SIAG/APDE LOOKING FORWARD SESSION
December 8, 2009

Minutes: Irene Fonseca chaired the session and initiated the discussion by introducing three distinct research directions:

- *Variational Methods in Image Processing*
- *Multiscale Problems for Damage and Fracture*
- *Pattern Formation and the Calculus of Variations*

Variational Methods in Image Processing
Andrea Bertozzi

Some case examples were presented:

High dimensional data

- Hyperspectral data and fusion
- Spectral detail from AVIRIS data
- Spectral preservation

Diverse data sets beyond imaging

- Human event activity
- Maximum penalized likelihood estimation

Combining statistical filtering (real time decisions) with spatial variational methods

- Example of boundary tracker – high dimensional data

Higher order geometric methods

- Example of segmentation with corners

Difficult inverse problems with spatio-temporal statistical variability

- Example of imaging through turbulence

Multiscale Problems for Damage and Fracture
Gianni Dal Maso

The following research directions were presented:

- Cohesive fracture versus brittle fracture

The variational theory of crack growth in the quasistatic regime is well developed only in the case of brittle cracks, where there is a sudden transition between the elastic behavior and the crack (originally considered by Griffith). In other models a force is still transmitted through the crack when the distance between the crack lips is small (Barenblatt). These are the so called cohesive zone models.

- Damage as macroscopic effect of micro-cracks

The presence of diffuse micro-cracks can lead to an elastic macroscopic response characterized by irreversible weakening of the elastic properties, called damage.

Pattern Formation and the Calculus of Variations
Rustum Choksi

Pattern Formation

- historically, territory of the dynamical systems community
- often variational – "energy-driven pattern formation": eg
- domain structures in magnetic materials, elastic materials,
- self-assembly in block copolymers, etc.
- develop direct/global methods to address properties of
- minimizers / energy functional
- methods: ansatz-free and work in higher-D

Short- vs long-range interactions

- many examples: c.f. Seul-Andelman, Science '95
- short-range _ perimeter
- long-range _ interaction kernel (Coulomb-like) negative Sobolev norm

Periodicity of the Ground State

- Discrete analogue: crystallization c.f. Theil ’05
- continuous problems: PDE techniques – too hard?
- modern techniques in calculus of variation: weaker
- formulations _ distribution of energy, decay of density
- variations c.f. Alberti-C-Otto ’09, Spadaro ’09
- to what extent can this type of analysis be furthered to
- describe minimizer ?
- can one rule out quasi-crystals?

Long-range interactions and local structure

Dynamics and Energy Landscapes

- highly non-convex energy landscape
- many local minimizers but also dynamic metastability
- even 1D exhibits complex landscape c.f. Otto and
- Westdickenberg
- can one extend analysis to higher D?
- ways of capturing local vs global?
- can mathematical understanding of energy landscape lead to
- novel numerical techniques for accessing the ground states
    - (“simulated annealing”)?

Irene Gamba initiated a discussion on

- Coupling of Multi-phase problems
- Coupling of atomistic continuum models
- Interfaces and free boundary problems
- Kinetic versa continuum theory
- Homogenization problems
- The role of boundary conditions associated with electrostatics

Gui-Qiang Chen initiated a discussion on major open questions
Evolution equations:

- The concept of “well-posedness” in nonlinear problems.

Blake Temple discussed the problem of

- *Periodic solutions of compressible Euler equations*
- *Shock waves and the Einstein equation*

Kirk Jordan discuss the concept of

- *"Multi-program versa Multi-data"*

Alfio Quarteroni pointed out that analyst often try to create a super-theory that can accommodate many different problems but this attempt often does not take care of the role of physical parameters in the problem.

Ricardo Nochetto identified two research areas that are of fundamental Importance in the modern world:

- *Nano-Technology/micro-technology*
- *Bio-technology-Medicine*

The question that he posed was:

- *What is the role of Mathematics?*

- Modeling
- Create patterns
- Control of devices
- The role of surface tension fundamental in applied problems

The role of Mathematics is to simplify and provide an understanding of the basics.