



Final Program

**SIAM Conference on
Applied Linear Algebra**

**April 26-29, 1982
Mission Valley Inn
Raleigh, North Carolina**

Supported by grants from the National Science Foundation
and the Army Research Office.

STANFORD, CALIFORNIA

JULY 19-23, 1982

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SIAM Conference on Applied Linear Algebra

April 26-29, 1982

Raleigh, North Carolina

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Selected Speakers	3
Contributed Papers and Poster Sessions	8
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The registration times for this conference are as follows:

	<u>FROM</u>	<u>TO</u>
Sunday, April 25	4:00 PM	9:00 PM
Monday, April 26	8:00 AM	5:00 PM
Tuesday, April 27	8:00 AM	5:00 PM
Wednesday, April 28	8:00 AM	5:00 PM
Thursday, April 29	8:00 AM	3:00 PM

Registration Area: LOBBY LOUNGE

SPECIAL NOTICE TO: Contributed Paper Authors and
 Chairpersons of Contributed Paper Sessions

Fifteen minutes are allowed for each contributed paper. Authors are requested to allow a maximum of 12 minutes for presentation of their paper, and to allow 3 minutes for questions and answers.

Chairpersons of contributed paper sessions are requested to adhere as closely as possible to the scheduled times to facilitate meeting participants moving from one session to another.

ATTENTION ATTENDEES:

THE SIAM COUNCIL PASSED A MOTION IN JUNE 1976 REQUESTING THAT PARTICIPANTS REFRAIN FROM SMOKING IN THE LECTURE ROOMS DURING LECTURES.

Conference Program Committee

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Carl D. Meyer (Chairman), North Carolina State University
Robert J. Plemmons, North Carolina State University
Hans Schneider, University of Wisconsin, Madison

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Future SIAM Meetings

July 19-23, 1982 at Stanford, CA
SIAM 30th Anniversary Meeting:
Symposia on the numerical solution of partial differential equations and their applications; the numerical solution of ordinary differential equations and their applications; control theory and optimization; biomathematics; methods in nonlinear analysis; and computer science

November 2-4, 1982 at Boston, MA
IEEE-SIAM Conference on Numerical Simulation of VLSI Devices

June 6-8, 1983 at Denver, CO
National Meeting: Symposia on inverse problems, parameter identification; signal processing; numerical solution of partial differential equations; Polya Prize

June 19-July 1, 1983 at Newark, DE
Special conference on numerical and statistical analysis

June 1983 at Cambridge, MA
Special conference on discrete mathematics and applications

November 7-9, 1983 at Norfolk, VA:
Fall Meeting: Symposia on computational aerodynamics; parameter identification for stabilization and control; parallel processing

November 10-11, 1983 at Norfolk, VA
International conference on parallel computing and parallel processing

March 1984 at Houston, TX
International conference on partial differential equations with focus on petroleum engineering, and in particular, exploration and extraction

June 18-29, 1984 at Newark, DE
Special conference on numerical and statistical analysis

June 1984 at Seattle, WA
National Meeting: Symposia on applied geometry (computer-aided design including graphic design); fluid dynamics (computational)

Final Program

Sunday, 4:00 PM - 9:00 PM Registration - Lobby Lounge	Monday, 3:30 PM Discrete Methods
Monday, 8:00 AM - 5:00 PM Registration - Lobby Lounge	Cameron-Mordecai Suite
Monday, 8:00 AM - 5:00 PM Book Exhibits - Rainey Suite	Chairperson: RICHARD BRUALDI, University of Wisconsin
Monday, 9:00 AM Opening Remarks. CARL MEYER, North Carolina State University, and HANS SCHNEIDER, University of Wisconsin, Madison	Food Webs, Competition Graphs, and Associated Matrices. HARVEY J. GREENBERG, Energy Information Administration; J. RICHARD LUNDGREN, University Colorado, Denver; JOHN S. MAYBEE, University of Colorado, Boulder
Monday, 9:30 AM Numerical Methods	Monday, 4:00 PM Numerical Methods. Contributed Papers. 1 and 2 Discrete Methods. Contributed Papers. 3 Poster Session. 1
Cameron-Mordecai Suite	Monday, 6:00 PM No-Host Cocktail Reception
Recent Developments in Numerical Linear Algebra. GENE H. GOLUB, Stanford University	Lobby Lounge
Monday, 10:30 AM Coffee	
Monday, 11:00 AM Numerical Methods	Tuesday, 8:00 AM - 5:00 PM Registration - Lobby Lounge
Cameron-Mordecai Suite	Tuesday, 8:00 AM - 5:00 PM Book Exhibits - Rainey Suite
Chairperson: ROBERT J. PLEMMONS, North Carolina State University	Tuesday, 8:30 AM Statistical Methods
Positive Definite Toeplitz Matrices, The Hessenberg Process for Isometric Operators, and Gaussian Quadrature on the Unit Circle. WILLIAM B. GRAGG, University of Kentucky	Cameron-Mordecai Suite
Monday, 11:30 AM Numerical Methods	Chairperson: DAN SOLOMON, North Carolina State University
Differential Equations and the Symmetric Eigenvalue Problem. <u>P. DEIFT</u> , T. NANDA and C. TOMEI, Courant Institute of the Mathematical Sciences	Linear Algebraic Methods in Statistics: A Review of Recent Contributions to Theory and Computations. C. RADHAKRISHNA RAO, Indian Statistical Institute and University of Pittsburgh
Monday, 12:00 Lunch	Tuesday, 9:30 AM Statistical Methods
Monday, 1:30 PM Discrete Methods	Computing Component Bounds in Errors-in-Variables Problems. DAVID M. GAY, Bell Laboratories
Cameron-Mordecai Suite	Tuesday, 10:00 AM Coffee
Chairperson: RICHARD BRUALDI, University of Wisconsin	Tuesday, 10:30 AM Numerical Methods
Path Problems. ROBERT E. TARJAN, Bell Laboratories	Cameron-Mordecai Suite
Monday, 2:30 PM Discrete Methods	Chairperson: DAN SOLOMON, North Carolina State University
Graph Coloring Using Eigenvalue Decomposition. BENGT ASPVALL, and <u>JOHN R. GILBERT</u> Cornell University	Numerical Solution of the Weighted Linear Least Squares Problem by G-Transformations. ERWIN H. BAREISS, Northwestern University
Monday, 3:00 PM Coffee	

*Underlining indicates the presenter of the
paper, if more than one author is given.

Tuesday, 11:00 AM
 Numerical Methods. Contributed Papers. 4
 Discrete Methods. Contributed Papers. 5
 Statistical Methods. Contributed Papers. 6
 Poster Session. 1(cont'd)

Tuesday, 12:30 PM
 Lunch

Tuesday, 1:30 PM
 Numerical Methods

Cameron-Mordecai Suite

Chairperson: DAVID McALLISTER, North Carolina State University

A Survey of Recent Results in Numerical Linear Algebra. RICHARD S. VARGA, Kent State University

Tuesday, 2:30 PM
 Numerical Methods

LU Decompositions of Generalized Diagonally Dominant Matrices. R. E. FUNDERLIC, Union Carbide Corporation, Nuclear Division; M. NEUMANN, University of South Carolina; and R. J. PLEMMONS, North Carolina State University

Tuesday, 3:00 PM
 Coffee

Tuesday, 3:30 PM
 Statistical Methods

Cameron Suite

Chairperson: DAVID McALLISTER, North Carolina State University

Linear Algebra in Digital Image Processing: Noise Smoothing and Image Compression. DIANNE P. O'LEARY, K. A. NARAYANAN, SHMUEL PELEG and AZRIEL ROSENFELD, University of Maryland

Tuesday, 4:00 PM
 Numerical Methods. Contributed Papers. 7
 Statistical Methods. Contributed Papers. 8
 Numerical Methods. Contributed Papers. 9
 Discrete Methods. Contributed Papers. 10
 Poster Session. 2

Tuesday, 6:00 PM
 Wine and Cheese Party

Ballroom-University Center, NCSU

Tuesday, 7:30 PM
 Southern Buffet

Ballroom-University Center, NCSU

Wednesday, 8:00 AM - 5:00 PM
 Registration - Lobby Lounge

Wednesday, 8:00 AM - 5:00 PM
 Book Exhibits - Rainey Suite

Wednesday, 8:30 AM
 Engineering/Control

Cameron-Mordecai Suite

Chairperson: HANS SAGAN, North Carolina State University

Displacement Ranks of Matrices and Fast Algorithms for Signal Processing. THOMAS KAILATH, Stanford University

Wednesday, 9:30 AM
 Engineering/Control

Controllability, Inertia, and Stability for Tridiagonal Matrices. DAVID CARLSON, Oregon State University

Wednesday, 10:00 AM
 Coffee

Wednesday, 10:30 AM
 Engineering/Control

Cameron-Mordecai Suite

Chairperson: HANS SAGAN, North Carolina State University

Studies in Reduced-Order Modeling and in Singular Dynamic Systems. GEORGE C. VERGHESE, Massachusetts Institute of Technology

Wednesday, 11:00 AM
 Engineering/Control. Contributed Papers. 11
 Operations Research. Contributed Papers. 12
 Core Linear Algebra. Contributed Papers. 13
 Poster Session. 2(cont'd)

Wednesday, 12:30 PM
 Lunch

Wednesday, 1:30 PM
 Operations Research

Cameron-Mordecai Suite

Chairperson: ELMOR PETERSON, North Carolina State University

The Role of Linear Algebra in Operations Research. ARTHUR F. VEINOTT, JR., Stanford University

Wednesday, 2:30 PM
 Operations Research

Growth Optimality for Branching Markov Decision Chains. URIEL G. ROTHBLUM, Yale School of Organization and Management; and PETER WHITTLE, University of Cambridge

Wednesday, 3:00 PM
 Coffee

Wednesday, 3:30 PM
 Operations Research

Cameron-Mordecai Suite

Chairperson: ELMOR PETERSON, North Carolina State University

Iterative Methods for Variational and Complementarity Problems. JONG-SHI PANG, The University of Texas at Dallas

Wednesday, 4:00 PM
 Engineering/Control. Contributed Papers. 14
 Operations Research. Contributed Papers. 15
 Core Linear Algebra. Contributed Papers. 16
 Poster Session. 3

Wednesday, 7:30 PM
 Informal Sessions

Thursday, 8:00 AM - 3:00 PM
 Registration - Lobby Lounge

Thursday, 8:00 AM - 3:00 PM
 Book Exhibits - Rainey Suite

Thursday, 8:30 AM
 Core Linear Algebra

Cameron-Mordecai Suite

Chairperson: ROBERT E. HARTWIG, North Carolina
 State University

Core Linear Algebra, ROBERT C. THOMPSON,
 University of California, Santa Barbara

Thursday, 9:30 AM
 Core Linear Algebra

Roots of M-Matrices and Generalizations. MIROSLAV
 FIEDLER, Czechoslovak Academy of Sciences,
 Czechoslovakia; and HANS SCHNEIDER, University of
 Wisconsin

Thursday, 10:00 AM
 Coffee

Thursday, 10:30 AM
 Core Linear Algebra

Cameron-Mordecai Suite

Chairperson: ROBERT E. HARTWIG, North Carolina
 State University

Nested Bounds for the Perron Root of a Nonnegative
 Matrix. EMERIC DEUTSCH, Polytechnic Institute of
 New York; and HELMUT WIELANDT, University of
 Tuebingen, West Germany

Thursday, 11:00 AM
 Core Linear Algebra. Contributed Papers. 17
 Applied Linear Algebra. Contributed Papers. 18 and
 Poster Session. 3 (cont'd)

Thursday, 12:30 PM
 Lunch

Thursday, 1:30 PM
 Core Linear Algebra

Cameron-Mordecai Suite

Chairperson: ROBERT E. HARTWIG, North Carolina State
 University

Combinatorial Aspects of Matrix Problems, CHARLES R.
JOHNSON, University of Maryland

Thursday, 2:00 PM
 Core Linear Algebra

Note on Global Perturbations of Hermitian Matrices
 and Related Problems, EDUARDO MARQUES DE SA,
 University of Aveiro, Portugal

Thursday, 2:30 PM
 Core Linear Algebra. Contributed Papers. 20
 Applied Linear Algebra. Contributed Papers. 21

Contributed Papers

Contributed Papers. 1
 Numerical Methods
 Monday, 4:00 PM - 5:30 PM

Haywood-Andrews Suite

Chairperson: WILLIAM STEWART, North Carolina
 State University

4:00
 Staircase Matrices and Systems. ROBERT FOURER,
 Northwestern University

4:15
 Matrix Computations of Hessenberg Matrices,
B. N. DATTA and KARABI DATTA, Northern Illinois
 University

4:30
 BLAS Implementation for the FPS164. BILL MARGOLIS
 Floating Point Systems, Inc.

4:45
 An Algorithm to Determine if Two Matrices Have an
 Eigenvalue in Common. K. DATTA, Northern Illinois
 University

5:00
 Memory Access Patterns in Vector Machines with
 Applications to Problems in Linear Algebra.
MICHAEL R. LEUZE, Duke University

5:15
 Pseudoinverting a Large, Almost-Block-Tridiagonal
 System. LEO P. MICHELOTTI, Fermilab.

Contributed Papers. 2
 Numerical Methods
 Monday, 4:00 PM - 5:45 PM

Cameron-Mordecai Suite

Chairperson: ROBERT E. WHITE, North Carolina State
 University

4:00
 The Impact of Microprocessor Technology on
 Computational Algorithms in Numerical Linear
 Algebra, VIRGINIA KLEMA, Massachusetts Institute
 of Technology

4:15
 Divergence-Free Bases for Finite Element Schemes in
 Hydrodynamics. KARL GUSTAFSON, University of
 Colorado; and ROBERT HARTMAN, Eastern Washington
 University

4:30
 VLSI Networks for Orthogonal Equivalence
 Transformations and Related Applications. DON E.
HELLER and ILSE C. F. IPSEN, Pennsylvania State
 University

4:45

The Bit-Complexity of Arithmetic Algorithms and Their Stability and Structural Complexity. VICTOR PAN, State University of New York at Albany

5:00

Asymptotic Methods for the Determination of Relaxation Convergence Rates. GARRY RODRIGUE, Lawrence Livermore National Laboratory; and RICHARD VARGA, Kent State University

5:15

Solutions of Higher Order Matrix Equations and Finite Element Methods. JOHN JONES, JR., CHARLES R. MARTIN and HALVOR A. UNDEM, Air Force Institute of Technology

5:30

The Stability of LU-Decomposition of Block Tridiagonal Matrices. ROBERT M. M. MATTHEIJ, Rensselaer Polytechnic Institute and Katholieke Universiteit, The Netherlands

Contributed Papers. 3

Discrete Methods

Monday, 4:00 PM - 5:30 PM

Hinsdale-Elmwood Suite

Chairperson: THOMAS W. REILAND, North Carolina State University

4:00

Estimation of Sparse Hessian Matrices and Graph Coloring Problems. TOM COLEMAN, Cornell University; and JORGE J. MORE, Argonne National Laboratory

4:15

Positivity Sets. ROBERTA S. WENOCUR, Drexel University

4:30

Interchanges and Upsets in Round Robin Tournaments. RICHARD A. BRUALDI, University of Wisconsin; and LI QIAO, China University of Science and Technology and University of Wisconsin

4:45

Some Extremal Markov Chains. JAMES E. MAZO, Bell Laboratories

5:00

A Matroid Abstraction of the Bott-Duffin Constrained Inverse. SETH CHAIKEN, State University of New York at Albany

5:15

The Characterization of Directed Graphs with a Break Vertex. ARAM K. KEVORKIAN, General Atomic Company

Contributed Papers. 4

Numerical Methods

Tuesday, 11:00 AM - 12:30 PM

Cameron-Mordecai Suite

Chairperson: C. T. KELLEY, North Carolina State University

11:00

Comparison of Acceleration Techniques for Iteration Schemes Whose Iteration Operator Is a Nonnegative Matrix. GEORGE AVDELAS, University of Ioannina, Greece; JOHN DEPILLIS, University of Karlsruhe, West Germany and University of California,

Riverside; APOSTOLOS HADJIDIMOS, University of Ioannina, Greece; and MICHAEL NEUMANN, University of South Carolina

11:15

On a Partitioning Technique for the Problem $AX + XB = C$. JOHNNY VRANCKEN and ADHEMAR BULTHEEL, Katholieke Universiteit Leuven, Belgium

11:30

Recursive Computation of Triangular 2-D Padé Approximants. ADHEMAR BULTHEEL, Katholieke Universiteit Leuven, Belgium

11:45

The Bandwidth Problem for Asymmetric Matrices. PAUL G. EITNER, General Electric Company-Space Division

12:00

Two-Dimensional Nonlinear Wave Propagation in a Shallow Tidal Estuary. LOKENATH DEBNATH, East Carolina University; and ANADI KUMAR CHATTERJEE, Calcutta Port Trust, India

12:15

The Lanczos Algorithm with Partial Reorthogonalization. HORST D. SIMON, University of California, Berkeley

Contributed Papers. 5

Discrete Methods

Tuesday, 11:00 AM - 12:30 PM

Hinsdale-Elmwood Suite

Chairperson: CARLA SAVAGE, North Carolina State University

11:00

Algebraic Representation of a Path Counting Measure. GABOR LASZLO, University of Calgary, Canada

11:15

Construction of Comma-Free Codes. LARRY J. CUMMINGS, University of Waterloo, Canada

11:30

A Matrix Test for Graph Symmetries. DAVID L. POWERS and MOHAMMAD M. SULAIMAN, Clarkson College of Technology

11:45

The Observability of Measured Electric Networks: Topological and Algebraic Theory. KEVIN A. CLEMENTS, GARY R. KRUMPHOLZ and PAUL W. DAVIS, Worcester Polytechnic Institute

12:00

Lower Bounds for Partitioning Graphs. E. R. BARNES and A. J. HOFFMAN, IBM Thomas J. Watson Research Center

12:15

The K-Domination Problem on Sun-Free Triangulated Graphs. GERARD J. CHANG and GEORGE L. NEMHAUSER, Cornell University

Contributed Papers. 6

Statistical Methods

Tuesday, 11:00 AM - 12:30 PM

Haywood-Andrews Suite

Chairperson: JOHN F. MONAHAN, North Carolina State University

11:00

On the Minres Method of Factor Analysis. FRANKLIN T. LUK, Cornell University

11:15

The Vec and Vech Operators and Their Applications to Statistics. SHAYLE R. SEARLE, Cornell University

11:30

A Probabilistic Price Index Model, CARY WEBB, Chicago State University

11:45

Correlation and Determinacy in Linear Systems and Networks. J. SCOTT PROVAN, National Bureau of Standards

12:00

The Use of Matrix Series Expansions in Computing Posterior Moments in Bayesian Inference. THOMAS W. F. STROUD, Queen's University, Canada

12:15

Analysis of Deterministic Trends in Gravity Gradiometer Data. RICHARD PHELPS, Sperry Rand; and CURTIS BRUNSON, Shorecomp Systems Inc.

