Ranking Methods in Machine Learning

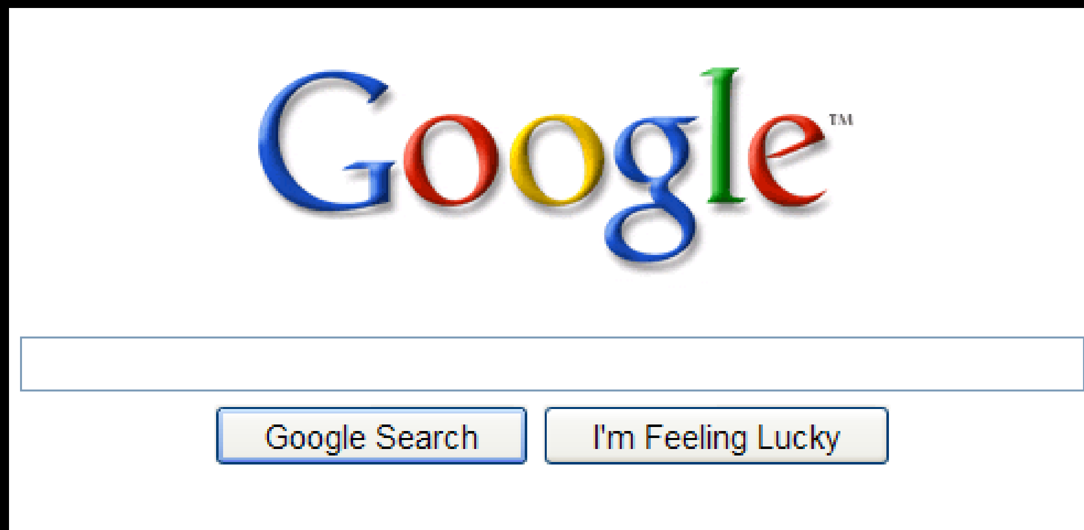
A Tutorial Introduction

Shivani Agarwal

Computer Science & Artificial Intelligence Laboratory
Massachusetts Institute of Technology

Example 1: Recommendation Systems

Example 2: Information Retrieval



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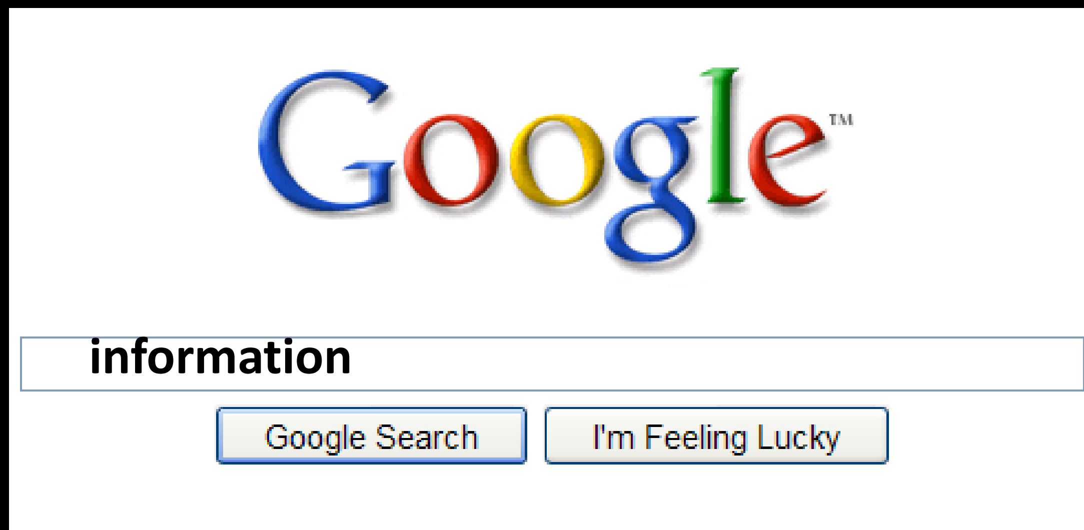

