions and Asymptotics

Consider the reproducing kernel $K_n(x, y) = \sum_{k=0}^{n-1} \varphi_k(x)\overline{\varphi_k(y)}$, where $\{\varphi_k\}_{k=0}^{n-1}$ forms an orthonormal basis for the space of rational functions with poles among $\{\alpha_1, \alpha_2, \ldots, \alpha_{n-1}\} \subset C \cup \{\infty\}$. In the first part of the talk we present Christoffel-Darboux-type formulae for K_n by exploiting its relation with so-called quasi-orthogonal rational functions. In the second part we use these formulae to obtain asymptotics for the reproducing kernel K_n for $n \to \infty$.

Karl Deckers
University of Lille, France
karl.deckers@math.univ-lille1.fr

MS40

Recent Asymptotic Expansions for Legendre Polynomial Expansions and Gauss-Legendre Quadrature

In this talk, we discuss some recent asymptotic expansions related to problems in approximation theory and numerical quadrature.

- We present a full asymptotic expansion, as $n \to \infty$, of the Legendre polynomial $P_n(x)$ for $|x| \leq 1 - \epsilon$, $\epsilon \in (0, 1)$.
- We present full asymptotic expansions, as $n \to \infty$, of Legendre series coefficients $a_n = (n + 1/2) \int_{-1}^{1} f(x) P_n(x) dx$, when $f(x)$ has arbitrary algebraic-logarithmic (interior and/or endpoint) singularities in $[-1, 1]$.
- We present a full asymptotic expansion (as the number of abscissas tends to infinity) for Gauss–Legendre quadrature formula $\sum_{i=1}^{n} w_{ni} f(x_{ni})$ for integrals $\int_{-1}^{1} f(x) dx$, where $f(x)$ is allowed to have arbitrary algebraic-logarithmic endpoint singularities.

Avram Sidi
Computer Science Department
Technion - Israel Institute of Technology
asidi@cs.technion.ac.il