Contributed Papers. 7

Numerical Methods

Tuesday, 4:00 PM - 5:45 PM

Cameron Suite

Chairperson: ROBERT J. PLEMMONS, North Carolina State University

4:00

Numerical Solution of Quadratic Regulator Problems. CHRIS PAIGE, McGill University, Canada

4:15

The Analysis of k-Step Iterative Methods for Linear Systems from Summability Theory. W. NIETHAMMER University of Karlsruhe, West Germany; and RICHARD S. VARGA, Kent State University

4:30

Numerically Stable Alternatives to Prony's Method. GEORGE CYBENKO, Tufts University

4:45

On the Stability of a Highly-Concurrent Parallel Algorithm for Matrix Inversion. J. F. EASTHAM, JR., University of Delaware

5:00

Updating the LU Factorization of the Basis Matrix in Sparse Linear Programming. DONALD GOLDFARB, The City College of New York

5:15

Isolating Error Effects in Solving Ill-Posed Problems. C. MARK AULICK, Louisiana State University-Shreveport; and THOMAS M. GALLIE, Duke University

5:30

The Computer Generated Symbolic Cayley-Hamilton

Solution of the Matrix Equation $dx/dt=Ax$ and Related Problems. JAMES N. HANSON, A. B. BENANDER and V. BENANDER, Cleveland State University

Contributed Papers. 8

Statistical Methods

Tuesday, 4:00 PM - 4:45 PM

Mordercai Suite

Chairperson: THOMAS M. GERIG, North Carolina State University

4:00

A Practical Approach to Variable Selection in Linear Regression. ALAN H. FEIVESON, Johnson Spa Center

4:15

Matrix Methods for Compartmental Model Estimation Demographic Health State and Mortality Transition. KENNETH G. MANTON and ERIC STALLARD, Duke University

4:30

Generalized Inverses and Scheffe's Theorem. KARTIK PATEL, Mississippi State University

Contributed Papers. 9

Numerical Methods

Tuesday, 4:00 PM - 5:00 PM

Haywood-Andrews Suite

Chairperson: RICHARD J. PAINTER, Colorado State University

4:00

Matrix Methods for Queueing Problems. LINDA KAUFMAN, Bell Laboratories

4:15

Some Extensions of an Algorithm for Sparse Linear Least Squares Problems. MICHAEL T. HEATH, Union Carbide Corporation-Nuclear Division

4:30

Iterative Methods for Solving Bordered Systems. TOI F. CHAN and YUCEF SAAD, Yale University

4:45

Numerical Solution of the Interior Neumann Problem Via Vortex Dipoles. RAUL MENDEZ, University of California, Santa Cruz

Contributed Papers. 10

Discrete Methods

Tuesday, 4:00 PM - 5:45 PM

Hinsdale-Elmwood Suite

Chairperson: CHARLES R. JOHNSON, University of Maryland

4:00

Analysis of Routing in Computer Networks. MARIANNE L. GARDNER, ROBERT E. KINICKI, MARK R. OHLSON AND STANLEY M. SELKOW, Worcester Polytechnic Institut

4:15

On Balance Points in Trees. WAYNE G. KELLNER, Northeastern University

4:30

An Algorithm for Finding Two Disjoint Paths with Minimum Sum of Weights Between Two Pairs of Vertices in a Graph. YONG S. FANG, Academia Sinica, China and Ohio University; and ZONG X. HU, Shandong University, China and Ohio University

4:45

Reduced Matrices and Combinatorics. PIERRE LEROUX, University of Quebec at Montreal, Canada

5:00

On Recovering Voter Preferences from Pairwise Votes. LAWRENCE C. FORD, Andrews University; and ALAN M. WOLSKY, Argonne National Laboratory

5:15

Graph Theoretical Approach to Qualitative Solvability of Linear Systems. RACHEL MANBER, University of Washington

5:30

The Distance-Domination Numbers of Trees. WEN-LIAN HSU, Northwestern University

Contributed Papers. 11

Engineering/Control

Wednesday, 11:00 AM - 12:30 PM

Hinsdale-Elmwood Suite

Chairperson: STEPHEN L. CAMPBELL, North Carolina State University

11:00

On the Computation of the Impulse Response Energy of a Linear Multivariable System. P. ROZSA, Technical University of Budapest, Hungary and McMaster University, Canada; and N. K. SINHA, McMaster University, Canada

11:15

Strict Positive Invariance. RON STERN, Concordia University, Canada

11:30

An Algebraically Derived System Model for Nonlinear Systems. ANDREW J. FISH, JR., University of Hartford; and DAVID JORDAN, University of Connecticut

11:45

An Alternative to the Classical Method of Computing Frequency Response Functions, Suitable for Large-Scale Systems. D. L. LUKES, University of Virginia

12:00

Lambda Matrices with Asymmetric Coefficients with Application to Vibration Problems. D. J. INMAN, State University of New York at Buffalo

12:15

Linear Dynamic Output Feedback and Algebraic Module Theory. JACOB HAMMER, University of Florida

Contributed Papers. 12

Operations Research

Wednesday, 11:00 AM - 12:30 PM

Haywood-Andrews Suite

Chairperson: DAVID CARLSON, Oregon State University

11:00

The Ellipsoid Algorithm for Linear Inequalities in Exact Arithmetic. SILVIO URSIC, University of Wisconsin

11:15

Pursuit on a Cyclic Graph - The Symmetric Stochastic Case. WILLIAM H. RUCKLE, Clemson University

11:30

A Trellis Shortest Path Problem Approach to Quadratic Integer Programming. HARRY H. TAN, University of California, Irvine; and T. Y. YAN, Jet Propulsion Laboratory

11:45

Matrix Theory Via Optimization Methods. TES RAGHAVAN, University of Illinois at Chicago

12:00

Comparison of Heuristic and Optimal Solution Models for a Truck Routing Problem. LEONARD R. FREIFELDER and RAYMOND J. MARRA, University of Connecticut

12:15

Search for an Object Moving Under General Conditions in Discrete Time. MARK CIANCUTTI, Pennsylvania State University, Beaver Campus

Contributed Papers. 13

Core Linear Algebra

Wednesday, 11:00 AM - 12:00 N

Cameron-Mordecai Suite

Chairperson: GEORGE BARKER, University of Missouri, Kansas City

11:00

On the Matrix Congruence $O^{p+1} = I_2 \pmod{p}$. BARRY ZASLOVE, Northeastern University

11:15

The Class MPW of Generalized Matrix Functions. LEROY B. BEASLEY, Utah State University

11:30

A Different Approach to the Field of Values. FRANK UHLIG, Institut für Geometrie und Praktische Mathematik, Aachen, West Germany

11:45

On the Exponent of a Primitive, Nearly Reducible Matrix II. JEFFREY A. ROSS, University of South Carolina

Contributed Papers. 14

Engineering/Control

Wednesday, 4:00 PM - 5:30 PM

Hinsdale-Elmwood Suite

Chairperson: JOSEPH M. MAHAFFY, North Carolina State University

4:00

Generalized Controllability, Observability and (A, B) Invariant Subspaces in Control Problems. S. P. BHATTACHARYYA, Texas A&M University

4:15

A Perturbation Theory for Linear Control Problems. DANIEL BOLEY, University of Minnesota

4:30
Analysis of Rigid Body Displacement by Linear Algebra Techniques. ALAN J. LAUB and GEOFFREY R. SHIFLETT, University of Southern California

4:45
The Algebraic Geometry of Stresses in Frameworks. NEIL L. WHITE, University of Florida; and WALTER WHITELEY, Champlain Regional College, Canada

5:00
Algebraic Aspects of Second Order Optimization of Practical Feedback Control Laws for Multivariable Time-Invariant Linear Dynamic Systems. MICHAEL J. ROSSI, Grumman Aerospace Corporation

5:15
Controllability and Observability for Generalized Linear Systems. E. BRUCE LEE, STANISLAW H. ZAK, JOHN N. CHIASSON, and STEPHEN D. BRIERLEY, University of Minnesota

Contributed Papers. 15
Operations Research
Wednesday, 4:00 PM - 5:45 PM

Haywood-Andrews Suite

Chairperson: AVI BERMAN, Technion Israel Institute of Technology

4:00
Eigenvalues for Multi-Module Markov Decision Processes. JEFFREY LEE POPYACK, Washington State University

4:15
Implementation of a Double-Basis Simplex Method for the General Linear Programming Problem. PAUL E. PROCTOR, University of Arizona

4:30
Solving General Nonlinear Programming Problems with an Ellipsoid Algorithm. J. G. ECKER and M. KUPFERSCHMID, Rensselaer Polytechnic Institute

4:45
A Survey of Semi-Infinite Programming. K. O. KORTANEK, Carnegie-Mellon University

5:00
Equivalent Linear Programming Problems. RALPH DEMARR, University of New Mexico

5:15
Linear Programming and Dynamic Programming. L. C. M. KALLENBERG, University of Leiden, The Netherlands

5:30
Principal Pivot Transforms of Nonlinear Functions. MICHAEL M. KOSTREVA, General Motors Corporation

Contributed Papers. 16
Core Linear Algebra
Wednesday, 4:00 PM - 5:30 PM

Cameron-Mordecai Suite

Chairperson: ROBERT E. HARTWIG, North Carolina State University

4:00
On the Matrix Function $AX + X^T A^T$. P. LANCASTER, University of Calgary, Canada; and P. ROZSA, McMaster University, Canada and Technical University of Budapest, Hungary

4:15
Generalized Matrix Functions and Pattern Inventory. RUSSELL MERRIS, California State University, Hayward

4:30
Construction of Matrices with Prescribed Submatrices and Eigenvalues. GRACIANO NEVES DE OLIVEIRA, University of Coimbra, Portugal

4:45
An Application of Matrix Multiplication in Recursive Functions. YUAN-HSI HSU, East Carolina University

5:00
A Module Theoretic Algorithm for Computing the Interlacing Inequalities of the Invariant Factors of an R-Matrix $M_{n \times m}$ by an R-Matrix $M_{(n+1) \times m}$. PETER M. JOYCE, University of Kentucky

5:15
Determinants of Nonprincipal Submatrices of Normal Matrices. RAPHAEL LOEWY, Texas A&M University

Contributed Papers. 17
Core Linear Algebra
Thursday, 11:00 AM - 12:30 PM

Cameron-Mordecai Suite

Chairperson: GEORGE POOLE, Emporia State Univ.

11:00
Reflexive Algebras and Reflexive Lattices. GEORGE PHILLIP BARKER, University of Missouri-Kansas City

11:15
Distance Matrices and Graph Realizations. CHRISTINA M. ZAMFIRESCU, CUNY-Hunter College

11:30
Behavior of the Perturbed Lanczos Algorithm. ANNE GREENBAUM, Lawrence Livermore National Laboratory

11:45
On the Discrete Lyapunov Matrix Equation. MINH TH. TRAN, NRC Corporation; KOUROSH RAHNAMAI and MAHMOUD EL-SAYED SAWAN, Wichita State University

12:00
Rank Additivity and Pairwise Orthogonality of Real Square Matrices. GEORGE P. H. STYAN, McGill University, Canada

12:15
Determinantal Formulae for Matrices with Sparse Inverses. WAYNE W. BARRETT, Brigham Young University; and CHARLES R. JOHNSON, University of Maryland

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Haywood-Andrews Suite

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11:00
Optimum Design and Control of Target Oriented
Systems. A. M. MEYSTEI, University of Florida

11:15
A Transformation Between Two Block Companion Forms.
J. MAROULAS, National Technical University of
Athens, Greece

11:30
Modelling with Integer Variables. R. G. JEROSLOW and
J. K. LOWE, Georgia Institute of Technology

11:45
Diagonal Equivalence of Matrices - A Survey. ABRAHAM
BERMAN, Technion-Israel Institute of Technology

12:00
Nonsymmetric Distance Matrices and Their
Realizations by Digraphs. J. M. S. SIMOES-PEREIRA,
Hunter College and the City University of New York

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Eigenvector Methods for Efficient Representation of
Waveforms in an Emulation of Infrared Detection.
RICHARD H. BURKHART, Boeing Computer Services

11:15
Solving Word Problems in Initial Algebras by Using
Complexity Classes Over N.
ALEX PELIN, Temple University

11:30
"Contravariant" and "Covariant" Linear
Transformations. DENNIS GLENN COLLINS,
Valparaiso University

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An Efficient Decoding Algorithm for a General Linear
Error-Correcting Code. LEV B. LEVITIN and CARLOS
R. P. HARTMANN, Syracuse University

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Detecting Mechanisms with Linear Algebra. WALTER
WHITELEY, Champlain Regional College, Canada

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A Transition Matrix for a Markov Chain. JAMES
WEAVER, University of West Florida

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University

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Scalar Polynomials Permitting the Matrices Over a
Field. J. V. BRAWLEY and GEORGE SCHNIBBEN, Clemson
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2:45
Singular Linear Maps Which Preserve Affine
Independence on Small Subsets.
R. E. JAMISON-WALDNER, Clemson University

3:00
Partitions of Vector Spaces and Single Error
Correcting Perfect Codes. PETER TANNENBAUM,
University of Arizona

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Identity Manipulation by Vector Spaces. IRVIN ROY
HENTZEL, Iowa State University

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State University

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Similarity with the Maximized Correlation. KUANG-HO
CHEN, California State University and Stanford
University Medical Center

2:45
Volume Correlation - With An Application to Air
Traffic Surveillance. AUBREY H. PAYNE, Raytheon
Company

3:00
Numerical Perturbation Methods for Degenerate
Nonlinear Systems. PETER NWOYE O. MBAEYI,
University of Tuebingen, West Germany

3:15
Concurrence Geometries. HENRY CRAPO, University of
Montreal

3:30
The Solution of the Cover Problem. A. C. ANTOULAS,
Swiss Federal Institute of Technology

Poster Sessions

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Linear Programming with an Array Processor. RONALD D.
COLEMAN and EDWARD J. KUSHNER, Floating Point
Systems, Inc.

High-Order, Fast-Direct Methods for Separable
Elliptic Equations. LINDA KAUFMAN, Bell

Laboratories; and DANIEL D. WARNER, Clemson
University

An Example of Path-Following in a Subspace. R. MEJIA,
National Institutes of Health

On the Solution of Almost Block Diagonal Linear
Systems. PATRICK KEAST, University of Toronto; and
GRAEME FAIRWEATHER, University of Kentucky

An Efficient Method to Invert Matrices with a Minimum Storage Requirement. KANAT DURGUN, University of Arkansas at Little Rock

An Algorithm to Obtain a Submatrix and/or Permutations of a Sparse Matrix. DANIEL B. SZYLD, New York University

Poster Session. 2

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Tucker Suite

Interval Arithmetic and Hansen's Problem 3 Revisited. MARIETTA J. TRETTER, Texas A&M University; and G. W. WALSTER, Lockheed Corporation

Computing the Probability That a Planar Graph is Balanced. FRANCISCO BARAHONA, University of Chile

Computing the Crossover Metric for Unordered Classification Trees. J. P. JARVIS, DOUGLAS R. SHIER AND JOHN K. LUEDEMAN, Clemson University

Statistical Computations Using Perpendicular Projections Operators. MARVIN S. MARGOLIS, Millersville State College

Estimating Parameters of Positive Semi-Definite Quadratic Form. ERIC STALLARD, MAX A. WOODBURY and KENNETH G. MANTON, Duke University

Poster Session. 3

Wednesday, 4:00 PM and Thursday, 11:00 AM

Tucker Suite

Practical Use of Krylov Subspace Methods for Solving Large Indefinite and Unsymmetric Linear Systems. YUCEF SAAD, Yale University

Singular Value Decomposition and the Solution of the Radiative Transfer Equation. ALAN H. KARP, IBM Scientific Center, Palo Alto

Solving Equations by Row and Column Reduction. NORMAN M. RICE, Queen's University, Canada

Dimensionality of Bi-Infinite Systems. P. W. SMITH, Old Dominion University; and HENRY WOLKOWICZ, University of Alberta, Canada

Application of a Homotopy Method to a Problem of Scarf's. ARNOLD LAPIDUS, Fairleigh Dickinson University

A Very Small Numerical Linear Algebra Algorithm Package. JOHN C. NASH, University of Ottawa

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Abstracts: Invited Speakers

Recent Developments in Numerical Linear Algebra

In this talk, some of the latest advances in numerical linear algebra will be described. In particular, the solution of sparse systems of equations by direct and iterative methods, the generalized eigenvalue problem and some problems arising from applications will be discussed. In addition, several important classes of unsolved problems will be given.

Gene H. Golub
Department of Computer Science
Stanford University
Stanford, CA 94305

Displacement Ranks of Matrices and Fast Algorithms for Signal Processing

In the final analysis, most signal processing problems reduce to solving a set of linear equations.

It takes $O(N^3)$ elementary operations to solve linear equations with an $N \times N$ coefficient matrix, which can be prohibitively large in many applications. While the assumption of sparseness often helps to reduce the computational burden, in signal processing applications the assumption of a Toeplitz coefficient matrix is more natural. It turns out that some classical function-theoretic results of Schur (1917) and Szego (1920) can be exploited to obtain a reduction in complexity to $O(N^2)$ -- via a recursive so-called Levinson Szego-Schur algorithm. Use of 'divide and conquer' ideas has also recently yielded a nonrecursive $O(N \log N \log N)$ algorithm (Brent, Gustavsson, Yu). Moreover, the recursive algorithm can be implemented by a so-called lattice filter, which has several properties that can aid both software and hardware realizations. This is valuable, but in many problems, it turns out that the coefficient matrices are not Toeplitz, though we would expect them to be close to Toeplitz in some sense (e.g., in least-squares problems, the coefficient matrix is often a product of rectangular Toeplitz matrices). It turns out that this notion can be quantified by introducing the concept of displacement ranks of matrices. This concept allows a natural extension of both the recursive and nonrecursive Toeplitz algorithms and their lattice filter implementations. These results will be discussed, along with recently discovered connections to the work of Livsic, Nagy-Foias and others on dilations of contractive operators and interpolation theory.

Thomas Kailath
Information Systems Laboratory
Stanford University
Stanford, CA 94305

Linear Algebraic Methods in Statistics: A Review of Recent Contributions to Theory and Computations

The following topics will be discussed:

- (i) Recent results on generalized inverse of matrices and the unification achieved in statistical theory and computations relating to linear estimation, multivariate normal theory, distribution of quadratic forms, regression, partial and multiple correlation coefficients.
- (ii) Separation theorems for singular values of a matrix, similar to the Poincaré separation theorem for the eigenvalues of a Hermitian matrix, and matrix approximations by minimizing unitarily invariant norms. The results will be applied to problems of approximating one random variable by another in a specified class. In particular, problems of canonical correlations, reduced rank regression, fitting of an orthogonal random variable and estimation of residuals in a Gauss-Markoff model will be discussed.
- (iii) Recent results on projections under seminorm and a generalized projection operator when the complementary spaces do not span the whole space will be used to discuss problems in linear estimation and analysis of variance.
- (iv) Generalization of the Kantorovich inequality and its application to estimation problems in linear estimation.

C. Radhakrishna Rao
University of Pittsburgh
Pittsburgh, PA 15261

Path Problems

Gaussian elimination and similar techniques of linear algebra solve not only systems of linear equations but a host of other problems. These problems include finding shortest paths in a graph, converting finite automata into regular expressions, and performing global flow analysis of computer programs. In this talk I will describe a unified framework for these path problems based on regular expressions, examine how various kinds of path problems fit into the framework, and describe a number of combinatorial techniques useful in solving such problems.

Robert E. Tarjan
Bell Laboratories
600 Mountain Avenue
Murray Hill, NJ 07974

This talk will survey some recent developments in linear algebra, examining results with evident degrees of "purity" but with enough unexpected structure to interest nonspecialists. Topics to be mentioned include doubly stochastic matrices and reflection groups, singular value inequalities, matrices of algebraic integers and their invariant factors, determinantal bounds, matrix norms, the triangle inequality, the numerical range, and the recent solutions to the van der Waerden problem. The selected topics, some of which are already in the published literature and some not, will be tied to one another with nontrivial degrees of tightness, and while each is "pure", it will be claimed that each has at least some applied significance.

R. C. Thompson
Department of Mathematics
University of California
Santa Barbara, CA 93106

A Survey of Recent Results in Numerical Linear Algebra

This talk will present a brief survey of recent results on the following topics in numerical linear algebra: i) M-matrix theory and applications; ii) generalizations of the Stein-Rosenberg Theorem; iii) an update on ω - and τ -matrices; iv) k-step iterative methods from the point of view of summability methods.

Richard S. Varga
Institute for Computational Mathematics
Kent State University
Kent, OH 44242

The ideas and methods of linear algebra and their interplay with geometry (especially convex analysis, graph theory, and other parts of algebra) permeate the theory and applications of nearly all areas of operations research. This is not just because the results of linear algebra are heavily used in operations research -- though they are, of course -- but more importantly because many fundamental questions arising in operations research have required the formulation and solution of new problems in linear algebra.

The role of linear algebra in operations research as well as some future needs will be illustrated with examples drawn from several areas under active development in recent years. In each case, the significance of the problem in operations research and its linear-algebraic aspects will be reviewed. The areas discussed will include linear programming (pivoting vs. iterative methods, polynomial-time algorithms, large-scale systems), equilibrium programming (complementary pivoting to achieve specified sign patterns in matrix element matrices with positive principal minors and other matrices for which the linear complementarity problem has a solution, polymatrix games, fixed points), combinatorial programming (graphs and their node-arc incidence matrices, independence and matroids, characterization of polyhedral sets having integral extreme points by unimodularity and total unimodularity of coefficient matrices and their discovery, computational complexity), parametric programming (matrices over the ordered field of rational functions), lattice programming (representation of polyhedral semisublattices and sublattices by linear inequalities having pre-Leontief and generalized-incidence transposed coefficient matrices, characterization of twice-differentiable subadditive functions by pre-Leontief Hessian matrices), and branching Markov decision chains (products of nonnegative matrices majorized by a polynomial in the number of matrices and their characterization in terms of spectral radii, linear inequalities, and positive similarity to class-substochastic matrices; characterization of degree of majorizing polynomial by linear inequalities and length of longest chain in class graph; expansions of Abel and Cesàro sums of matrix powers in terms of projection and deviation matrices).