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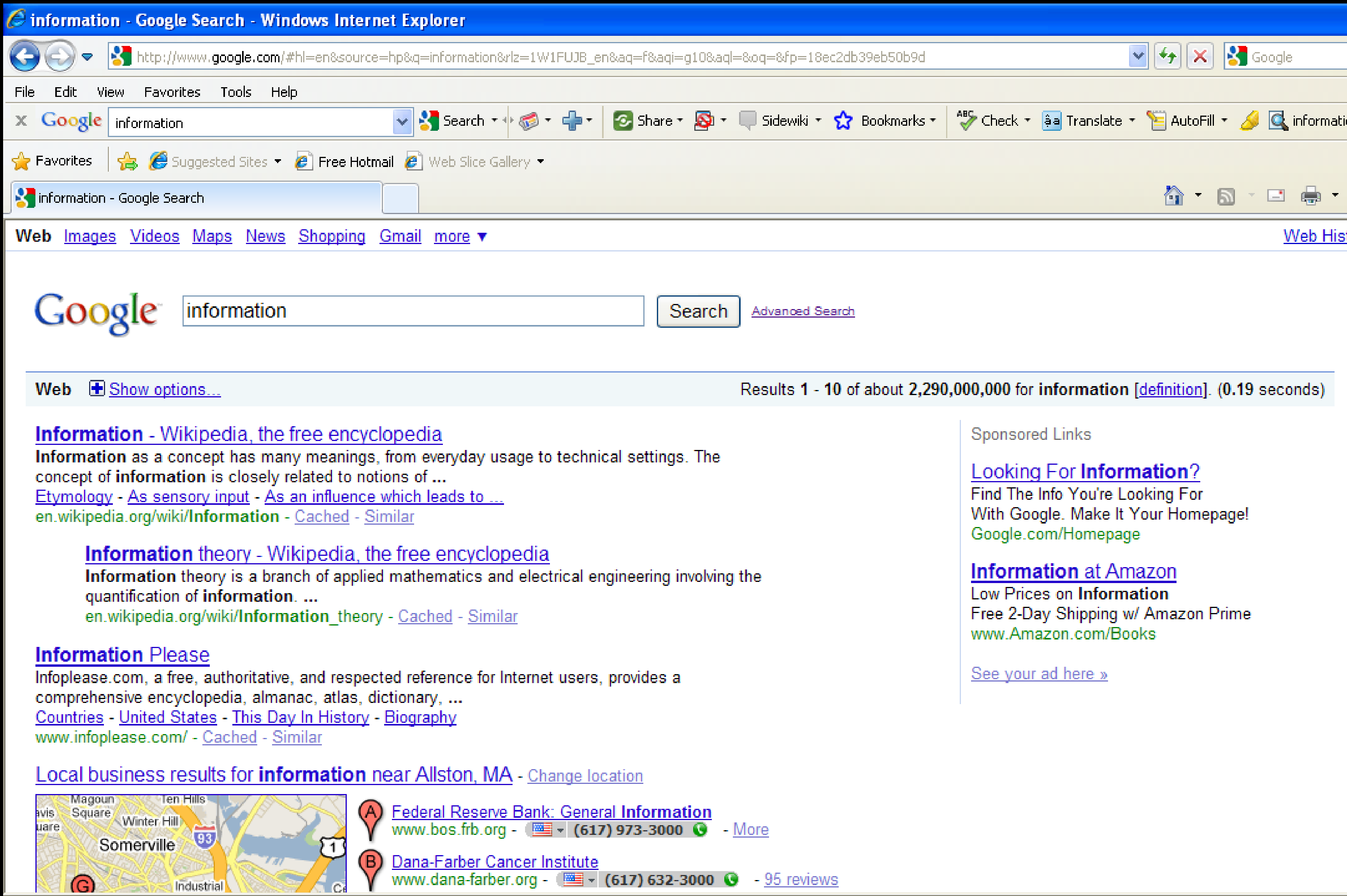