Arthur F. Veinott, Jr.
Stanford University
Stanford, CA 94305

Abstracts: Selected Speakers

Graph Coloring Using Eigenvalue Decomposition

Determining whether the vertices of a graph can be colored using k different colors so that no two adjacent vertices receive the same color is a well-known NP-complete problem. Graph coloring is also of practical interest (for example, in estimating sparse Jacobians and in scheduling), and many heuristic algorithms have been developed. We present a heuristic algorithm based on the eigenvalue decomposition of the adjacency matrix of a graph. Eigenvectors point out "bipartite-looking" subgraphs that are used to refine the coloring to a valid coloring. The algorithm optimally colors complete k -partite graphs and certain other classes of graphs with regular structure. Using perturbation arguments, we argue that the algorithm colors a wide range of graphs well.

Bengt Aspvall

John R. Gilbert

Department of Computer Science
Cornell University
Ithaca, NY 14853

Numerical Solution of the Weighted Linear Least Squares Problem by G-Transformations

Based on a new type of orthogonal transformations (the H-transformations) a family of algorithms (the G-algorithms) is presented which solve the weighted least squares problem $(Ax-b)'W(Ax-b)=\rho^2$ for x such that ρ^2 is minimal. A sequence of G-transformations triangularizes A and yields ρ^2 as a by-product, quite similar to the Householder-Golub procedure for the unweighted problem, but the transformation matrices are sparser, and no square roots are required. J.E. Hansen (MS-Thesis, Northwestern University 1981, 163pp.) compared the *Basic G-Algorithm* with the Householder, modified Gram-Schmidt, LU-factorization (and the normal equation) methods: "The results indicate that the use of G-transformations to solve the linear least squares problem yields the same accuracy as the best available methods, requires fewer operations, provides greater flexibility such as adding rows or columns, and is storage efficient."

The *partial G-transformations* operate only on a partition of a matrix and with reduced efficiency. If the partitions are always two rows, they become Gentleman's Fast Givens Methods.

The G-transforms have many other applications.

Prof. Erwin H. Bareiss

Departments of Electrical Engineering and Computer Science, Engineering Science and Applied Mathematics

The Technological Institute
Northwestern University
Evanston, Illinois 60201

Controllability, Inertia, and Stability for Tridiagonal Matrices

Known results on controllability, inertia, and stability of tridiagonal matrices are surveyed. In particular, criteria are given for the controllability of certain pairs of tridiagonal matrices. These criteria are used, with the Chen-Wimmer Theorem, to obtain a variety of stability and inertia results. Also, we discuss a characterization of nonsingular tridiagonal matrices with certain principal minors nonnegative which are positive stable, and a characterization of the real D-stable tridiagonal matrices.

David Carlson

Mathematics Department
Oregon State University
Corvallis, Oregon 97331

Note on Global Perturbations of Hermitian Matrices and Related Problems

1. Given Hermitian matrices A and B with eigenvalues $\alpha_1 \geq \alpha_2 \geq \dots \geq \alpha_n$ and $\beta_1 \geq \beta_2 \geq \dots \geq \beta_n$. We give complete descriptions of the eigenvalues of the $n \times n$ Hermitian matrices of the form $A + UBU^*$ (where U runs over the set of unitary matrices), in the following cases:
 - (a) When the speed of B is "small" with respect to A , more precisely, when

$$\beta_1 - \beta_n \leq \min_{1 \leq i < n} (\alpha_i - \alpha_{i+1});$$
 - (b) When A and B are partial isometries.
2. The relationship with similar problems is stressed. These problems involve singular values of complex matrices and invariant factors of integral matrices. Some open problems are stated.

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Differential Equations and the Symmetric Eigenvalue Problem

Let $(a_k(t), b_k(t))$ be the solution of

$$(1) \quad \frac{d}{dt} a_k(t) = 2(b_k^2 - b_{k-1}^2), \quad 1 \leq k \leq n,$$

$$\frac{d}{dt} b_k(t) = b_k(a_k - a_{k+1}), \quad 1 \leq k \leq n-1,$$

$(b_0 \equiv b_n \equiv 0)$ with initial conditions

$a_k(0) = a_k, b_k(0) = b_k$. Then the tridiagonal matrix

$$\begin{pmatrix} a_1(t) & b_1(t) & & & 0 \\ b_1(t) & a_2(t) & b_2(t) & & \\ & \ddots & \ddots & \ddots & \\ & & \ddots & \ddots & b_{n-1}(t) \\ & & & b_{n-1}(t) & a_n(t) \\ 0 & & & & \end{pmatrix},$$

converges to $\text{diag}(\lambda_1, \dots, \lambda_n)$,

where $\lambda_1 > \lambda_2 > \dots > \lambda_n$ are the eigenvalues of the initial matrix

$$\begin{pmatrix} a_1 & b_1 & & & 0 \\ b_1 & a_2 & b_2 & & \\ & \ddots & \ddots & \ddots & \\ & & \ddots & \ddots & b_{n-1} \\ 0 & & & b_{n-1} & a_n \end{pmatrix}.$$

In other words we have an algorithm to calculate the eigenvalues $\{\lambda_1, \dots, \lambda_n\}$ of a tridiagonal matrix

$$\begin{pmatrix} a_1 & b_1 & & & 0 \\ b_1 & a_2 & b_2 & & \\ & \ddots & \ddots & \ddots & \\ & & \ddots & \ddots & b_{n-1} \\ 0 & & & b_{n-1} & a_n \end{pmatrix} :$$

solve (1) subject to $a_k(0) = a_k, b_k(0) = b_k$.

Then $a_k(t)$ converges as $t \rightarrow \infty$ to λ_k ,

$1 \leq k \leq n$.

The QR algorithm can also be understood as (an isospectral) flow. In general there is a correspondence between algorithms to calculate eigenvalues and vector fields on the isospectral manifold of tridiagonal matrices with fixed spectrum.

The method extends to general (not necessarily tridiagonal, not necessarily self-adjoint) matrices. Numerical comparisons of (1) with RATQR of EISPACK are given and are very encouraging.

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Nested Bounds for the Perron Root of a Nonnegative Matrix

It is known that for a nonnegative matrix A the smallest row sum $R'(A)$ and the largest row sum $R''(A)$ provide lower and upper bounds, respectively, for the Perron root of A . These bounds are generalized for a partitioned nonnegative matrix A . The new bounds are better than $R'(A)$ and $R''(A)$ and they get better when one switches to a refined partitioning. Known monotonicity and convergence properties of $R'(A)$ and $R''(A)$ are generalized for the new bounds.

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Roots of M-Matrices and Generalizations

We introduce generalizations of Z-matrices and M-matrices associated with certain cones K matrices which we call positivity cones. Thus each K we define the sets $\tilde{Z} = \tilde{Z}(K)$ and $\tilde{M} = \tilde{M}(K)$.

Theorem: Let K be a positivity cone and let $A \in \tilde{M}$. Let p be a real number, $p \geq 1$. Then $B = A^{1/p}$ is the unique matrix for which $B^p = A$. B, B^2, \dots, B^m belong to \tilde{M} , where m is the integer satisfying $p/3 \leq m < (p/3) + 1$.

Corollary. Let $A \in \tilde{M}$ and let $1 \leq p \leq 3$. Then $A^{1/p}$ is the unique matrix in \tilde{M} for which $B^p = A$.

If \tilde{M} is the set of positive definite matrices, roots within \tilde{M} are unique. By means of an example we show that for $n \geq 3$ and $p > 12$ there exist distinct non-singular $n \times n$ M-matrices B and C such that $B^p = C^p$ is again an M-matrix. Als

give an example of positivity cone K such that for the corresponding set M there is an $A \in M$ with at least $2^{(n-1)}$ matrices $B \in M$ with $BA = A$, provided $p > 3$.

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LU Decompositions of Generalized Diagonally Dominant Matrices

Using the simple vehicle of M-matrices, the existence and stability of LU decompositions of matrices A which can be scaled to diagonally dominant (possibly singular) matrices are investigated. Bounds on the growth factor for Gaussian elimination on A are derived. Motivation for this study is provided in part by applications to solving homogeneous systems of linear equations $Ax = 0$, arising in Markov queueing networks, input-output models in economics and compartmental systems, where A or $-A$ is an irreducible, singular M-matrix.

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Computing Component Bounds in Errors-in- Variables Problems

Linear regression (least-squares) problems arise in many contexts. In these problems, one has a design matrix, X , and a right-hand side, y , and one wishes to find a parameter vector, $\beta = X^+y$, that minimizes the sum of the squares of the components of $y - X\beta$. Often the rows of X represent exogenous variables or the control variables in an experiment, and the components of y represent corresponding response variables. The components of X and y thus are often measured data and are subject to error. In contexts where reliable bounds on the errors in these data are known but no reasonable stochastic assumptions about these errors can be made, it may be of

interest to determine the range values possible for the components of β . That is, given appropriate Cartesian products X and y , it may be of interest to compute bounds on the components

of $\{X^+y | X \in X, y \in y\}$. This paper describes an efficient method for doing so and reports some computational experience with it.

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Positive Definite Toeplitz Matrices, The Hessenberg Process for Isometric Operators, and Gaussian Quadrature on the Unit Circle

We show that the well-known Levinson algorithm for computing the inverse Cholesky factorization of positive definite Toeplitz matrices can be viewed as a special case of a more general process. The latter process provides a very efficient implementation of the Hessenberg process when the underlying operator is isometric. This is analogous with the case of Hermitian operators where the Hessenberg matrix becomes tridiagonal and results in the Hermitian Lanczos process. We investigate the structure of the Hessenberg matrices in the isometric case and show that simple modifications of them move all their eigenvalues to the unit circle. These eigenvalues are then interpreted as abscissas for analogs of Gaussian quadrature, now on the unit circle instead of the real line. The trapezoidal rule appears as the analog of the Gauss-Legendre formula.

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Food Webs, Competition Graphs, and Associated Matrices

Recently, Roberts and Cohen have investigated the dimension of ecological phase space by studying the competition graph of a food web. Mathematically we represent a food web by an acyclic digraph D . Then, if A is the adjacency matrix for D , the competition graph is $RG(A)$, the row graph studied recently by the authors. Most competition graphs arising from food webs are interval graphs, but the reason for this has not been explained either mathematically or ecologically. This ecology problem raises several interesting mathematical questions which we investigate here. What graphs are competition graphs, and are almost all competition graphs interval graphs? $CG(A)$, the column graph of A , can be interpreted ecologically as a common enemy graph and adds a duality to the theory. Is there a special significance to the case where both $RG(A)$ and $CG(A)$ are interval graphs? To answer

these questions, we determine the structure of the adjacency matrix A of an acyclic digraph D , and, more generally, we investigate the relationship between D and the structure of A , $RG(A)$, and $CG(A)$.

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Combinatorial Aspects of Matrix Problems

Analysis of a growing list of problems in matrix theory involves a major combinatorial feature. Certain of these combinatorial aspects are to be expected (e.g. sparse matrix problems, qualitative matrix theory and other problems involving sign patterns), and others are rather unexpected. Working interest in all such problems seems decidedly on the increase, due to the nature of motivating applications, tastes and most recently a desire to better understand the nature of and connection among such problems.

Here we give an informal survey of a variety of such work. This includes 1) the characterization of $+$, $-$ sign patterns which occur among the inverses of positive matrices; 2) refined spectral location results based upon the graph of a nonnegative matrix; 3) determinantal optimization problems which rely upon reduction to a special (finite) class of matrices; 4) determination of those singular M -matrices have L - U factorizations into M -matrices, which turns out to depend only upon the O -pattern outside the singular components; and 5) special determinantal formulae for matrices with sparse inverses. These are only a few examples from a long list.

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Linear Algebra in Digital Image Processing: Noise Smoothing and Image Compression

A digitized black and white image may be considered as a matrix with positive integer entries corresponding to measured gray levels. Such measurements are often noisy, and two main problems in image processing are the filtering of noise and the economization of storage.

Two algorithms for noise smoothing, one based on an optimization approach and one based on a relaxation algorithm, will be described. Implementation issues, including multiresolution iterations ("multigrid") will be discussed and sample results shown.

Storage can be reduced through approximation of an image by an outer product expansion of low rank. The expansion which minimizes the sum of squares of deviations over all approximations of given rank is the truncated singular value decomposition, but the range of elements in the outer product vectors makes storage expensive. A step-wise procedure has been developed which still uses the least squares criterion, but which restricts the vectors to have elements equal to $+1$, -1 , or 0 only. Both the decomposition and the reconstructed image can be computed in moderate cost, and examples will be given showing compression to 1 bit per matrix element with minimal loss of resolution.

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Iterative Methods for Variational and Complementarity Problems

In this paper, we study both the local and global convergence of various iterative methods for solving the variational inequality and the nonlinear complementarity problems. Included among such methods are the Newton and several successive overrelaxation algorithms. For the most part, the study is concerned with the family of linear approximation methods. These are iterative methods in which a sequence of vectors is generated by solving certain linearized subproblems. Convergence to a solution of the given variational or complementarity problem is established by using three different yet related approaches. Finally, several convergence results are obtained for some nonlinear approximation methods.

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Growth Optimality for Branching Markov Decision Chains

This paper considers a (multiplicative) process called branching Markov decision chains in which the output at the end of the N -th period equals the product of N nonnegative matrices chosen at the beginning of periods $1, \dots, N$, respectively, times a positive (fixed) terminal reward vector. It is assumed that the above transition matrices are drawn out of a finite set of matrices given in product form (i.e., the rows of the matrices can be selected independently out of finite sets of nonnegative row vectors). For each coordinate s we define the geometric and algebraic growth rates, respectively, of the s -th coordinate of the stream of output. The main result of this paper is the constructive establishment of the existence of a transition matrix whose repeated use will guarantee, for each coordinate, the achievement of the best geometric growth rate and the best algebraic growth rate subject to the geometric growth rate at maximum, among all potential sequences of transition matrices.

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Studies in Reduced-Order Modeling and in Singular Dynamic Systems

Recent research in two areas of systems and control theory has yielded a variety of interesting results in linear algebra. The first area involves reduced-order modeling of large dynamic system models of the form $\dot{x}' = Ax$, to reproduce to a desired accuracy a few selected modes of the large system, with models involving only a small subset of the original state variables that has been identified as significant in constructing the selected modes. We present ideas on partitioning A for the above purpose, and results on various properties of the "Schur complement" of the partitioned form of $sI - A$, leading finally to iterative procedures for constructing the desired reduced-order models. The second area concerns models of the form $E\dot{x}' = Ax + Bu$, $y = Cx$, where E is square but singular and $sE - A$ is nonsingular, and where u, y are vectors of system inputs and outputs. Such "singular models" arise naturally in practice. We show that they may contain natural frequencies at infinity, corresponding to infinite zeros of $sE - A$; the system then exhibits free-response behavior that is impulsive. The structure of such systems is exposed, and the importance in system theory of structure at infinity in the generalized eigenvalue problem is emphasized.

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Abstracts: Contributed Papers and Poster Sessions

The Solution of the Cover Problem

Given a field k , let U, W, Y be subspaces of k^n , and F a k -endomorphism of k^n . Find all subspaces V of k^n such that the following inclusions are satisfied:
 (*) $FV \subseteq V + U$; (**) $W \subseteq V$; (***) $V \subseteq Y$.

Subspaces which satisfy (*) are called F-invariant modulo U; they are a generalization of the classical F-invariant subspaces. First introduced about a decade ago, they proved of central importance in linear system theory.

If $U = 0$, the solution of (*) - (***) is well known. It is based on the fact that F-invariant subspaces form a sublattice of the lattice of all subspaces of k^n (under subspace addition and subspace intersection). If $U \neq 0$, the problem becomes non trivial. The difficulty arises from the fact that F-invariant subspaces modulo U, do not form a sublattice of the lattice of all subspaces of k^n .

We will show, that if (*) - (***) is solvable, V is a solution iff it is the state space of some realization of a sequence of matrices, which is constructed from the data F, U, W, Y .

The problem is of great interest because it is a typical example of how a system-theoretic concept, in this case the concept of realization, provides the key to the solution of a linear algebraic problem, namely (*) - (***) .

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Isolating Error Effects in Solving Ill-Posed Problems

Many ill-posed problems are reduced to a matrix equation, usually very ill-conditioned, which is then solved using the smoothing techniques of regularization. Any such smoothing will introduce bias into the calculated solution in the sense that if the data were exact, the calculated solution will not be the "exact" solution. Since this calculated solution is also affected by error in the data, we show how these two error effects may be isolated and considered separately. Using a very general form of the regularization technique, we derive exact formulas for each error component which

illustrates the dependence of each upon the different variables and parameters of the problem.

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Comparison of Acceleration Techniques for Iteration Schemes whose Iteration Operator is a Nonnegative Matrix

For a first degree iteration scheme

$$x_i = Bx_{i-1} + c, \quad (1)$$

where $B \geq 0$ and where the spectral radius of B , $\rho(B)$, is less than 1, two parameter dependent acceleration techniques can be considered:

$$y_i = [(1-\alpha)I + \alpha B]y_{i-1} + \alpha c \quad (2)$$

and

$$z_i = (1-\beta)Bz_{i-1} + \beta z_{i-2} + \beta c. \quad (3)$$

These are known as the extrapolation and the sec degree acceleration schemes for (1), respectively. We compare the relative optimal performance of (2) and (3) via the following observation: If C_1, C_2

are two $n \times n$ nonnegative matrices such that $\rho(C_1 + C_2) < 1$, then

$$\rho(C_1 + C_2) \leq \rho \begin{pmatrix} C_1 & C_2 \\ I & 0 \end{pmatrix}.$$

It is shown that if the index of cyclicity of B is greater or equal to 3, scheme (2) will have a least as favourable optimal asymptotic convergence rate as scheme (3).

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Computing the Probability that a Planar Graph is Balanced

Given a planar graph, a stochastic model is considered in which each edge may take on either of two signs: positive or negative. The state of an edge is a random event that is independent of the state of any other edge. The positive sign is taken with probability P and the negative sign with probability $1-P$. The resulting signed graph is called balanced if each of its cycles includes an even number of edges with negative sign. A polynomial algorithm is presented to compute the probability that the graph is balanced.

For reliability problems, this measure gives a lower bound to the probability that a graph is disconnected.

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Reflexive Algebras and Reflexive Lattices

We begin with the direct sum of algebras and show that the direct sum of reflexive algebras is reflexive. As a corollary we note that a semisimple algebra is reflexive. Some specialized results on commutative algebras are obtained. Finally we give an algebraic, rather than functional analytic proof, of a result on complemented subspace lattices which is due to Harrison and Longstaff.

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Lower Bounds for Partitioning Graphs

Let G be an undirected graph having n nodes and E edges. We consider the problem of partitioning the nodes into k nonoverlapping sets S_1, \dots, S_k of given sizes $|S_i| = m_i, i = 1, \dots, k$, in such a way that the number of edges connecting nodes in different sets is minimized. Let E_c denote the number of such edges. Let a_{ij} be the number of edges connecting nodes i and j for $i \neq j$, and let $a_{ii} = -\sum_{j \neq i} a_{ij}$. Let A denote the $n \times n$ matrix (a_{ij}) . We obtain lower bounds on E_c in terms of a few eigenvalues and eigenvectors of A . Let $\lambda_1 \leq \dots \leq \lambda_n = 0$ denote the eigenvalues of A in ascending order. Let u_1, \dots, u_n denote a corresponding set of ortho-

normal eigenvectors. We show that

$$E_c \geq -\frac{1}{2} \sum_{i=2}^k \lambda_i m_i - \frac{1}{2} \sum_{i=2}^k \lambda_i (m_{i-1} - m_i) \frac{m_1 + \dots + m_k}{n},$$

where the notation is chosen such that $m_1 \geq m_2 \geq \dots \geq m_k$. This bound depends only on

$\lambda_1, \dots, \lambda_k$ and u_1 . If more eigenvalues and eigenvectors of A are used sharper bounds can be obtained.

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Determinantal Formulae for Matrices with Sparse Inverses

The determinant of matrix R is expressed in terms of certain of its principal minors by a formula which can be "read off" from the graph of the inverse of R . The only information used is the zero pattern of the inverse and each zero pattern yields one or more corresponding formulae for $\det R$. The main tool used is the formula for minors of the inverse matrix (see Chantal Shafroth, A Generalization of the Formula for Computing the Inverse of a Matrix, The American Mathematical Monthly, Volume 88, Number 8, 1981, pp. 614-616 or Gantmacher, Matrix Theory, vol I, p. 21). This generalizes a determinant formula in Barrett and Feinsilver, Inverses of banded matrices, Linear Algebra and its Applications, to appear.

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The Class MPW of Generalized Matrix Functions

Let S_n denote the set of all permutations of $\{1, 2, \dots, n\}$. Let G be a subgroup of S_n and let χ be an irreducible character of G . The generalized matrix function, d , is defined on an $n \times n$ matrix $A = (a_{ij})$ by

$$d(A) = \sum_{\sigma \in G} \chi(\sigma) \prod_{i=1}^n a_{i, \sigma(i)}.$$

We say that d is in class MPW if $d(A) \det(A^{-1}) = d(A^{-1}) \det(A)$ for every invertible matrix A .

THEOREM: d is in class MPW if and only if either
 i) $G = S_n$ and χ is unsaturated (i.e., χ corresponds to a partition (p_1, \dots, p_s) with

$2 \geq p_1 \geq p_2 \geq \dots \geq p_s$); ii) for some $\sigma \in S_n$
 $\sigma^{-1}G\sigma = S_{n_1} \oplus S_{n_2}$ with $n_1 + n_2 = n$ and $\chi = \mathcal{E}$,
 the signum; or iii) for some $\sigma \in S_n$, $\sigma^{-1}G\sigma =$

$\langle S_{n/2} \oplus S_{n/2}, \tau \rangle$, where $\tau(i) = n/2 + i$ for
 $i \leq n/2$ and $\tau(j) = j - n/2$ for $j > n/2$, and χ is
 essentially the signum.

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Diagonal Equivalence of Matrices - A Survey

Two matrices, A and B , are diagonally equivalent if $B = DAE$, where D and E are nonsingular diagonal matrices. Scaling of a matrix A , that is, choosing a matrix B which is diagonally equivalent to it, is of importance in solving linear equations or linear programs.

When $E = D^{-1}$ we say that A and B are diagonally similar. This equivalence is of interest, for example, in eigenvalue problems.

In data adjustment, say in transportation problems, one wants to scale a nonnegative matrix to one with prescribed row and column sums. In other applications, which will be described in the talk, one wishes to scale a given matrix to a symmetric or orthogonal one.

In this talk we shall describe the matrix and graph theoretical aspects of diagonal equivalence, diagonal similarity and optimal scaling, and survey several algorithms, classical and recent, derived from them.

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Generalized Controllability, Observability and (A, B) Invariant Subspaces in Control Problems

The concept of generalized controllability introduced by Carlson and Hill "Generalized Controllability and Inertia Theory" (Lin. Alg. and Its Applic., 15, 177-187 (1976)) is extended here to apply to a triple $(\underline{C}', \underline{A}, \underline{B})$ with $\underline{C}' = \{C'_i/C'_i \in \mathbb{C}^{m \times n}, i = 1, 2, \dots, p\}$, $\underline{A} = \{A_i/A_i \in \mathbb{C}^{n \times n}, i = 1, \dots, q\}$, $\underline{B} = \{B_i/B_i \in \mathbb{C}^{n \times r}, i = 1, \dots, s\}$. The generalized controllable and unobservable subspaces of this triple are defined and calculated and some of their properties presented. The concept of $(\underline{A}, \underline{B})$ invariant subspaces is introduced as a natural extension of the notion of (A, B) invariant subspaces introduced by Wonham (Linear Multivariable

Control: A Geometric Approach, Springer-Verlag 1979). The role of these objects in the solution of control problems subject to structured parameter perturbations is illustrated by solving the parameter invariant disturbance rejection problem which extends previous results on this problem (Bhattacharyya "Parameter invariant observers" Int. J. Control, Vol. 32, no. 6, 1127-1132, Dec 1980).

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A Perturbation Theory for Linear Control Problems

A perturbation theory is sketched for a method to compute the controllable part of a linear system of the form

$$\dot{x} = Ax + Bu,$$

where A, B are matrices, u, x are the input control vector and state vector, respectively. The well-known method based on reducing A to pseudo-upper-Hessenberg form, is extremely sensitive to small perturbations in the coefficients. A posteriori bounds on the sensitivity of results obtained by this method are developed, giving sufficient conditions for such results to be robust, i.e., insensitive to perturbations in the initial parameters.

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Scalar Polynomials Permitting the Matrices Over a Field

Let F denote an arbitrary field. Let $F_{n \times n}$ denote the algebra of $n \times n$ matrices over F . Each scalar polynomial $f(x) \in F[x]$ defines, under substitution, a function from $F_{n \times n}$ to $F_{n \times n}$. This paper determines necessary and sufficient conditions on $f(x)$ in order that it define a permutation of $F_{n \times n}$.

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Interchanges and Upsets in Round Robin Tournaments

We investigate the class of tournaments $T(R)$ having monotone score vector $R = (r_1, \dots, r_n)$. A graph whose vertices are the tournaments in $T(R)$ is defined, and some of its properties are determined. For given n , upper and lower bounds on the minimum number of upsets and on the maximum number of upsets for tournaments in $T(R)$ with R strong are obtained, and the cases of equality are characterized. Some special score vectors, notably $(1, 1, 2, \dots, n-2, n-2)$, $(\frac{n-1}{2}, \dots, \frac{n-1}{2})$ for n odd, and $(\frac{n-2}{2}, \dots, \frac{n-2}{2}, \frac{n}{2}, \dots, \frac{n}{2})$ for n even, are investigated. For the first, we calculate the cardinality of $T(R)$; for the last two, we obtain a lower estimate for the cardinality of $T(R)$. The paper concludes with some problems and conjectures for future research.

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Recursive Computation of Triangular 2-D Padé Approximants

Recently recursive computation of rectangular Padé approximants in two variables has been considered. The procedure proposed is exactly Rissanen's algorithm to solve block Hankel matrices. Thus a system of $n \times n$ blocks, each of size $m \times m$ is solved in $m^3 n^2$ operations. For triangular approximants one also has to solve a large system of linear equations that has a low displacement-rank feature. Thus it can be solved with reduced complexity. Based on the ideas used for rectangular approximants, a recursive procedure is described to compute the approximants in a fast way.

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Eigenvector Methods for Efficient Representation of Waveforms in an Emulation of Infrared Detection

Arrays of infrared sensors are being designed to detect warm and possibly closely spaced objects in a cold space environment. One problem in the design of a hard-wired emulator is the representation of the variations in the peak-shaped, 2D waveforms produced when a sensor scans an object. Only a limited number of easily calculated variations are permitted, and they should significantly affect the algorithm for resolving closely spaced objects. Which ones should be chosen? A nonlinear manifold of low dimension offers the greatest efficiency in approximating the full, infinite-dimensional space of waveforms. The algorithmic effects, weighted by physical likelihood, are incorporated as prior (Bayes) information in a parametric, maximum likelihood model of surface fitting, leading to a Riemannian metric on the manifold. The eigenvector-eigenvalue (curvature) analysis of the metric provides a simple, nearly optimal method for selecting representative waveforms. It is also used to stabilize and speed up the surface fitting algorithm. The result of this expository presentation is a general method of cluster analysis and optimization wherein linear algebra is the key to nonlinear analysis.

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A Matroid Abstraction of the Bott-Duffin Constrained Inverse

Let (E_1, E_2, G) be a linking system with linking function γ as defined by Schrijver (Matroids and Linking Systems, Math. Centre, Amsterdam 1978) or bimatroid. I.e., there is a matroid on the disjoint union of E_1 and E_2 whose bases are E_1 and $(E_1 \setminus X) \cup Y$ for $(X|Y) \in G$. G abstracts to matroid theory some properties of the non-singular minors of a matrix and γ abstracts the submatrix rank function. For $i=1,2$ let M_i be a matroid on E_i with rank function r_i and bases B_i . Suppose $r_1(E_1)=r_2(E_2)=R$ and there are bases B_i in M_i such that $(B_1|B_2) \in G$. We show (E_2, E_1, T) is a linking system where $(Y|X) \in T$ iff there exist $F_1 \subseteq E_1$ s.t. $F_1 \cap X = F_2 \cap Y = \emptyset$, $F_1 \cup X \in B_1$, $F_2 \cup Y \in B_2$ and $(F_1|F_2) \in G$. The linking function $\tau(Y, X) = \min_{F_1 \subseteq E_1, F_2 \subseteq E_2} [r_1(F_1 \cup X) + \gamma(F_1^c, F_2^c) + r_2(F_2 \cup Y)] - R$ for T and Schrijver's extension of Edmond's intersection theorem are used in the proof.

When M_i is coordinatized by cycle space C_i and G is coordinatized by matrix G (suitably generic) and they satisfy the above conditions, the Bott-Duffin inverse problem to find v for i_0 such that $v \in C_2^\perp$ and $Gv - i_0 \in C_1$ has a unique solution $v = T i_0$, and conversely. We show then that matrix T coordinatizes linking system (E_2, E_1, T) .

Applications to matroid, graph, and electrical network theory are given.

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The Observability of Measured Electric Networks: Topological and Algebraic Theory

An electric utility's control center relies upon remotely transmitted data to estimate the power flow in every line of the entire system. Neither the measurement set nor the network topology are fixed. Is the available measurement set sufficient to permit a linear mean square estimate of the state of the system as currently configured?

Answering this observability question is equivalent to determining the rank of the Jacobian matrix of the measurement model; the latter consists of the relations between line and bus measurements and branch power flows as well as Kirchhoff's Voltage Law. Avoiding a floating point rank computation, we prove that this Jacobian has maximal rank if and only if the network's graph admits a spanning tree with an association of each tree branch to a distinct, adjacent measurement. A striking feature of this result is its combinatorial characterization of a purely algebraic property of the measurement matrix.

A polynomial-time algorithm for constructing such a tree is described briefly.

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Linear Programming with an Array Processor

Two linear programming subroutines using the revised simplex algorithm were implemented on the FPS-164 Array Processor. The highly parallel and pipeline architecture of the FPS-164 gives high performance for vector and matrix computations at relatively low costs. One version of the subroutine requires the full-coefficient matrix and the other requires only the nonzero elements. The full matrix version was coded in AP-FORTRAN with all the vector and matrix computations performed by existing APMATH64 library routines. Several new APAL64 (assembly language) routines were written for the sparse version. Equivalent FORTRAN routines of both versions were coded for comparison on the VAX-11/780 and the IBM 4341. An actual problem from the wood products industry with 147 constraints, 535 variables, and a nonzero element density of 0.048 was used for a benchmark. The execution times of the full matrix version for the FPS-164, VAX, and IBM were 287, 7333, and 6927 seconds respectively giving relative times of 1.0, 25.6, and 24.1 respectively. The sparse matrix versions were about 10 times faster.

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Estimation of Sparse Hessian Matrices and Graph Coloring Problems

Recently, Coleman and More demonstrated that heuristic graph coloring procedures can be useful when estimating large sparse Jacobian matrices by finite differences. In this talk I will describe our current efforts to adapt this work to the symmetric case. In particular, it will be shown that the direct estimation of a sparse Hessian by 'column partitioning' is equivalent to a restricted coloring problem on the adjacency graph. In addition, a practical indirect method will be described. This method involves the unrestricted coloring of 'lower triangular intersection graphs'.

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"Contravariant" and "Covariant" Linear Transformations

This expository talk explains how the introduction of the terms "contravariant" and "covariant" linear transformation might be able to remove confusion for students by conforming beginning work to the notation of tensors and differential forms. With the conventions that 1) an object is determined as contravariant or covariant by its coefficients, 2) contravariant vector = vector = column vector = upper indices = multiplication of matrices on left, versus covariant = row vector = lower indices = mult. of matrices on right, and 3) basis elements transform opposite to their coefficients; there results: To each matrix there correspond two linear transformations, a contravariant transformation $A: V \rightarrow W$ and a covariant transformation $B: W^* \rightarrow V^*$ (where "*" denotes dual and $B \neq A^*$). Standard treatments cast work with change of basis into the "covariant" format (basis elements listed as a column) and work with coefficients into the "contravariant" format, where one format can be changed into the other by taking transposes.

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Concurrence Geometries

Recent application of combinatorial geometry to research on structure and form in architecture and structural engineering has revealed a number of fascinating problems in pure projective geometry and linear algebra. We deal with such a problem: to describe combinatorially the variety of configurations c of hyperplanes which can be constructed, given that the section of the configuration c on a fixed

Iterative Methods for Solving Bordered Systems

In numerical continuation methods, one often encounters linear systems with matrices B of the form:

$$B = \begin{bmatrix} | & A & b & | \\ | & & & | \\ | & T & & | \\ | & c & d & | \end{bmatrix}$$

where A may be nearly singular but the vectors b and c are such that B is nonsingular. If A is large and sparse, as for example, when solving nonlinear elliptic eigenvalue problems, use of iterative methods seems appropriate. In this paper, we will consider conjugate gradient type methods, applied either directly to B , or to A through a block-elimination algorithm. Often, a good preconditioning for A is available and we show various ways for exploiting it. A number of numerical experiments will be reported.

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The k -domination Problem on Sun-Free Triangulated Graphs

The k -domination problem is to find a minimum k -dominating set in a graph, i.e., a minimum vertex set D such that each vertex in the graph is within distance k from some vertex in D . For every fixed k , the k -domination problem is NP-complete even for bipartite and triangulated graphs. A linear algorithm and duality results are known for forests. In this paper a polynomial algorithm and duality results are given for sun-free triangulated graphs. These graphs are a significant subset of triangulated graphs and include forests, interval graphs, and C -forests, which are graphs in which all bi-connected components are complete.

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Similarity with the Maximized Correlation

The interpretation of the migration history reconstructed from the genetics study of surname frequencies of counties requires information of the various history in literature, different travel distances between each pair of counties, etc.. One way to determine the importance hierarchy of that information is to determine the linear weights, with or without constraint, which give the maximized correlation coefficient between the

reconstructed history and the linear combination of the information. The general scheme has been designed as follows: Two sets of variables have their coefficients of the respective linear combinations determined by maximizing the correlation coefficient of these two combinations. The numbers for the variables of two sets are not necessarily equal. It is possible that the two sets may be either exclusive or overlapped. This scheme is also applied to analyze spouse data, newlyweds data, physical attractiveness, relations of physical and psychological traits, and various other data, together with the treeness analysis, the analysis of the canonical correlation coefficient, etc.

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Search for an Object Moving Under General Conditions in Discrete Time

The paper deals with search theory. A single object moves among a finite number of cells in a finite number of time periods according to a variety of stochastic processes. Search effort is divided among the cells in each time period in to locate the object.

Using a simple transition matrix approach a particular allocation of search effort is presented which yields a simple expression for the probability of object survival for a wide variety of Markov and non-Markov processes for object movement. The probability of describing the object movement from time t to time $t+1$ may be a function of the search effort and object positions in time period prior to t . Moreover, these processes may not be specified as long as they may be in transition matrix form as described in the paper. Finally, this particular allocation of search effort can be used in linear programming formulation of the problem under conditions of variable amounts of search effort.

The history of this problem has been to find an optimal policy of search effort for the object using algorithms under the conditions of a known Markov process for object movement and a fixed supply of search effort.

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hyperplane H be a given geometric figure. We begin with a detailed study of plane configurations of lines, then give the natural generalization of this theory to configurations of hyperplanes in higher-dimensional spaces. We show how the construction of a concurrence geometry solves the related problem of deriving from any vector geometry G a geometry G' whose points are the circuits (minimal dependent sets of vectors) in G . Applications to statics and mechanics advance the exemplary work begun in the 1800s by J. C. Maxwell and Luigi Cremona, under the title «graphical statics».

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Construction of Comma-Free Codes

Block comma-free codes with words of length n form an important class of synchronizable codes which are used in telemetry systems and computer design. By considering comma-free codewords as vertices in the De Bruijn graph and studying the incidence matrix of this graph, representatives of all equivalence classes of these codes can be constructed and their synchronization delays determined.

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Numerically Stable Alternatives to Prony's Method

An important problem in the numerical study of diffusion-type systems is the fitting of data by sums of positive exponentials. Once some initial estimates are available, one may solve this problem iteratively by using variable projection algorithms or methods for semi-infinite programming.

Historically, the first method for obtaining starting values was due to Prony in the 1700's. It was shown in the early 1950's by Householder that this method has poor numerical properties since it involved the explicit computations of the zeroes of a real polynomial.

In this paper, we shall present methods with improved accuracy properties. If the initial data involves N equally spaced points, one method uses the symmetric Lanczos algorithm to reduce the problem to a symmetric tridiagonal

eigenvalue problem in $O(N^2)$ operations after which the Francis QR algorithm is applied. Numerical examples will be presented.

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Matrix Computations of Hessenberg Matrices

In this talk, we would like to present several mathematical algorithms for matrix computations of various types based on Hessenberg reduction of matrices. These include computing a polynomial matrix, determining if two matrices A and B have an eigenvalue in common, solving the matrix equation problem $AX + XB = C$, testing controllability or observability of a pair of matrices (A, B) and locating the eigenvalues of a matrix and the zeros of a polynomial in certain regions of the complex plane.

Some of these algorithms are new and some of them are reformulations of existing ones. The algorithms are all simple, general-purpose and efficient. An interesting result on the matrix equation $XA + BX = R$, where A is a $n \times n$ Hessenberg matrix, B is a $k \times k$ arbitrary matrix, and R is a $k \times n$ matrix having the first $(k-1)$ rows zero, play an important role in the derivations of these algorithms.

The paper is expository in nature.

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An Algorithm to Determine if Two Matrices have an Eigenvalue in Common

We consider here the problem of knowing if two matrices A and B have an eigenvalue in common without computing the eigenvalues of these matrices. This problem arises in mathematical control theory. It is well known that A and B have a common eigenvalue iff the Resultant of the characteristic polynomials of A and B is different from zero; alternatively, the Kronecker matrix product $(I \otimes A - B \otimes I)$ is non-singular. This result does not suggest a practical approach for the solution of the problem.

We propose in this paper a simple practical and efficient algorithm. Starting from two n -square matrices A and B , where A is Hessenberg and B is arbitrary, we construct a symmetric matrix X of order n such that X is non singular iff A and B do not have an eigenvalue in common. X being a symmetric, its nonsingularity can be checked by means of the efficient triangular decomposition of $X = LDL^T$.

An implementation of the algorithm in parallel is also discussed.

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Construction of Matrices with Prescribed Submatrices and Eigenvalues

This paper deals with methods for constructing a square matrix with two principal complementary prescribed submatrices and prescribed eigenvalues.

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Two-Dimensional Nonlinear Wave Propagation in a Shallow Tidal Estuary

A study is made of a two-dimensional mathematical model of nonlinear wave propagation in a shallow tidal estuary using an unconditionally stable numerical scheme. The alternative direction implicit scheme is used to investigate the simulation of the flow pattern of the estuary, and to examine the effects of the changes in the bed topography either due to dredging or due to construction of spurs and guidewalls in tidal flows. The Coriolis force due to rotation of the earth is taken into account in the present two-dimensional model. It is shown that the Coriolis force is responsible for the existence of the transverse component of the flow field. The proposed theoretical model is applied to investigate the flow structure in the Hooghly estuary. The computed results are then compared with the observed values in the Hooghly estuary. A stability analysis of the alternative direction implicit scheme is also included.

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Equivalent Linear Programming Problems

In the problem of maximizing an objective function subject to constraints one may regard the data for the problem to be a coefficient matrix A . The solution is obtained by finding a matrix R such that $RA = M$, where M is a

maximizing reduced form (both the objective row and right-hand column are nonnegative). For suitable matrices L and H we may define $B = LAH$. We say that B is equivalent to A if we can find a matrix S such that $SB = N$, where N is a maximizing reduced form which has the same basic columns as M . The advantage in doing this is that the pivot operations on B will tend to give nonnegative columns which can then be removed as indicated in the paper of Thompson, Tonge and Zions (Management Science, 1966).

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An Efficient Method to Invert Matrices with a Minimum Storage Requirement

Let M be a nonsingular matrix of order m . A sufficient condition to invert this matrix on its own array with m^3 multiplications and divisions is that M be a definite matrix. It is shown that an arbitrary matrix can be inverted with the same number of operations but with a $m^2 + 2m - 1$ storage requirement.