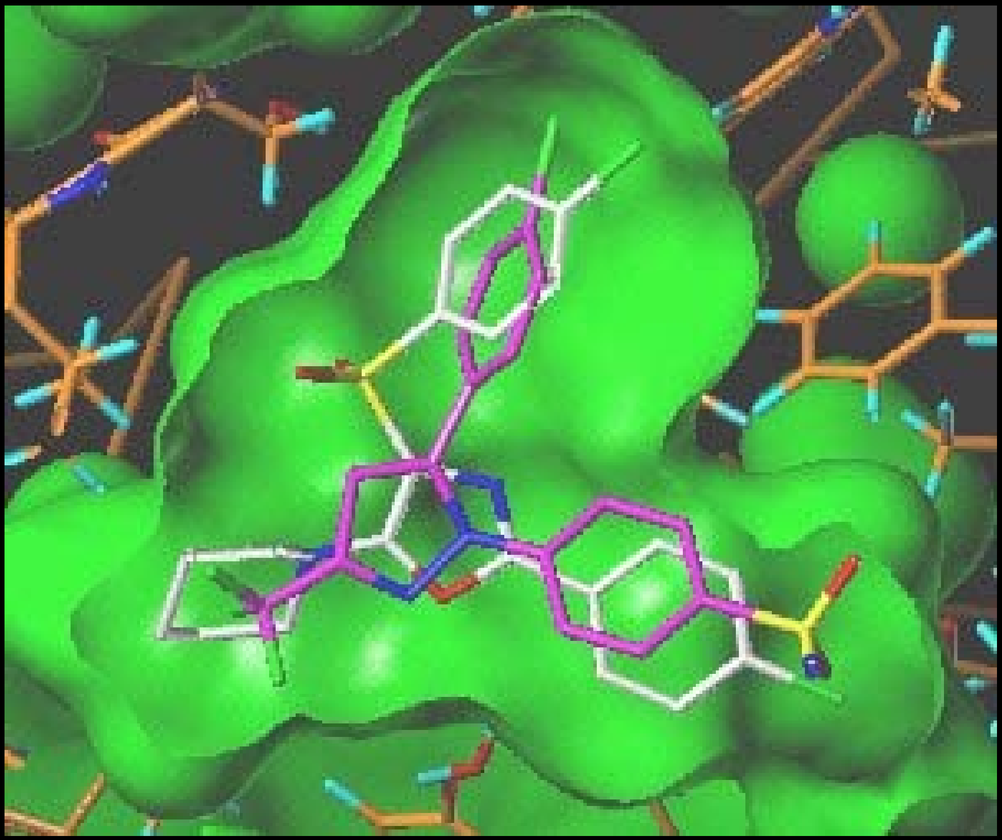
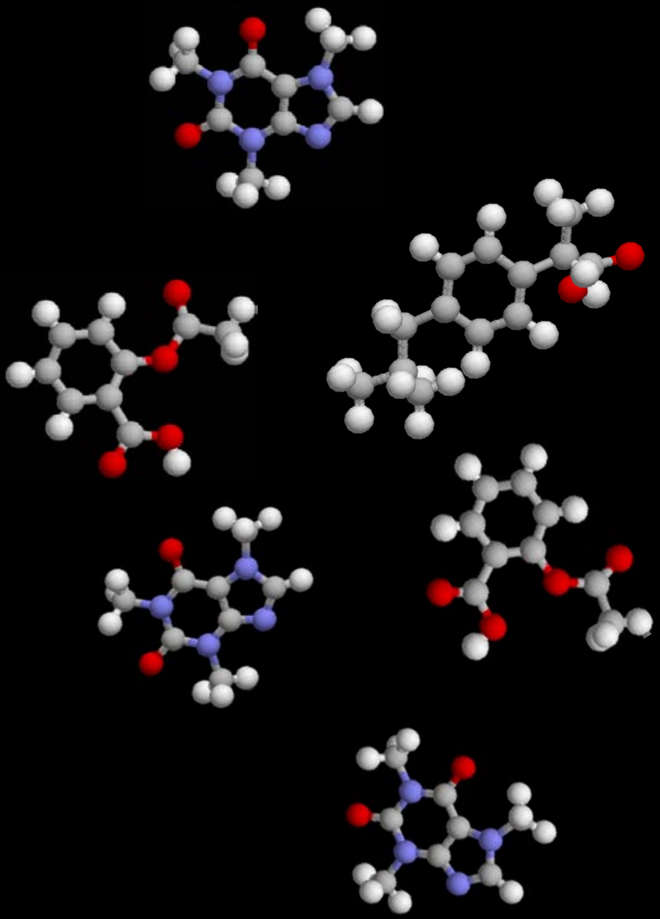