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On the Stability of a Highly-Concurrent Parallel Algorithm for Matrix Inversion

We consider the stability of an algorithm proposed by Kant and Kimura for the inversion of an $n \times n$ matrix on a mesh-connected system of microprocessors. Emphasis will be given to weakening the requirement of strong non-singularity of the matrix and to estimates of the precision needed for reliable inversion.

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Solving General Nonlinear Programming Problems with an Ellipsoid Algorithm

Use of an ellipsoid algorithm in solving convex and nonconvex mathematical programming problems is discussed. A comparison of our variant of the ellipsoid algorithm with other general purpose nonlinear programming algorithms, including GRG2 and subroutine E04VAF of the NAG Library, is presented. This study shows the ellipsoid algorithm to be reasonably competitive and extremely robust. In addition it provides very precise solutions and it is extremely easy to use.

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The Bandwidth Problem for Asymmetric Matrices

The symmetric bandwidth problem has been studied extensively using graph-theoretical methods (see for example Chvatalova, Dewdney, Gibbs, and Korfhage, The Bandwidth Problem for Graphs, University of Western Ontario Department of Computer Science Research Report, 1975). This paper undertakes a similar investigation of the bandwidth problem for asymmetric matrices, which may be stated as follows: given a sparse, $n \times n$ matrix A , find permutation matrices P and Q such that the bandwidth of PAQ is small. Results will be presented which provide bounds on the bandwidth of special classes of matrices. Relative merits of various algorithms for

asymmetric bandwidth reduction will be discussed.

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An Algorithm for Finding Two Disjoint Paths with Minimum Sum of Weights Between Two Pairs of Vertices in a Graph

In the view of both theoretical and practical importance the path problems are of particular interest in the areas of network and applied graph theory. Finding two disjoint paths with minimum sum of weights between two pairs of vertices in a graph (STPP in short) is an unsolved problem. The STPP is precisely as follows: Given an edge-weighted graph $G(V, E)$ and four distinct vertices s_1, t_1, s_2, t_2 , determine

whether or not G admits two vertex-disjoint paths p_1 and p_2 such that (i) p_1 and p_2 connect s_1 with t_1 and s_2 with t_2 , respectively, and (ii) $D(p_1) + D(p_2) \rightarrow \min$ where $D(p_i)$ ($i = 1, 2$) is the sum of the weights of the edges in p_i , and find such paths if they do exist.

The authors propose an effective algorithm to solve the STPP. As background, several theorems on the shortest path in a graph are introduced. Among these theorems, three cover theorems are the key points. Basically, the main idea of the algorithm is to construct a 'growing tree' from a given graph. With the exception of the case of 'strict-repetition' on the growing tree, the algorithm is proved to be $O(n^2 \cdot md) - O(n^3 \cdot md)$ complete, where $n = |V|$, md - the maximum degree of vertices. An effective computer program is designed for the above algorithm. The computational efficiency is demonstrated by making use of examples. An enriched matrix technique is developed in programming. It takes less space of memories to store the same size graph in the computer than the incidence matrix or adjacent matrix does.

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A Practical Approach to Variable Selection in Linear Regression

In the classical variable selection problem in linear regression, one is asked to select a subset of size k from n independent variables such that R^2 for the regression of a given dependent variable on the members of the subset is maximized. In particular, given a candidate subset S_1 with corresponding R_1^2 , one would want to know if there were any other subset S_2 with corresponding $R_2^2 > R_1^2$; which at the present state of the art requires a search which is combinatorial in scope. In this paper, given $\Delta > 0$, we present a criterion which if satisfied, is a sufficient condition for there being no subsets with R^2 exceeding $R_1^2 + \Delta$. As Δ is increased the condition becomes closer to necessity in the sense that the likelihood of it being satisfied increases if there actually is no subset with $R^2 > R_1^2 + \Delta$. Examples show that Δ does not have to be trivially large for the criterion to be useful. Its use can prevent time-consuming searches for regressions which are only slightly better than a given one.

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An Algebraically Derived System Model for Nonlinear Systems

This paper develops a nonlinear system model that can be used to model nonlinear systems that may or may not have a state space representation. The model is derived, through algebraic operations, from the system's basic equations. The algebraic operations are restricted to a linear vector space. The vector space is constructed from an abelian group and a field. The abelian group is generated by the system's variables and the field is generated by the system's linear operators.

This construction yields a system model that consists of a primitive equation and an output equation. The primitive equation relates the system's prime variables and the system controls. And the output equation equates the remaining system variables to the prime variables and controls. The system's prime variables, unlike state variables, are not necessarily determined by their value at a given point in time and the system controls. They may be determined by their past history as well. This model is applicable to analog, discrete, stochastic, delay and hybrid systems.

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On Recovering Voter Preferences from Pairwise Votes

Given n alternatives there are $n!$ strict preference orderings a voter can express and $\binom{n}{2}$ pairwise comparisons between alternatives. There are many proposed solutions to the problem of achieving a consensus ranking once V voters have revealed their individual rankings. Some (Borda voting, Kemeny's median) depend only on the pairwise votes, while others depend on the number of votes each preference list receives. Given only the pairwise votes we would like to determine the minimum number of voters necessary to produce that outcome, and given V voters who could have produced those pairwise votes, find all possible ways they could have voted.

Define a profile N to be a nonnegative integer vector of dimension $n!$ where the k th component is the number of votes for permutation k ; let R be a certain $\binom{n}{2}$ by $n!$ Matrix with entries ± 1 .

Given a dimension $\binom{n}{2}$ vector P of pairwise votes we want to determine the minimum possible number of voters V , and want to find all profiles N subject to $N \cdot (1, 1, \dots, 1) = V$ such that $RN = P$. We discuss the problem of finding V for any n , and describe an efficient computer implementation of the entire problem for $n=4$.

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Staircase Matrices and Systems

Diverse computational methods -- including discretization of boundary-value and control problems, approximation by splines, and multi-period linear programming -- give rise to linear equations that have a familiar sequential structure. The variables of these equations fall into a natural series of partitions; each equation links at most the variables of two adjacent partitions. Such equations comprise a "staircase system," because their coefficients form a "staircase matrix" whose nonzero elements are confined to rectangular blocks on and just below the diagonal.

This paper surveys properties of staircase matrices and methods for solving staircase systems. Particular attention is placed upon results that are recent or little-known, and upon methods that take full advantage of the sparsity of nonzeros within staircase matrices. Major topics include the following:

1. Characterizations of structural and algebraic nonsingularity of staircase matrices.
2. Direct solution of staircase systems, particularly through perfect-elimination and recursive reorderings of the staircase.
3. Adaptation of sparse-elimination methods

to solve staircase systems that are very sparse within the staircase blocks.

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Comparison of Heuristic and Optimal
Solution Models for a Truck Routing Problem

Two company plants must service a set of m distributors with n trucks. The objective is to develop routes that minimize the total mileage for all trucks combined. Limitations on the length of each individual route and the storage capacity of each truck are considered. There is no restriction that the truck must return to the plant at the end of the run.

The optimal total mileage figure for all routes is established by solving a set covering model. All partial and complete routes satisfying the problem constraints are generated by implicit enumeration. A heuristic solution procedure based on the minimal spanning tree model is used to develop another set of n -routes that satisfy the problem constraints.

Several routing problems are solved with both approaches to determine the relative effectiveness of the heuristic procedure. Preliminary results indicate the heuristic will produce an optimal solution in some cases.

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Analysis of Routing in Computer Networks

Given a network of communication channels with deterministic demands and messages decomposed into packets, several possible criteria for optimality will be discussed.

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Updating the LU Factorization of the
Basis Matrix in Sparse Linear Programming

Several methods have been proposed for updating the LU factorization of the basis matrix B in the simplex method when one of B 's columns is replaced. In this talk we describe how to extend the sparsity-exploiting capabilities of a method due to Reid. Specifically we show how cyclic permutations more general than those introduced by Reid can be used to further avoid eliminations, and hence fill-ins. The relationship between our algorithm and the problem of finding the strong component decomposition of the digraph corresponding to B is also discussed.

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Behavior of the Perturbed Lanczos Algorithm

The Lanczos algorithm uses a three-term recurrence to construct polynomials orthogonal with respect to a measure that puts weight on the discrete set consisting of the eigenvalues of the matrix being considered. The roots of these polynomials are taken as approximate eigenvalues of the matrix. If the recurrence is perturbed slightly, as, for example, by roundoff errors, then the polynomials generated will not be orthogonal with respect to the desired measure. It is demonstrated, however, that the polynomials are orthogonal with respect to a slightly different measure -- a measure that puts weight on many points, all within about ϵ of the eigenvalues, where ϵ is the size of the perturbation. This description is used to explain observed behavior of the perturbed Lanczos algorithm.

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Divergence-Free Bases for Finite
Element Schemes in Hydrodynamics

In the numerical solution of the Navier-Stokes equations of incompressible hydrodynamics, a number of finite element schemes satisfying the incompressibility condition

$$\text{div } u = 0$$

have been devised. As pointed out in Temam: Navier-Stokes Equations, see in particular the discussions on pp. 58, 138, and 494, a difficulty has been the lack of explicit bases for those schemes. In several cases, even the dimension of the divergence-free subspace was unknown. In the

present work we resolve those questions, employing, for both conceptual and algorithmic purposes, graph-theoretic methods.

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Linear Dynamic Output Feedback and Algebraic Module Theory

The effectiveness of output feedback in improving the characteristics of linear control systems has been a widely accepted principle for nearly five decades. This principle originated mainly from qualitative and single variable considerations, and it was not until recent years that multivariable output feedback (as opposed to state feedback) phenomena have been directly investigated. The purpose of this lecture is to expose the fundamentals of a very recently introduced mathematical framework for the study of multivariable output feedback, and to present some of the results obtained through it. This framework employs algebraic module theory as its basic tool, and is in consistency with the classical Kalman realization theory. Using it we shall construct the minimal output feedback compensator that is needed to achieve a prescribed design goal. The minimality of the feedback compensator has the advantage of reducing both the sensitivity to parameter variations and the design complexity of the final system. Internal stability is ensured.

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The Computer Generated Symbolic Cayley-Hamilton Solution of the Matrix Equation $dx/dt = Ax$ and Related Problems

PL/I-Formac has been used to produce a computer generated symbolic polynomial solution of the matrix equation $dx/dt = Ax$ for an arbitrary n by n matrix A and arbitrary initial conditions on x . The characteristic polynomial of A is generated by the Levrier of Krylov methods, whose time complexities are of order n^4 and n^3 , respectively. The characteristic polynomial is then used recursively to generate $\exp(At)x(0)$. The algorithm was tested for $n = 10, 50, 100, 200$ for randomly generated A and A

for which the solution is known. The precision of x versus t is observed. CPU time in minutes for polynomials of order n is roughly $(n/50)^4$. The method was also tested on the companion matrix, certain stochastic processes and on coupled mechanical system. The method was further generalized to the inhomogeneous case and, subsequently, to the case of variable A . Finally, the Newton-Kantorovich Theory is used to solve the nonlinear case, $dx/dt = f(x, t)$.

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Some Extensions of an Algorithm for Sparse Linear Least Squares Problems

In this paper several algorithms are developed which extend the method of George and Heath for sparse linear least squares problems to include rank deficient problems, linear equality constrained problems, and updating of solutions. An application of these methods to the solution of sparse square nonsymmetric linear systems is also presented.

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VLSI Networks for Orthogonal Equivalence Transformations and Related Applications

A linearly connected VLSI parallel processor array consisting of a few types of simple processors is shown to serve as the integral component of a special purpose device for the QR and QL decompositions and for least squares computations. For matrices A of bandwidth w each network requires less than w^2 processors independently of the order, n , of A . Computation time is between $2n$ and $4n$ steps, subject to the number of codiagonals. Combining two linear meshes results in a systolic array for the orthogonal reduction in time $O(nw^2)$ of an arbitrary matrix to bidiagonal form A_b or a symmetric matrix to tridiagonal form A_t . Furthermore, it accomplishes one iteration of the singular value computation for A_b or one iteration of the implicitly shifted QR algorithm for A_t in $O(n)$ steps. An elimination order due to Rutishauser assures preservation of the bandwidth and thus $2w$ processors per array. The iterations can be carried out to required accuracy by a simple system comprising the systolic

array, an internal memory, and a scalar unit which takes care of convergence tests, shifts and implicit deflation.

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Identity Manipulation by Vector Spaces

Using a computer and representation theory, we identify identities of a nonassociative algebra with a vector space of N-tuples. From this vector space we can tell instantly the various implications of the identities. Examples: 1. We compute the representation of a set of given identities. If an unknown is in the vector space, then the unknown is an identity and is provable from the given identities. 2. We can tell if an unknown is a consequence of given identities or not. When it is not a consequence, we can tell how close it is to being true. 3. We can take a list of identities and determine which are essential and which are nonessential. We can tell which are equivalent. Using this technique, we have sorted out the varieties appearing in the literature. We have established which varieties are subvarieties of others, which are equivalent, and which are neither. 4. We can answer questions like whether the associators, the alternators, or any other specified set is an ideal. 5. We can compute the meet and join of varieties. 6. We can tell which characteristics must be avoided.

Professor Hogben and I have this system running at Iowa State University and have used it successfully to analyze identities.

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The Distance-Domination Numbers of Trees

The P-center problem is to locate P centers in G so that the maximum distance between centers and non-centers is minimized. A related problem is to determine the maximum number of vertices that can be "covered" (within a distance of α) by a vertex set of cardinality P in G. In this paper we describe an $O(n^3P)$ algorithm for solving the maximum coverage problem on trees. The algorithm generates a list $L(v)$ of information for each vertex v of the tree. This list contains the results of the maximum coverage problem by vertex sets of cardinality $1, \dots, P$ in the subtree $T(v)$ with v as the root. The information from the son lists are used to obtain the information on the father list. The main idea is to separate a complicated dominating relationship on the original tree into a number of independent dominating relationships on son subtrees. We also apply the same ideas to

solve the P-median problem on trees, which is to find a vertex set A of size P in a tree such that $\sum_{v \in V} d(v, A)$ is minimized.

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An Application of Matrix Multiplication in Recursive Functions

In applied mathematics, often we see the following recursive functions:

$$A, f_k = a_k + b_k f_{k-1}$$

$$B, f_k = a_k f_{k-1} + b_k f_{k-2}$$

where a and b are given.

In order to find f_n , the conventional approach requires to compute f_0, f_1, \dots, f_{n-1} . When n is large, the computation may be very time-consuming. This paper provides a faster way to obtain f_n through the application of matrix multiplication.

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Lambda Matrices with Asymmetric Coefficients with Application to Vibration Problems

This work derives sufficient conditions for the asymmetric lambda matrix, $D_2(\lambda) = \lambda^2 A + \lambda B + C$, to be similar to a symmetric lambda matrix. Here, A is assumed to be a real $n \times n$ symmetric, positive definite matrix, while B and C are real $n \times n$ asymmetric matrices. The matrices A, B and C are further restricted so that $A^{-1}B$ and $A^{-1}C$ have real eigenvalues. These conditions are then used to derive sufficient conditions for the latent values of $D_2(\lambda)$ to have negative real parts. In addition, these results are applied to a class of linear vibration problems of the form, $M\ddot{x} + D\dot{x} + Kx = 0$, where the matrices M, D and K satisfy the same conditions as A, B and C above, respectively. The application of the theory developed for $D_2(\lambda)$ yields a new stability condition for the related asymmetric mechanical system. Numerical examples are provided.

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Singular Linear Maps which Preserve Affine Independence on Small Subsets

Often in geometry one would like to collapse or project a configuration of points onto a lower dimensional space without unnecessarily destroying the lower dimensional freedom of the configuration. The existence of such projections may be insured in some cases by a simple counting argument and in other cases (i.e., in \mathbb{R}^d) by a Baire category argument. The goal of this talk is to exhibit such projections based on considerations of linear algebra. As illustrations of the method, applications will be given of the results to visibility graphs, to the representation of finitary matroids, and to higher dimensional analogues of a problem on slopes posed by P. R. Scott (Am. Math. Monthly 77 (1970), 502-505).

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Modelling with Integer Variables

We give several results which characterize MIP-representability of sets and functions, and the finite union of MIP-representable sets. These results extend earlier ones due to R. R. Meyer. We provide several applications of this work, including representations whose linear relaxations have efficient formulations, while retaining maximum accuracy to the problem modelled.

Key Words:

- 1) Integer programming
- 2) MIMM
- 3) Modelling

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Computing the Crossover Metric for Unordered Classification Trees

Undirected binary trees, or dendrograms, have been used in numerical taxonomy for describing the evolution of species. The crossover (nearest neighbor interchange) metric is a measure of the similarity between two alternative classification schemes for a particular set of species. Since the number of distinct dendrograms grows factorially with the number of species to be classified, there are formidable computational problems in computing this metric. Counterexamples to published results concerning properties of the metric are presented in conjunction with computational experience and procedures employed in computing the metric.

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Solutions of Higher Order Matrix Equations and Finite Element Methods

The main purpose of this work is to establish necessary conditions and sufficient conditions for the existence of solutions of higher order Riccati matrix type equations of the form

$$AX + XB + C + DX + XE + FX = 0.$$

Results obtained are then used in determining optimal location of nodes in finite element methods of solutions of elliptic partial differential equations. The coefficient matrices A, B, C, D, E, F will have elements belonging to the ring of polynomials in n-variables with complex coefficients, $R = C[z_1, z_2, \dots, z_n]$. Applications

of results obtained can be made to multidimensional system theory, optimal control theory, reducibility of differential systems containing parameters, stability of transport processes, the Wiener theory of filtering and prediction of stationary stochastic processes and elsewhere. Use is made of the notion of weak generalized inverses of a matrix over $R = C[z_1, z_2, \dots, z_n]$.

This work extends that of J. Jones, Jr. (Matrix Equations, Proc. Amer. Math. Soc. 1972) and others.

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A Module Theoretic Algorithm for Computing the Interlacing Inequalities of the Invariant Factors of an R-Matrix $M_{n \times m}$ by an R-Matrix $M_{(n+1) \times m}$

An algorithm for computing quotients of finitely generated R-modules by cyclic sub-modules where R is a principal ideal domain is given using elementary arguments.

Translating the algorithm to matrix augmentation by a row or a column yields Thompson's interlacing inequalities from elementary module theory.

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Linear programming and dynamic programming

Linear programming and dynamic programming are two important features of operations research. This paper deals with a Markov decision process over a finite horizon. It is well known that this problem can be solved by dynamic as well as by linear programming.

We show that the linear programming approach implemented by a special block-pivoting simplex algorithm is equivalent to Bellman's backward recursion algorithm of dynamic programming. Furthermore, we discuss the computation of optimal policies for problems with a number of side constraints.

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Singular Value Decomposition and the Solution of the Radiative Transfer Equation

The transfer of radiation in a planetary atmosphere is governed by a first order, linear, integro-differential equation. One method used to solve this equation is the Discrete Ordinates Method of Chandrasekhar in which the integral is written as a quadrature sum. The resulting system of equations can be solved in a number of ways. One method is to integrate numerically, a method that becomes unstable when the atmosphere gets thick. Another approach is to assume an exponential solution and find the roots of the characteristic polynomial, a method that becomes unstable when the atmosphere does not absorb much radiation. A third method constructs the solution for thickness $2x$ from the solution for thickness x , a method that is expensive since the starting value must be very small. Only recently have linear algebra techniques been applied. We have developed an algorithm using the SVD that is

always stable. An error analysis will be presented comparing our algorithm with others currently in use.

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Matrix Methods for Queuing Problems

Analytic techniques do not seem to be amenable to solving queuing problems arising from models of non-Jackson networks. Thus the probability distribution associated with these models is usually found by determining the null vector of the matrix derived from the detailed balance equations. These matrices are very large, sparse, highly structured, singular, nonsymmetric matrices. We have been very successful in using iterative methods usually applied to nonsingular matrices arising during the solution of partial differential equations to these singular matrices. Our computational experience on two very different methods and a theoretical treatment of the iterative methods will be presented.

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High-Order, Fast-Direct Methods for Separable Elliptic Equations

The Rayleigh-Ritz-Galerkin method with tensor product B-splines yields high-order discretizations for elliptic partial differential equations. For smooth problems the resulting linear system of equations is both smaller and denser than the corresponding systems for lower-order discretizations. However, several fast-direct methods are known for solving these low-order systems when the partial differential equation is separable. In this talk we show how to extend the matrix decomposition technique to yield a fast-direct method for high-order, finite-element discretizations.

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On the Solution of Almost Block Diagonal Linear Systems

Almost block diagonal linear systems arise in several finite element methods for two-point boundary value problems. In particular, in the popular code COLSYS(1), which is based on the method of spline-collocation at Gaussian points, the general routine SOLVEBLOK(2) is used to solve such systems. Based on the usual form of Gaussian elimination with partial pivoting, SOLVEBLOK introduces fill-in. There are, however, pivotal strategies which can be used to avoid fill-in, and produce more efficient routines. In this paper, we discuss implementations of two pivotal strategies suggested by Lam and Varah(3), and report on some numerical comparisons.

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- (3) J. M. Varah, Alternate row and column elimination for solving certain linear systems, SIAM J. Numer. Anal. 13(1976), 71-75.

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On Balance Points in Trees

A nonnegative number called the unbalance at a point v of a tree is defined in terms of a minimization process applied to the sum of signed first-moments of the branches at point v . The balance points of a tree are points of least unbalance. Properties of balance points are demonstrated, and although the balance of a tree is distinct from the familiar concepts of center and centroid, it is shown that, like the center and centroid, the balance of a tree consists of either one point or two adjacent points. Generalization to k th-moments for positive integer k , allows the consideration of moment characterization of trees.

Reference: F. Harary (1969), Graph Theory, Addison-Wesley, Reading, Mass.

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The Characterization of Directed Graphs with a Break Vertex

If the directed graph G of a matrix with a nonzero main diagonal has a break vertex, then G is a perfect elimination graph. In this paper we give a necessary and sufficient condition for an arbitrary directed graph to have a break vertex. Our result offers an algorithm that constructs a break vertex whenever such a vertex exists. The number of computation steps required by the algorithm to construct a break vertex of a directed graph with n vertices is proportional to the square of n in the worst case. We show that for any $n \times n$ matrix A with term rank n and with exactly two nonzero entries per row, there exists a permutation matrix Q such that every connected component of the directed graph of AQ has a break vertex.

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The Impact of Microprocessor Technology on Computational Algorithms in Numerical Linear Algebra

Recent innovations in microprocessor technology include many features that are particularly useful for numerical algorithms for matrix factorizations and the solution of least squares problems. The proposed IEEE Standard for Floating Point Arithmetic is implemented in the Intel 8087 numeric co-processor chip for the Intel Microprocessor Development System. The use of this configuration and its possibilities for expansion to provide concurrent computing capability will be described.

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A Survey of Semi-Infinite Programming

Several examples of semi-infinite programs are given from the fields of air pollution abatement, experimental regression design, and equilibrium models in economics and engineering mechanics. The role of generalized finite sequences is illustrated in these models and in the derivation of nonlinear systems of equations for numerical treatment. Recent results include an equivalence between a class of saddle value problems having a general kind of separability and functional bilinearity in the saddle function and

a dual pair of (separably) infinite linear programs. Comparisons will be given to the generalized Fenchel duality theory.

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Principal Pivot Transforms of
Nonlinear Functions

The class of at most 2^n functions which are combinatorially equivalent to a given function through a pivotal transform on a principal subfunction is studied for the P-functions of Moré and Rheinboldt. By means of closure under principal pivot transforms such functions share complementary solutions. Under certain circumstances these solutions are pigeonholed -- each is located in its own orthant. Algorithmic implications for complementarity problems are significant.

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On the matrix function $AX + X^T A^T$

We study properties of the matrix functions $G(X) = AX + X^T A^T$ and $H(X) = AX - X^T A^T$ defined on real $n \times m$ matrices X . (Matrix A is a fixed real $m \times n$ matrix.) The spectrum of G when $m = n$ was obtained by Taussky and Wielandt (Arch. Rat. Mech. Anal. 9 (1962), 93-96). Here, we focus on description of the kernels, $\text{Ker } G$ and $\text{Ker } H$ as well as explicit solution of the equations $G(X) = C$, $H(X) = S$. Extension to the maps

$\tilde{G}(X) = AX + X^* A^*$, $\tilde{H}(X) = AX - X^* A^*$ defined for corresponding complex matrices are considered. An application in the calculation of eigenvectors for a matrix M which is H -selfadjoint ($H^* = H$, $\det H \neq 0$ and $HM = M^* H$) is discussed.

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Application of a Homotopy Method to a
Problem of Scarf's

A system of equations associated with a Walrasian market equilibrium price vector is solved by application of a homotopy method of H. Keller.

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- (1) H.B. Keller, Global Homotopies and Newton Methods, in Recent Advances in Numerical Analysis (ed. by De-Boor) Academic Press, New York (1978) 73-94
- (2) H. Scarf and T. Hansen, The Computation of Economic Equilibria, Yale University Press (1973)

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Algebraic Representation of a Path Counting
Measure

Let S be a finite set, equipped with some binary relation \leq . Let $\Omega_0 = S^+$, when $\mathbb{N}_+ = \{0, 1, 2, \dots\}$. The elements of Ω_0 are called *paths*. $\omega \in \Omega_0$ is said to be *admissible* when $\omega(t) \leq \omega(t+1)$ for all $t \in \mathbb{N}_+$. Let Ω be the set of admissible paths of Ω_0 . Let F be the σ -field on Ω , generated by cylinder sets of the form

$$A = \{\omega \in \Omega: \omega(i) \in V_i, i = 0, 1, \dots, n\}, \text{ where } \forall V_i \subseteq S, \forall n \in \mathbb{N}_+.$$

Let π be the set of all additive set functions on F . This is a linear vector space over the set of real numbers.

Two families of linear operators, namely $\{T_t, t \in \mathbb{N}_+\}$, $\{E_V, V \subseteq S\}$, and a linear functional κ^* are defined on π . These in the following sense can represent any $\gamma \in \pi$.

$$\gamma(\{\omega \in \Omega: \omega(t_1) \in V_1, \omega(t_1+t_2) \in V_2, \dots,$$

$$\omega(t_1+t_2+\dots+t_n) \in V_n\}) =$$

$$\sum_{n=0}^{\infty} T_n \dots E_V T_1 \gamma,$$

where on the right-hand side we proceed from left to right with the operations.

An important element of π , a path counting measure is defined and the construction of (T_t, E_V, κ^*) , the so-called algebraic representation is given for it.

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A new method, based on a purely linear algebraic approach, is presented for determining, in an elegant closed-form fashion, the general displacement parameters from initial and final position data of a rigid body. These displacement parameters involve a (proper) rotation and translation. Explicit formulas are given for the rotation and translation in terms of: (1) initial and final positions of three (non-collinear) points fixed in the rigid body, or (2) the initial position and velocity at that position of three fixed points. Special rotation angles (with respect to various coordinate systems) or other information can subsequently be calculated if desired. Previous solutions have relied on designation of one of the three points as a reference point which can lead to ambiguities in the case of imprecise or noisy measurements depending on the particular choice of reference. A certain matrix approximation technique is suggested here for the case of imprecise data. Finally, details concerning a microprocessor implementation are discussed.

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Controllability and Observability for Generalized Linear Systems

We consider generalized linear systems (polynomial systems) which can be represented by the triple of polynomial matrices $(A(z), B(z), C(z))$ or by the transfer function $H(s, z) \equiv$

$C(z)(sI - A(z))^{-1}B(z)$. Necessary and sufficient conditions are derived for algebraic controllability and observability in terms of zero coprimeness and factor coprimeness of polynomials in two indeterminates. The basic idea is to compute the resultant of two polynomials whose coefficients are from a unique factorization domain. The relationship between the resultant corresponding to the given system and its Hankel matrix is given. Using the obtained results the spectral controllability and observability of linear time delay systems is investigated in the frequency domain. The relationship between spectral controllability and observability and appropriately defined relative primeness of some elements in the transfer function matrix is also given. An interpretation and some applications of the obtained results are demonstrated for 2-D systems also.

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Minimum distance decoding (MDD) for a general error-correcting linear code is a hard computational problem which recently has been shown to be NP-hard.¹ The complexity of known decoding algorithms is determined by $\min(2^k, 2^{n-k})$, where n is the code length and k is the number of information digits. An algorithm is suggested which reduces substantially the requirements on storage and computational complexity of MDD. The algorithm is based essentially on the group structure of a linear code and makes use of a new concept of zero-neighbours codewords. Only these codewords (which can be computed in advance) should be stored and used in the decoding procedure. The number of zero neighbours is shown to be very small in comparison with $\min(2^k, 2^{n-k})$ for $n \gg 1$ and a wide range of k/n values. For example, for $k/n \approx 0.5$ this number grows approximately as a square root of the number of codewords.

¹E.R. Berlekamp, R.J. McEliece and H.C.A. van Tilborg "On the Inherent Intractability of Certain Coding Problems," IEEE Trans. Inform. Theory, IT-24 (1978), pp.384-386.

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Reduced Matrices and Combinatorics

Let $0 \leq m \leq n$ be integers. It is well known that reduced $m \times n$ matrices (i.e. row-reduced echelon, of rank m) over a field K can be used to codify m -dimensional subspaces of K^n and to enumerate them when $K = GF(q)$. It is less often observed that reduced matrices are closed under matrix multiplication and that this multiplication contains all the information about the inclusion of subspaces: if R and T are reduced matrices and if $W(R)$ denotes the row space of R , then $W(R) \subseteq W(T)$ if and only if there exists a (unique) reduced matrix S such that $R = S \cdot T$. In this paper we show that many sub-classes of reduced matrices, closed under multiplication, correspond to familiar objects in combinatorics such as finite subsets and set partitions of finite sets, and number partitions and compositions. We also introduce generalizations of reduced matrices and their multiplication in order to include permutations, Laguerre configurations, Dowling lattices, and colored bipartite graphs which provide a combinatorial model for the so-called Fibonacci coefficients. In each case, a partial order relation can be defined, using the matrix multiplication as above, and Möbius inversion techniques as well as generating functions can be introduced directly.

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Memory Access Patterns in Vector Machines with Applications to Problems in Linear Algebra

The performance of a vector algorithm on a vector machine is frequently determined not so much by the speed of the vector processor but rather by the rate at which vector operands can be accessed from memory. Almost all existing vector machines are limited to performing operations on vectors which can be constructed from logically contiguous storage locations. Our analysis predicts that the addition of the capability to access the memory of a vector machine in a variety of simple patterns will dramatically decrease the time required to solve many important linear algebra problems. These predictions, however, do not take into account the problems of memory module conflicts in an interleaved memory. A simulator which steps through a sequence of vector instructions resolving memory conflicts nevertheless indicates that improvement in execution time of linear algebra problems would be significant if multiple access patterns were possible.

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Determinants of Nonprincipal Submatrices of Normal Matrices

Let A be an $n \times n$ normal matrix, and let $1 \leq m < n$. Let $\alpha, \beta \in Q_{m,n}$, the set of increasing integer sequences of length m chosen from $1, 2, \dots, n$. Suppose α and β have exactly k common entries, denoted by $|\alpha \cap \beta| = k$, and suppose $k \leq m-1$. Marcus and Filippenko obtained an upper bound for $|\det A[\alpha|\beta]|$, which depends on k and the moduli of the eigenvalues of A . Their proof depends on the quadratic Plücker relations and combinatorial arguments. We use a different approach to improve their bound. Marcus and Moore defined $\rho_k(A) = \max_{U \in U_n} |\det(U^*AU)[\alpha|\beta]|$, where U_n is the group of all $n \times n$ unitary matrices. They conjectured that if A is semi-definite hermitian then $\rho_0(A) \leq \rho_1(A) \leq \dots \leq \rho_m(A)$. We prove this conjecture.

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On the Minres Method of Factor Analysis

A basic problem in factor analysis is the resolution of n observed variables (z) in terms of a smaller number m of common factors (f).

The following linear model is assumed:

$$\tilde{z} = A\tilde{f} + D\tilde{u},$$

where A is an $n \times m$ matrix consisting of unknown parameters called common-factor loadings, and $D\tilde{u}$ represent the error. Once A has been found, the fundamental theorem of factor analysis states that the matrix \hat{R} of reproduced correlations is given by

$$\hat{R} = AA^T,$$

under the assumption of uncorrelated factors. The strategy is therefore to determine A so that the reproduced correlation matrix \hat{R} is a best fit to the observed correlation matrix R^0 . Harman and others proposed to solve this problem through the minimization of the objective function

$$f(A) \equiv \sum_{j=1}^n \sum_{\substack{k=1 \\ k \neq j}}^n (r_{jk}^0 - \sum_{p=1}^m a_{jp} a_{kp})^2,$$

subject to the constraints

$$\sum_{p=1}^m a_{jp}^2 \leq 1 \quad \text{for } j = 1, 2, \dots, n.$$

They developed a block Gauss-Seidel technique called the minres method. Their numerical experiments indicated that this method is superior to several other procedures in terms of work. In this talk, we shall show how the minres method can be efficiently and reliably implemented on the computer using some well-known linear algebraic techniques. We shall also prove that, under certain assumptions, the minres method always converges to a minimum of the objective function.

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An Alternative to the Classical Method of
Computing Frequency Response Functions, Suitable
for Large-Scale Systems

Control engineering, based upon the equation $\dot{x} = Ax + Bu$, $x \in \mathbb{R}^n$ and A, B real matrices, is concerned with the frequency dependence of the steady-state response of the i th coordinate of x to a sinusoidal input into the j th coordinate of u ; i.e., with $a_{ij}(\omega)$ called the amplitude response and $\theta_{ij}(\omega)$ called the phase shift in the formula $x_{ij}^s(t) = a_{ij}(\omega) \sin[\omega t + \theta_{ij}(\omega)]$. The classical approach to computing these functions, based upon the Laplace transform, computes $F(i\omega) = [i\omega I_n - A]^{-1}B$ from which the $a_{ij}(\omega)$ and $\theta_{ij}(\omega)$ are then derived by complex arithmetic.

The paper reviews this classical approach and points out, from the view of computational complexity, undesirable aspects of this method for n and the range of ω large. An alternative approach, which applies if the eigenvalues of A are available, is presented and is shown to allow exploitation of the progress in numerical linear algebra made during recent years. The formulas of the alternative approach are derived from a nonstandard matrix representation of the solution to the uncontrolled equation which can be found in Ref[1]: Lukes, D. L. Differential Equations: Classical to Controlled. Academic Press, to appear.

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Matrix Method for Compartmental Model
Estimation of Demographic Health State
and Mortality Transitions

A basic problem in assessing the health state of a population is the fact that primary incidence and total prevalence for most chronic diseases are unobserved. Specialized compartment model techniques have been developed for analyzing age specific time series of cause specific mortality data to generate estimates of transition intensity functions for unobserved health states. Since differential exposure and endowment lead to considerable heterogeneity in individual transition risks, the form of each transition function is modified to allow certain parameters to be distributed in the population. Mathematically, the problem is to generate estimates of the parameters of multiple convoluted waiting time distributions where the waiting times are generated from transition functions with distributed parameters. The fact that multiple convolutions and heterogeneity in transition functions are taken into account distinguishes these methods from other multi-state multi-region projection strategies where populations are necessarily assumed homogeneous, where chronological age is the only temporal dimension of the process and where maximum likelihood procedures are presently unavailable. Consideration is made of the translation of the integral forms for the multi-health-state, multi-demographic-state matrices of transition functions into the corresponding discrete time matrix analogs.

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Graph Theoretical Approach to
Qualitative Solvability of Linear Systems

A linear system, $Ax = b$, is sign-solvable if both its solvability and the sign-pattern of its solution x are determined by the sign-patterns of A and b ; when all coordinates of x are nonzero, the system is strongly sign-solvable. The structure of such systems is studied here by a refinement of the graph-theoretic approach first suggested by Maybee. Both sign-solvability and strong sign-solvability are characterized in terms of an associated digraph, and this leads to a polynomial-time algorithm for recognizing strong sign-solvability. Under fairly general conditions on the sign-pattern of A , it is possible to determine all sign-patterns for b which render the system $Ax = b$ sign-solvable.

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BLAS Implementation for the FPS164

The well-known BLAS (Basic Linear Algebra Subroutines) of LINPACK have been coded in APAL64, the assembly language for the FPS164—a highly parallel, large word size (64 bit floating point), fast cost efficient, attached processor.

This paper will give performance results and comparisons, and will examine those speed-up techniques used in the implementation, of general applicability.

In particular, the technique of pipeline programming is extended from loops with straightline code to loops including IF-THEN-ELSE structures. This technique is illustrated in the implementation of ISAMAX, resulting in a 2 cycle per element timing.

Also, loop unrolling techniques useful for accessing and distributing output to different memory modules are shown.

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Statistical Computations Using Perpendicular Projections Operators

The perpendicular projection operator approach to statistical computations in the general linear model offers some important insights into data manipulation. This exposition first summarizes the matrix properties of the projection operator and then applies the operator to some diverse linear models. These applications include orthogonal regressions, the analysis of variance, and restricted least squares. The goal of the exposition is to fit many different statistical computations into the mold of the perpendicular projection operator. In all our examples we derive both concrete formulas and vector geometrical diagrams. In the process, we illustrate the Gram-Schmidt orthogonalization procedure, the perpendicular projection on subspaces of ellipsoids of concentration, and duality in certain subspaces of the sample space.

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A Transformation Between Two Block Companion Forms

In linear control theory the solution of the matrix equation $XC=C^T X$ (1) where, C is a block companion matrix and C^T is its block transpose, is expressed in the form $X = \sum_{i=1}^{\ell} c_i B \Theta_i$ (2) where $\ell = \dim(X)$: solution of (1),

$$B = \begin{bmatrix} A_{n-1} & \dots & A_1 & I \\ A_1 & I & & \\ I & 0 & & 0 \end{bmatrix} \text{ and } \Theta_i = \begin{bmatrix} E C^{i-1} \\ E C^i \\ \vdots \\ E C^{n+i-2} \end{bmatrix} \quad C = \begin{bmatrix} 0 & I & \dots & I \\ \vdots & & & \\ -A_n & \dots & A_2 & -A_1 \end{bmatrix} \quad E = [I, 0, \dots, 0]$$

The form (2), is also expressed in terms of suitable Bezoutian matrices Z_{i-1} of polynomial matrices $A(\lambda) = I \lambda^n + A_{n-1} \lambda^{n-1} + \dots + A_1 \lambda$, $B(\lambda) = I \lambda^{n-1}$ since $Z_{i-1} = B \Theta_i$. This is a generalization of the result obtained by Howland (Univ. of Ottawa 1962).

Moreover, an approach for the determination of ℓ in (2) is to transform the eq. (1) in the form $JY=YJ$ where J is the associated Jordan matrix of C , as defined by I. Cohberg et al. (LAA v.20 pp.1-44). This is achieved using a block confluent Vandermonde matrix.

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The Stability of LU-Decomposition of Block Tridiagonal Matrices

In solving boundary value problems of (systems of) ordinary differential equations by methods like collocation or multiple shooting, one obtains a sparse linear system of equations for the approximants of the solution on some grid.

Rather than using a standard algebraic method one usually partitions this matrix such that it can be looked upon as a block tridiagonal matrix for which a sparseness preserving block LU-decomposition method is employed. Though its stability is not appreciated, in general, it gives remarkably good results. A simple explanation is given here. We first indicate a particular LU-decomposition strategy which uses pre and post multiplication by suitable block diagonal matrices (generalizations of permutations). Under the (weak) assumption that the original boundary value problem is physically well conditioned, we show that this is a stable method. This result is used to show that other LU-decomposition methods are stable, if only they exist.

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Some Extremal Markov Chains

Using a Markov chain model for the motion of a particle through a V-node network one may introduce a figure-of-merit N , which is the average number of steps that the particle requires to travel from the originating node to the destination node. These nodes are included in the averaging. We consider both unrestricted Markov chains or those corresponding to random routing (choose available lines with equal probability) and study which networks or, equivalently, which transition matrices, minimize or maximize N . We show that for random routing the complete graph has $N = (V-1)$ and is the minimizing graph. The maximizing graph is unknown, but we establish that the worst behavior of N increases at least with the cube of the number of vertices, but no worse than the 3.5 power. Properties of the class of graphs known as bar-bells are useful here. The minimizing unrestricted chain corresponds to placing the nodes on a circle and proceeding unidirectionally from one node to the next. Here, $N = V/2$.

Many old, and some new, matrix and eigenvalue inequalities form the basis of the proofs.

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Numerical Perturbation Methods for Degenerate Nonlinear Systems

Numerical determination of periodic solutions and bifurcations are, using perturbation techniques, considered for systems of the following type:

$$\dot{x} = A(\mu)x + g(x, \mu) \quad (1)$$

$$x \in \mathbb{R}^n, \mu \in \mathbb{R}^m, \det A(\mu) = 0 \text{ for } \mu \in \mathcal{D}$$

$\subset \mathbb{R}^k$, $g(x, \mu)$, $\mathbb{R}^n \times \mathbb{R}^m \rightarrow \mathbb{R}^n$ has no discontinuities and is regularizable. By transforming system (1) into the system

$$\dot{y} = A'(\mu, \epsilon)y + g'(y, \mu, \epsilon) \quad (2)$$

$\epsilon \in \mathbb{R}^k$, $k \geq 1$, $\det A'(\mu, \epsilon) \neq 0$, $A'(\mu, \epsilon) \rightarrow A(\mu)$, $\|\epsilon\| \rightarrow 0$, such that systems (1) and (2) are structurally equivalent:

$$\|y - x\| \rightarrow 0 \text{ \& \& } \|\dot{y} - \dot{x}\| \rightarrow 0, \|\epsilon\| \rightarrow 0,$$

for all $\mu \in \mathcal{D}$; (multigrid) iterations are developed using exponential matrices, subject to the following additional constraints:

$$\epsilon \in Q^k, Q^k \subset \mathbb{R}^k, Q^k = Qx \text{ ----- } xQ$$

k-times Cartesian product of the set of rationals

$Q \subset \mathbb{R}$. Equations of type (1) are important for phase transition problems.

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An Example of Path-Following in a Subspace

A path-following procedure is described for studying solutions to a large, sparse system of nonlinear equations. Such systems arise in the solution of convection-diffusion equations described by R. Mejia and J. L. Stephenson, J. Computational Phys. 32(1979), 235. A path is followed either as a function of an arbitrary or model parameter; see R. Mejia and J. L. Stephenson, in Numerical Methods for Engineering, G.A.M.N.I.2, DUNOD, Paris (1980), 1003. We show that when a path between solutions exists as a function of a sequence of parameters, this path is attainable by continuation in a subspace: namely, we find the roots of $F(g_1, g_2, \dots, g_m, a) = 0$, with independent vectors g_j and parameter a , by iteratively solving a sequence of problems $F_j(g_j, g_m, a) = 0$, $1 \leq j \leq m-1$, with low dimension for $g_j(g_m, a)$ and a larger system $F_m(g_m, a) = 0$. Turning points and simple bifurcations of $F_m^{-1}(0)$ are handled

using methodology due to M. Kubiček, ACM TOMS 2(1976), 98 and H.B. Keller, in Applications of Bifurcation Theory, Academic Press (1977), 359.

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Numerical Solution of the Interior Neumann Problem via Vortex Dipoles

Vortex methods applied to viscous flow problems containing immersed boundaries of arbitrary configuration require the superposition of a potential flow to secure the satisfaction by the resulting flow of the normal boundary condition, to calculate this potential a Neumann problem must be solved. In this work we introduce a numerical technique for solving the interior Neumann problem for domains with irregular boundaries with corners. The method is intended for use in regions in which direct methods are difficult to implement. The solution is derived from the potential of a collection of discrete vortex dipoles (vortex pairs of opposite strengths) collocated along the boundary.

The dipoles axii are tangent to the boundary, and the vortices' strengths are found by solving the resulting Fredholm integral equation of the second kind iteratively.

The method has been used on a region which is a coarse approximations to an aortic sinus, to solve a Neumann problem whose data is the normal component of the flow induced by a collection of vortices. Results compare favorably against those yielded through the approximation of a double layer potential introduced by Smith.

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Generalized Matrix Functions and Pattern Inventory

Let G be a subgroup of S_m . Two functions $\alpha, \beta: \{1, 2, \dots, m\} \rightarrow \{1, 2, \dots, n\}$ are equivalent modulo G if there is a g in G such that $\alpha g = \beta$. The Pólya-Redfield theorem enumerates the equivalence classes modulo G . The concept of distance between equivalence classes is developed and shown to be related to familiar constructions in multilinear algebra. Applications, examples, and extensions are available.

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Optimum Design and Control of Target Oriented Systems

In many control problems encountered in Target Oriented Systems (TOS) which include positioning devices, mobile systems, robot-manipulators, missiles, aircrafts with STOL and V/STOL systems, the problem of Optimum Design (OD) and Control (OC) may be addressed in terms assuming an application of the linear theory techniques. An explicit separation of the conceptual and analytical parts within the OD and OC iterative procedures, is helpful in formalization both of these parts with the acceptable level of theoretical rigidity.

The conceptual part of both OD and CD though seeming exotic within modern control theory nevertheless allows an application of some developed areas of analysis /1-4/. On the other hand, the analytical design and control which seem to be more appropriate in the framework of above mentioned separation, pose some interesting problems presented in the paper.

The redundant initial model may be reduced into the minimum order equivalent robust model /5/ by a special orthonormalization and decoupling technique which may be typical for TOS. This simplifies the application of known optimization techniques /6,7/. The multiple inversion within the decoupled TOS in order to obtain control program may be shown to be trivial although it changes the approach to the construction of a compensator. The theory is illustrated by examples in robotic area.

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Pseudoinverting a Large, Almost-Block-Tridiagonal System

A general procedure is developed for constructing the Moore-Penrose pseudoinverse of large, almost-block-tridiagonal matrices, Ω , whose null spaces are known. The work was motivated by the problem of laying out and surveying a ring of monuments for Fermilab's new superconducting accelerator, Tevatron, and the application to its solution is presented as an example. When the ring has maximum symmetry Ω becomes block circulant. The pseudoinverse can then be constructed semi-analytically by carrying out a singular value decomposition. In the general case, of course, one must rely on machine computation. The substantial round-off errors which are generated can be reduced by iteratively refining the answer. Within the context of the original problem, a conjecture is presented that if the null space of Ω has dimension four, then the ring must be maximally symmetric.

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A Very Small Numerical Linear Algebra Algorithm Package

For computing systems which lack or cannot support quality software libraries for numerical linear algebraic tasks, an alternative to purchasing suitable programs is to implement algorithms which are very compact and suitably inter-related. This presentation describes a suite of algorithms for linear equations, least squares and eigenproblems based on a singular value decomposition. Extensions to nonlinear least squares and function minimization are discussed. Various implementations are considered with sample performance figures to indicate the properties of the algorithms chosen. Particular attention is paid to the human costs of implementing and using such codes and to the necessity of adequate documentation which should also be compact yet complete.

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The Analysis of k-Step Iterative Methods for Linear Systems from Summability Theory

Using the theory of Euler methods from summability theory, we investigate general iterative methods for solving linear systems of equations. In particular, for a given Euler method p and for iteration matrices T whose

spectra are compatible with p , we derive upper and lower bounds for the associated asymptotic iterative rates of convergence. As special cases, k -step iterative methods are deduced, and techniques of "embracing" spectra are discussed. Similar ideas in the case $k = 2$ have appeared in de Pillis (1980) and Manteuffel (1977).

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Numerical Solution of Quadratic Regulator Problems

The discrete time quadratic regulator problem can be formulated as a linearly constrained linear least squares problem. This formulation leads easily to reliable numerical algorithms for solving even the most general of such problems. The numerical stability of some excellent algorithms by Kailath and others as well as Chandrasekhar-type algorithms can be examined via this formulation.

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The Bit-complexity of arithmetic algorithms and their stability and structural complexity

B , the bit-operation complexity of arithmetic problems such as matrix multiplication (MM), DFT, vector convolution (VC), solution of simultaneous linear equations (SLE) relates those problems with combinatorial ones and characterizes the stability of their solution. (If A is the number of arithmetic operations used then large B/A would indicate the instability.) It is shown that $B = O(n^{s+\epsilon} \log(M/E))$ in the case of $n \times n$ MM and simultaneously $O(n^2 \log(Mn/E))$ bits suffice for the storage space. Here M, E bound the magnitudes of the inputs (the entries of given matrices) and of the absolute errors respectively. ($A = O(n^s)$, $\epsilon > 0$ arbitrary, s can be chosen less than 2.5.) Similar results are obtained for DFT and VC. The benefits of the use of residue arithmetic are shown for VC algorithms with real parameters. In case of well-conditioned SLE, $B = O(n^2 \log(M/E))$ if $n = o(\log(M/E))$ but the problem is open otherwise. In addition to the bit-complexity, a combinatorial parameter is introduced and estimated. The parameter measures the structural complexity (asynchronicity) of algorithms for MM, VC, and DFT.

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Generalized Inverses and Scheffé's Theorem

In the theory of least squares, generalized inverses of matrices are well known to be useful tools for analysis of full as well as nonfull rank linear models. In this note, in particular it is proved that the spaces generated by the rows of matrices A and B have only null vector in common if and only if $A'A(A'A + B'B)^-A'A = A'A$ where $(A'A + B'B)^-$ is a generalized inverse of $(A'A + B'B)$. This result in some sense provides a different characterization of a theorem proved by Scheffé.

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Volume Correlation - with an Application to Air Traffic Surveillance

Given an observation in real n -space, the problem is to classify it into one of a set of nonoverlapping subspaces (regions).

One approach is to decompose the regions into n -simplexes and pre-classify the simplexes by region. It is then sufficient to know the simplex of a new observation to know its region. The method is particularly efficient when successive observations are estimates of points on a continuous curve such as a trajectory. Most of the work then consists of confirming that a new observation lies in the same simplex as the preceding one -- by evaluating $n+1$ determinants of order $n+1$.

A procedure for decomposing an arbitrary simple polygon into triangles is demonstrated by an application of mathematical induction.

A set of algorithms for applying the methods to Air Traffic Surveillance is written in a version of "FORTRAN 77."

A numerical example is included.

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Solving Word Problems in Initial Algebras by Using Complexity Classes Over N

Heterogeneous Algebras were introduced by Birkhoff and Lipson in 1970. They are used to represent different structures in mathematics and computer science. Given the signature S and the set of equations E of an algebra we are interested in determining if $t_1 = t_2$, where t_1 and t_2 are terms in the free algebra $T(S)$ generated by S , and $=$ is the smallest congruence on $T(S)$ induced by E . The set of representatives of the congruence classes together with S form the initial algebra $I(S, E)$. The problem of determining the equality of two arbitrary terms in an initial algebra has been studied

by Knuth and Bendix, Rosen, Huet, Lankford, Sielman and others. Our approach consists of attaching to each term of the free algebra $T(S)$ natural number called its complexity. Those complexity functions are used in generating a program for $R:T(S) \rightarrow T(S)$, which maps each term into the representative of the congruence class to which it belongs. We study the relationship between the axioms and the linear complexity function, i.e., the complexity defined as $c(f(t_1, \dots, t_n)) = c(f) + \dots + cnc(t_n)$ for all functions f in S .

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Analysis of Deterministic Trends in Gravity Gradiometer Data

Modern techniques in applied linear algebra are used in the analysis of deterministic trends in Gravity Gradiometer Instrument (GGI) data. The theory and practice of deterministic modeling are discussed with particular attention to the problems of large data sets (over half a million points), colored noise and exponential trends in GGI test data. Topics include: The solution of least squares problems using QR decomposition; the implementation of Gauss-Markov estimation for large data sets; and the solution of non-linear least squares problems using the technique of separable least squares. Background information is given on the Bell Aerospace Textron and Charles Stark Draper Laboratory GGIs. Test data from these instruments are analyzed using the computational methods described. The paper is based on work done by the authors at Sperry Systems Management.

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Eigenvalues for Multi-Module Markov Decision Processes

This presentation shall consider the computation of eigenvalues and eigenvectors for a special class of Markov Decision Process (MDP), known as the Multi-Module Markov Decision Process (3MDP). Solution of the 3MDP with M modules depends on the solution of several subglobal MDP's. In this paper, it is shown how the eigenvalues and eigenvectors for the 3MDP may be determined without the computational expense of forming and solving the subglobal MDP's.

These results are derived in part from well-known results concerning Kronecker products and eigenvalues.

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A Matrix Test for Graph Symmetries

Let G be a graph with vertex set $V = \{1, 2, \dots, n\}$ and adjacency matrix A . Define a partition P_k of V by putting two vertices together in the same class iff the corresponding rows of

$$W_k = [e_k, Ae_k, \dots, A^{n-1}e_k]$$

are identical (e_k is the k th column of the identity). If Γ_k is the group of automorphisms of G

that fix k , then the partition of V into the orbits of Γ_k is finer than P_k .

This partitioning can be used to study the automorphisms of trivalent, vertex-transitive graphs.

- (1) Such a graph is 0-symmetric if the group Γ_1 (all the Γ_k are isomorphic) is trivial - its orbit partition is all singletons. Sufficient conditions for 0-symmetry, in terms of P_1 , are easily derived and tested. Experimental results are encouraging.
- (2) If a graph is edge- and vertex-transitive, it is said to be regular. Necessary conditions for regularity can also be phrased in terms of P_1 and easily tested.

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Implementation of a Double-Basis Simplex Method for the General Linear Programming Problem

G O
The basis is factored as DP Q,
H I

where D is the basis at the last reversion, and P and Q are permutations. Computational results for two implementations on problems of up to about 1000 constraints are given. One implementation utilizes sparse LU factorization for G ; its speed is about that of the standard simplex method. The second implementation uses an explicit g^{-1} . Detailed breakdowns of run times and nonzero growth are analyzed. The implementations are based on Marsten's modularized XMP package which in turn employs Reid's LA05 routines. Standard method routines are replaced by appropriate double-basis

routines. Multiplications by the basis inverse are performed without explicit reference to H . Updating techniques are presented.

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Correlation and Determinacy in Linear Systems and Networks

Let the variables $x = (x_1, \dots, x_n)$ be subject to the system of linear constraints $Ax = b$. For B a basis of columns of A , form the equivalent system $B^{-1}Ax = x_B + Nx_N = B^{-1}b$. Set $M = \begin{pmatrix} I & N \\ 0 & -I \end{pmatrix}$. We define the row (column, resp.) correlation between x_i and x_j with respect to B to be the scalar product of the rows (columns, resp.) of M corresponding to x_i and x_j , and we say that this correlation is strong if every non-zero term in the scalar product has the same sign. We call x_i and x_j determinate, in any of the above four senses, if their correlation has the same sign or is zero over all bases of the system. The four definitions of correlation correspond directly to statistical correlation, rates of substitution in economics, sensitivity in linear programming, and sign solvability in linear algebra. Further, the property of determinacy is independent of which definition of correlation is used. Finally, in systems related to networks, good characterizations of determinacy can be derived in terms of properties of the underlying network.

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Matrix Theory via Optimization Methods

The paper surveys certain applications of game theoretic and linear programming methods to the theory of M -matrices, non-negative and power positive matrices, P matrices and doubly stochastic matrices. The main tools are the minimax theorem for completely mixed games, the duality theorem of linear programming and the Kuhn-Tucker theorem of non-linear programming. In the process some new results besides some of the classical results on such matrices are also obtained.

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Solving Equations by Row and Column Reduction

Any $n \times m$ matrix A can be reduced by row and column operations to the direct sum of an identity matrix whose size r equals the rank of A , and a zero matrix. The reduction is equivalent to pre- and post-multiplication by invertible matrices R and C . Let C be partitioned into r and $(m-r)$ columns $C = [S, N]$ and R be partitioned into r and $(n-r)$ rows $R^t = [T^t, M^t]$. Theorem 1 An equation $Ax = b$ has a solution if and only if $Mb = 0$, and in this case the general solution is $x = STb + Nz$ with z free. Theorem 2 The matrix ST is a generalized inverse of A (actually a $[1,2]$ -inverse). Theorem 3 (a). The matrix R may be chosen so that $TM^t = 0$, and in this case ST is a $[1,2,3]$ -inverse of A ; (b) In addition C may be chosen so that $S^tN = 0$, and then ST is the Moore-Penrose pseudoinverse of A .

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Asymptotic Methods for the Determination of Relaxation Convergence Rates

Garabedian, [1], interpreted the SOR solution of the difference equation approximating the Dirichlet problem for the Laplace equation as the solution, by difference methods, of a time dependent problem $du/dt = L(u, h)$ where $L(u, h)$ is an operator different from that of Laplace. The great value of this contribution is its applicability to various finite difference analogs of Laplace's equation regardless of whether or not they possess property A. By taking $\omega = 2/(1+ch)$, the usual asymptotic convergence rates could be obtained. In this paper, we exploit this idea to 1) Show how the usual asymptotic rates of convergence for iterative methods other than SOR can be obtained, and 2) Derive new pre-conditioning systems for the biharmonic problem along with their corresponding convergence rates.

[1] Math Tables and Aids to Comp. 10, 1956, 183-185

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On the Exponent of a Primitive, Nearly Reducible Matrix II

A nonnegative matrix is called nearly reducible provided it is irreducible and the replacement of any positive entry by zero yields a reducible matrix. The purpose of this article is to investigate the exponent $\gamma(A)$ of an $n \times n$ primitive, nearly reducible matrix A . Our principal result is that $\gamma(A) \leq n+s(n-3)$, where s is the length of a shortest circuit in the directed graph associated with A . It is an easy application of this result to find gaps in the exponent set of $n \times n$ primitive, nearly reducible matrices. We also show that for integers n, k satisfying $n \geq k-1 \geq s$ there exists an $n \times n$ primitive, nearly reducible matrix with exponent k . The proofs are carried out by means of directed graphs.

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Algebraic Aspects of Second Order Optimization of Practical Feedback Control Laws for Multivariable Time-Invariant Linear Dynamic Systems

The addition of practically required constraints on the feedback matrix to the standard linear-quadratic-Gaussian problem in optimal control theory severely hampers the attainment of a solution. Practical examples illustrate nonconvexity of the objective function even in a tiny neighborhood of a strong minimum. Evaluation of the objective function and its first and second derivatives is shown to depend on solving: $A' X_k + X_k A = Y_k' Y_k$ and its adjoint, where A' is the transpose of the A matrix. The relationship of the constrained solution to the unconstrained one is given in terms of (unknown) Lagrange multipliers. Motivation for constraints is developed based on ever-present bandwidth limitations on sensors, effectors, and noise as well as on extreme ill-conditioning of the Hessian matrix at strong minima. The latter phenomenon indicates a large payoff in reliability for a small price in performance. This mathematical problem is pivotal in coping with such distributed control areas as: large space structures, fusion engineering, and aircraft divergence and flutter suppression.

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On the Computation of the Impulse Response Energy of a Linear Multivariable System

In obtaining reduced models of high order systems, an important approach is to retain the modes which contribute most to the impulse response energy of the system. Thus, given the multivariable system described by

$$\dot{x} = Ax + Bu, \quad \text{where all the components of } u(t) \text{ are unit impulses}$$

$$y = C^T x$$

The impulse response energy is given by $E = b^T Q b$ where b is the sum of the columns of B and Q is the solution of the matrix equation

$$A^*Q + QA = -CC^T$$

where A^* denotes the conjugate transpose of A .

A method for solving the above equation is presented which requires the left and right eigenvectors of A and the use of Kronecker products. The system is first transformed to the Jordan form and an explicit solution is obtained for each block of the transformed Q matrix. Finally an expression is obtained for directly evaluating the component of the impulse energy due to each eigenvalue of A . The result is quite general and even multiple complex eigenvalues are considered.

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Pursuit on a Cyclic Graph - The Symmetric Stochastic Case

In the cyclic pursuit game the evader, BLUE, and the pursuer, RED, choose a vertex of an n point cyclic graph at discrete time 1. If they initially choose the same vertex, BLUE receives payoff one. At each subsequent time BLUE may remain where he is or move to an adjacent vertex. RED has the same capability. At no time do RED and BLUE know the other's location. The game ends when RED and BLUE meet. BLUE then receives a payoff equal to the time of his arrival.

In this paper we solve the game when both RED and BLUE are restricted to stochastic strategies for which each moves right or left with equal probability. To do this we obtain the transition matrix A for an isomorphic game and then determine the payoff function in terms of this matrix. Using arguments concerning bounds on the eigenvalues of A and analysis of the payoff function we derive formulas for the value of the game and the optimal strategies for RED and BLUE. We then construct computer programs for constructing solutions in the case when the cyclic graph has 10 or fewer points.

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Practical Use of Krylov Subspace Methods for Solving Large Indefinite and Unsymmetric Linear Systems

The main purpose of the talk is to show how to develop stable versions of some algorithms based on Krylov subspace methods. As for the SYMMLQ algorithm in the symmetric case, our algorithms are based upon stable factorizations of the banded Hessenberg matrix representing the projected part of the matrix A on the Krylov subspace. We show how an algorithm similar to Paige and Saunders' SYMMLQ can be developed for unsymmetric problems but we will also describe a more economical algorithm based upon the LU factorization with partial pivoting. In the particular case where A is symmetric, our new algorithm is theoretically equivalent to SYMMLQ but is more economical. An advantage of the method is that unsymmetric or indefinite or both unsymmetric and indefinite systems of linear equations can be handled by a unique algorithm. Some numerical experiments will be reported.

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The Vec and Vech Operators and their Applications to Statistics

The vec operator converts a matrix to a vector by stacking all its columns, one beneath another, to form a single, long vector; and vech \tilde{X} , for square \tilde{X} , does the same thing except for starting each column at its diagonal element. Both operators occur spasmodically in the literature, under a multitude of names and notations; a review is given. Both operators are useful for solving matrix equations of the form $\sum_{i=1}^m \tilde{A}_i \tilde{X} \tilde{B}_i + \sum_{j=1}^n \tilde{X}' \tilde{E}_j = \tilde{C}$ for \tilde{X} , and for developing Jacobians of one-to-one transformations: $J(\tilde{X} \rightarrow \tilde{Y})$ is $\|\partial \text{vec} \tilde{X} / \partial \text{vec} \tilde{Y}\|$ for non-symmetric matrices and $\|\partial \text{vech} \tilde{X} / \partial \text{vech} \tilde{Y}\|$ for symmetric matrices. Extension to any other patterned matrix is shown, using the general operator $\text{vecp} \tilde{X}$ for \tilde{X} of pattern p . Applications in statistics arise not only from Jacobians but also from matrix differentiation where the use of vec and vech show that, contrary to customary usage, each of $\partial \text{tr}(\tilde{A} \tilde{X}) / \partial \tilde{X}$ and $\partial \log |\tilde{X}| / \partial \tilde{X}$ have a value for symmetric \tilde{X} different from that for non-symmetric \tilde{X} . The operators are also extremely convenient in providing succinct and tractable notation for multivariate analysis and enable multivariate data to be analyzed as a special form of univariate data. They also provide tools for developing new results; e.g., fourth moments in a general linear model with non-zero kurtosis parameters.

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Nonsymmetric Distance Matrices and their Realizations by digraphs

A non-symmetric distance is a distance which no longer satisfies the axiom $d(x,y) = d(y,x)$. Non-symmetric distance matrices may be realized by digraphs. Similarities and differences with known results for distance matrices and their graph realizations will be presented.

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The Lanczos Algorithm with Partial Reorthogonalization

The Lanczos Algorithm is becoming accepted as a powerful tool for finding the eigenvalues of a matrix and for solving linear systems of equations. Any practical implementation of the algorithm suffers however from roundoff errors, which usually cause the Lanczos vectors to lose their mutual orthogonality. In order to maintain some level of orthogonality, full reorthogonalization (FRO) and selective orthogonalization (SO) have been used as a remedy in the past. Here, partial reorthogonalization (PRO) is proposed as a new method for maintaining semiorthogonality among the Lanczos vectors. PRO is based on a simple recurrence, which allows us to monitor the loss of orthogonality among the Lanczos vectors directly without computing the inner products. Based on the information from the recurrence, reorthogonalizations occur only when necessary. Thus, substantial savings are made as compared to FRO.

In some numerical examples, the Lanczos algorithm with PRO is used for the solution of large sparse symmetric systems of linear equations. The results indicate that the algorithm is especially useful when the matrix vector product dominates other costs, or when the system has to be solved for several right-hand sides.

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Dimensionality of Bi-infinite Systems

Suppose that A, B are two $m \times n$ real matrices. Let K be the solution set of bi-infinite vectors $\{x_j\}$ satisfying

$$Ax_j + Bx_{j+1} = 0.$$

In this paper we study the dimensionality of K . In particular, we characterize the case of finite dimensionality of K using an

A,B-invariant pair of subspaces. Relationships with the Kronecker canonical form of the pencil $A + \lambda B$ and applications to spline interpolation are discussed.

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Estimating Parameters of Positive Semi-Definite Quadratic Form

Positive semi-definite quadratic forms are frequently encountered in multivariate analysis, primarily as a result of the widespread use of the multivariate normal distribution. Consideration is given to the constrained estimation problem of maintaining the property of positive semi-definiteness of the parameter matrix in a quadratic form involving measured, known variate values. Examples of such matrices are the covariance matrix in the multivariate normal distribution or the difference between two covariance matrices in a dynamic model. The proposed estimation method is a quasi-Newton search algorithm which involves simultaneous diagonalization by rotation of the parameter matrix and the Newton direction matrix. Motivation for dealing with this particular rotation is due to the difficulty of imposing appropriate constraints on the Newton direction matrix to guarantee preservation of the Gramian properties of the updated parameter matrix. In this rotated coordinate system, both the $n \times n$ Newton direction matrix and the $n \times n$ parameter matrix have at most n non-zero elements all of which are on the diagonal of the matrix. Positive semi-definiteness can be maintained by employing Kuhn-Tucker procedures to maintain non-negative diagonal elements in the updated parameter matrix. If the objective function is a likelihood, the parameter covariance matrix can be obtained by pseudo-inversion of the singular information matrix.

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Strict Positive Invariance

Let A be a real $n \times n$ matrix, and let $C \subset \mathbb{R}^n$ be a proper cone. It is proven that if C is strictly positively invariant (that is, $e^{tA}(C/O) \subset \text{int} C \quad \forall t > 0$) then there exists a certain direct-sum decomposition of \mathbb{R}^n into A -invariant subspaces. Our results lead to a characterization of the set of initial points whose trajectories reach C under the differential equation $\dot{x} = Ax$. An application in stability is offered as well.

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The Use of Matrix Series Expansions in Computing Posterior Moments in Bayesian Inference

Let Ψ be a random diagonal matrix; let A_1, A_2, \dots represent non-random symmetric matrices. Expectations of products such as $E(\Psi A_1 \Psi A_2 \Psi A_3 \Psi)$ appear as terms in the series expansion of any of various posterior moments in certain Bayesian inference problems. For example, the posterior mean of a regression coefficient in a separate-regressions model can be expressed in terms of $E(\Psi^{-1} + A)^{-1} = E(\Psi) - E(\Psi A \Psi) + E(\Psi A \Psi A \Psi) - \dots$ which is valid under certain eigenvalue conditions. We present a formula for computing the components of the matrix product $\Psi A_1 \Psi A_2 \dots \Psi A_n \Psi$ as homogeneous polynomials in the components of Ψ , thus facilitating the taking of expectations.

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Rank Additivity and Pairwise Orthogonality of Real Square Matrices

Let $A(1), \dots, A(k)$ be real square matrices, not necessarily symmetric, and let $A = A(1) + \dots + A(k)$. Suppose that the $A(i)$ are pairwise orthogonal in the sense that $A(i)A(j) = 0$ for all $i \neq j$. Then Khatrı (*Sankhyā Ser. A*, 1968) has shown that if A is idempotent and each $A(i)$ has index 1, viz. $\text{rank}[A(i)^2] = \text{rank}[A(i)]$, then rank is additive: $\text{rank}(A) = \text{rank}[A(1)] + \dots + \text{rank}[A(k)]$. We extend this by proving

that when the $A(i)$ are pairwise orthogonal then the set of nonzero eigenvalues of A coincides with the set of all the nonzero eigenvalues of all the $A(i)$; furthermore, the nonzero eigenvalue λ of A is regular if and only if λ is a regular nonzero eigenvalue of each $A(i)$. When, in addition, rank is additive, then the eigenvalue 0 of A is regular if and only if 0 is a regular eigenvalue of each $A(i)$, i.e., A has index 1 if and only if each $A(i)$ has index 1. It follows that when the $A(i)$ are pairwise orthogonal and rank additive then $[A(1)]^p, \dots, [A(k)]^p$ are rank additive for any positive integer p . For related developments see T.W. Anderson & Styan (*Statistics and Probability: Essays in Honor of C.R. Rao*, North-Holland, Amsterdam, 1981) and Takemura (Tech. Rep. 44, Dept. of Statistics, Stanford University).

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An Algorithm to Obtain a Submatrix
and/or Permutations of a Sparse Matrix

An algorithm to obtain a submatrix from a sparse matrix is presented. If the rows and columns of the submatrix correspond to rows and columns in the same order in the original matrix, the submatrix obtained has its rows and columns in order. The same algorithm can be used to obtain row and column permutations of a sparse matrix. Execution times comparing this algorithm and the fastest published so far [HALFPERM by F. Gustavson, TOMS, 4, 1978, pp. 250-268], on a variety of sparse matrices taken from the literature and from the Institute for Economic Analysis' Data Base, are presented. The new algorithm is faster, but it leaves the rows (if the matrix is stored columnwise) unordered. If the rows are needed in order, the additional time to use MC20B [from I. Duff's MA28 set of subroutines] makes the use of HALFPERM twice, run faster.

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A Trellis Shortest Path Problem Approach
to Quadratic Integer Programming

This paper is concerned with the integer quadratic program

$$\min_{\underline{u}} \quad \underline{u}^T \underline{R} \underline{u} - 2 \underline{u}^T \underline{c}$$

where the variables \underline{u} are constrained to belong to a given set of discrete values. This quadratic

integer program is shown to be equivalent to a problem of finding the shortest path in a particular directed graph called a trellis when \underline{R} is a positive-definite symmetric banded matrix. An efficient procedure for solving this shortest path problem is presented which allows the solution of the integer quadratic program. This method is particularly effective when the half-bandwidth of \underline{R} is significantly smaller than its dimension.

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Partitions of Vector Spaces and Single
Error Correcting Perfect Codes

Let $V = F^n$ denote the space of all n -tuples over a field F . A partition of V is a collection of subspaces V_1, V_2, \dots, V_k such that $V = V_1 \cup \dots \cup V_k$ and $V_i \cap V_j = \{0\}$ for all $i \neq j$. It is known that when F is a finite field a partition of V induces a perfect linear code which is single error correcting. We discuss conditions for the existence of such partitions in the special case when $F = GF(2)$.

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On the Discrete Lyapunov Matrix Equation

The Lyapunov equation is of particular interest to the control engineers. Many papers have been published concerning the bound of the solution of the Lyapunov matrix equation. More, Fukuma and Kuwahara derived a lower bound for the determinant of the solution to the continuous Lyapunov matrix equation (IEEE Transactions on Automatic Control, Vol. AC-26, No. 4, August 1981). Since discrete-time systems have become more and more exposed in the recent years, we present here a result that provides a bound for the solution to the discrete Lyapunov matrix equation having the form $P = A^T P A + B$. The result gives a lower bound for the determinant

of the matrix P . It turns out that this lower bound depends on the determinant of the matrix B , the order and the trace of the matrix A .

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Interval Arithmetic and Hansen's
Problem 3 Revisited

This paper reconsiders the interval arithmetic matrix computations of Hansen's Problem 3 (Eldon Hansen, "Interval Arithmetic in Matrix Computations, Part I"; SIAM Jnl. Numer. Anal., Ser B, vol. 2, #2, 1965, pp. 308-320.) using the M77 Fortran Interval Arithmetic compiler recently released by the University of Minnesota. Problem 3 involves the approximate inversion of a matrix with bounds on round off error.

The modified M77 Fortran compiler is perhaps the most efficient software implementation of interval arithmetic that is available for large mainframe computers. Interval arithmetic performed using M77's XOTHER variables has been found to be only 4 to 7 times slower than non-interval arithmetic versions of the same calculations.

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A Different Approach to the Field of Values

The field of values $W(A)$ of a matrix A is of great theoretical and practical importance. We propose to study $W(A)$ via the polar decomposition of $A : A = VP$ with $VV^H = I$, $P = P^H \geq 0$ and their fields of values $W(V)$ and $W(P)$. Since V and P are both normal, $W(V)$ and $W(P)$ can easily be determined as the convex hulls of their respective eigenvalues. The main ideas for this investigation stem from the 2×2 case where $W(A)$ and

the polar factors P and V of A can be computed explicitly. In the easiest case, if $\text{tr } A = 0$, then the midpoint of $W(P) \in \mathbb{R}$ gives the length of the major halfaxis of $W(A)$ while the length of $W(P)$ gives the length of its minor axis. $W(V)$ is a chord of the unit circle in direction of the major axis of the ellipse $W(A)$. For $n > 2$, no explicit descriptions of $W(A)$ or P and V are known. For $A \in \mathbb{C}^{nn}$ we test the conjectures that the location and size of $W(P) \in \mathbb{R}$ determine the minimal and maximal diameters of $W(A)$ and that the 'direction' of $W(V)$ determines that of the largest diameter of $W(A)$ in several examples.

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The Ellipsoid Algorithm for Linear
Inequalities in Exact Arithmetic

We present a modification of the ellipsoid algorithm, capable of finding an exact feasible solution to $Ax = b$, $x \geq 0$, in polynomial time. All the necessary rational arithmetic is performed exactly. The bulk of the computations consists of a sequence of linear least squares problems, each a rank one modification of the preceding one. We use the Continued Fractions Jump to compute some of the coordinates of a feasible point. For problems with N variables, the number of rank one updates on the solution of a linear system, needed to find a feasible point, if any exists, is as follows:

- 1) In exact arithmetic, $O(N^3 \log d)$, where d is the largest integer entry, in absolute value, in A and b .
- 2) In approximate arithmetic, $O(N^2 \log(Nu))$, where u is the reduction in the uncertainty, in each variable, of a feasible point.

Both approaches degenerate, for $N = 1$, into a standard dichotomous search on a line segment. As an extra bonus, the rank one updates to be performed are fill-less, in the sense that they do not alter the sparsity of the linear systems to be solved.

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On a partitioning Technique for the
Problem $AX + XB = C$

A slight variant of the Bartels-Stewart algorithm or the Hessenberg-Schur algorithm for the above problem is described. The idea is to transform A and B to upper Hessenberg form first and then use the QR algorithm to produce zeros on the subdiagonal at some preset locations. The problem can then be partitioned in subproblems of smaller size. In this way one can eventually save in auxiliary storage location and in the number of operations without affecting the numerical stability. The implication of this divide and conquer strategy on the computation of eigenvalues of a matrix and on the solution method of the matrix Lyapunov equation is also considered.

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A Transition Matrix for a Markov Chain

I establish that for fixed positive integers $\alpha > 1$ and n , $\lambda_t = (\alpha^t n_t) / (\alpha n_t)$ for $0 \leq t \leq n$ are all the $n+1$ eigenvalues of the $(n+1) \times (n+1)$ matrix P whose $(i+1, j+1)$ -entry is $P_{i+1, j+1} = \binom{(n-i)\alpha}{n-j} \binom{i\alpha}{j} / \binom{n\alpha}{n}$, $0 \leq i, j \leq n$, where $\binom{n}{0} = 0!$ and 0^0 are defined to be 1. $n_t = n(n-1)\dots(n-t+1)$ for $1 \leq t \leq n$, $n_0 = 1$, and $n_t = 0$ for all the other integer values of t . Explicit formulas for all the corresponding eigenvectors X_t are given as determinants in Theorem 2. The $(i+1)$ th entry $x_{i,t}$ of X_t is a polynomial of degree t , having coefficients that are functions of n , t , and α but not of i .

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A Probabilistic Price Index Model

Suppose M is a real symmetric $n \times n$ matrix, the diagonal entries of M are positive, the off-diagonal entries are non-positive, the sum of every row is zero, and M is not a direct sum (i.e., M does not consist of two or more blocks on the diagonal with zeroes elsewhere). Then, Theorem. M has rank $n-1$.

This theorem leads to necessary and sufficient conditions on the data that guarantee the uniqueness of the LS estimates for the coefficients of a probabilistic price index model. The application of the model is to markets where trading is not continuous (e.g., real estate, collectibles).

The model is a development of ideas first considered in [A. Court, "Hedonic Price Indexes with Automobile Examples," in *The Dynamics of Automobile Demand*, New York, General Motors Corp., 1939] and in [M. Bailey, R. Muth, H. Nourse, "A Regression Method for Real Estate Price Index Construction," *J. of Amer. Stat. Assoc.*, vol.58 (1963), 933-942].

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Positivity Sets

Combinatorial properties of certain classes of sets can be established by employing linear algebra methodology. These collections of sets are important in the study of empirical measures.

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The Algebraic Geometry of Stresses in Frameworks

A bar-and-joint framework, with rigid bars and flexible joints, is said to be generically isostatic if it has just enough bars to be infinitesimally rigid in some realization in Euclidean n -space. We determine the equation that must be satisfied by the coordinates of the joints in a given realization in order to have a non-zero stress, and hence an infinitesimal motion, in the framework. This equation, called the pure condition, is expressed in terms of certain determinants, called brackets. The pure condition is obtained by choosing a way to tie down

the framework to eliminate the Euclidean motions, computing a bracket expression by a method due to Rosenberg, and then factoring out part of the expression related to the tie down. A major portion of this paper is devoted to proving that the resulting pure condition is independent of the tie-down chosen.

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Detecting Mechanisms with Linear Algebra

When is a framework, with bars and universal joints, a mechanism (with a non-trivial finite motion) and when is it rigid? For many interesting examples this is an unsolved problem. The traditional first step has been the study of infinitesimal motions (the velocities which might extend to a finite motion) using the linear algebra of the rigidity matrix. While recent extensions of these motions (e.g. the n th order motions of Connelly) use non-linear algebra, we refine the infinitesimal motions, by iterated steps of linear algebra, to select out n -step motions. The method follows a naive vision of a finite motion as an infinite sequence of infinitesimal motions, made precise in terms of algebraic geometry. We construct a sequence of matrices, for a given graph, starting with the rigidity matrix and adding new rows at each stage. When the rank of the matrix becomes constant for a realization, then this rank determines the rigidity of the framework or the presence of a mechanism. We illustrate with some new classes of mechanisms and rigid frameworks with simple graphs in the plane and in space.

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Distance Matrices and Graph Realizations

Distance matrices are (traditionally) square matrices with real non-negative entries d_{ij} , such that for all i, j, k : $d_{ii} = 0$ and $d_{ij} + d_{ik} \geq d_{kj}$.

They can be realized by graphs. Bounds for the number of non-tree-realizable principle submatrices of a non-tree-realizable distance matrix will be discussed. Non-square distance matrices will be also introduced.

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On the Matrix Congruence

$$\underline{0}^{p+1} \equiv I_2 \pmod{p}$$

This is an exposition of a result of the Scottish geometer, W. L. Edge. (Canadian Journal of Mathematics, 8(1955), 371-2.)

Let $\underline{0}$ be of the form $\begin{pmatrix} a & b \\ -b & a \end{pmatrix}$, $a, b \in GF(p)$. Let j_2 satisfy: $j^2 + 1 \equiv 0$, either \pmod{p} or $\pmod{p^2}$, to be specified, p a rational prime. Define the complex units: $z \equiv a + bj$, $z^* \equiv a - bj$ such that $zz^* \equiv 1$. We prove the following theorem: (A) There exists $q \in \mathbb{Z}^+$ such that $\underline{0}^q \equiv I_2$ iff $z^q \equiv z^{*q} \equiv 1 \pmod{q}$. (B) Now, specify p : $p \equiv 3 \pmod{4}$. In this case, j , z , z^* not in $GF(p)$ but in $GF(p^2)$, a "quadratic extension" of $GF(p)$. Then, $z^p \equiv z^*$ and so $z^q \equiv 1 \equiv z z^* \equiv z^{p+1}$, all modulo p . Conclusion: Combining (A) and (B), any $\underline{0}$ defined with coefficients in a field of order p , where $p \equiv 3 \pmod{4}$ satisfies $\underline{0}^{p+1} \equiv I_2 \pmod{p}$.

Next, we determine two j 's explicitly in terms of primitive roots over the fields $GF(7)$ and $GF(11)$, respectively and eliminate in either case between the complex forms z and z^* to obtain explicitly the entries of the matrices satisfying:

$$\begin{pmatrix} -2 & -2 \\ 2 & -2 \end{pmatrix}^8 \equiv I_2 \pmod{7}, \quad \begin{pmatrix} 3 & -5 \\ 5 & 3 \end{pmatrix}^{12} \equiv I_2 \pmod{11}.$$

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