Example 3: Drug Discovery





Problem: Millions of structures in a chemical library. How do we identify the most promising ones?

Example 4: Bioinformatics

nature

Searching for genetic determinants in the new millennium

N.J. Risch

Human genetics is now at a critical juncture. The molecular methods used successfully to identify the genes underlying rare mendelian syndromes are failing to find the numerous genes causing more common, familial, non-mendelian diseases . . .

Nature **405**:847–856, 2000

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Types of Ranking Problems

Instance Ranking

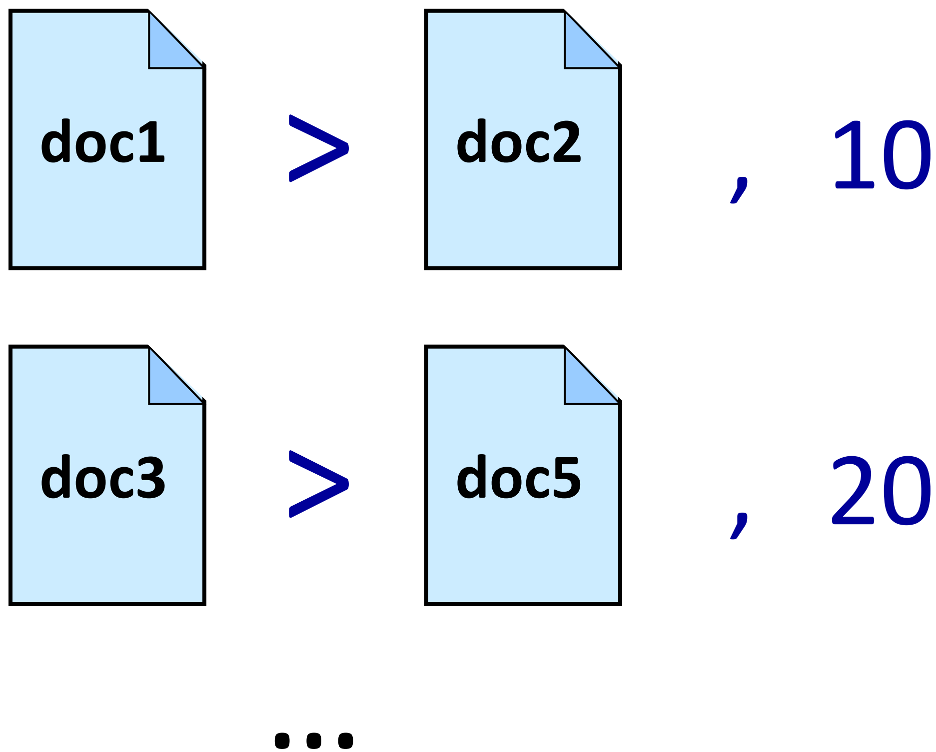
Label Ranking

Subset Ranking

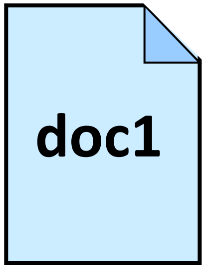
Rank Aggregation

?

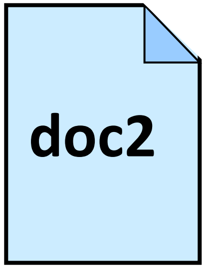
Instance Ranking



Label Ranking



sports > politics
health > money
...



science > sports
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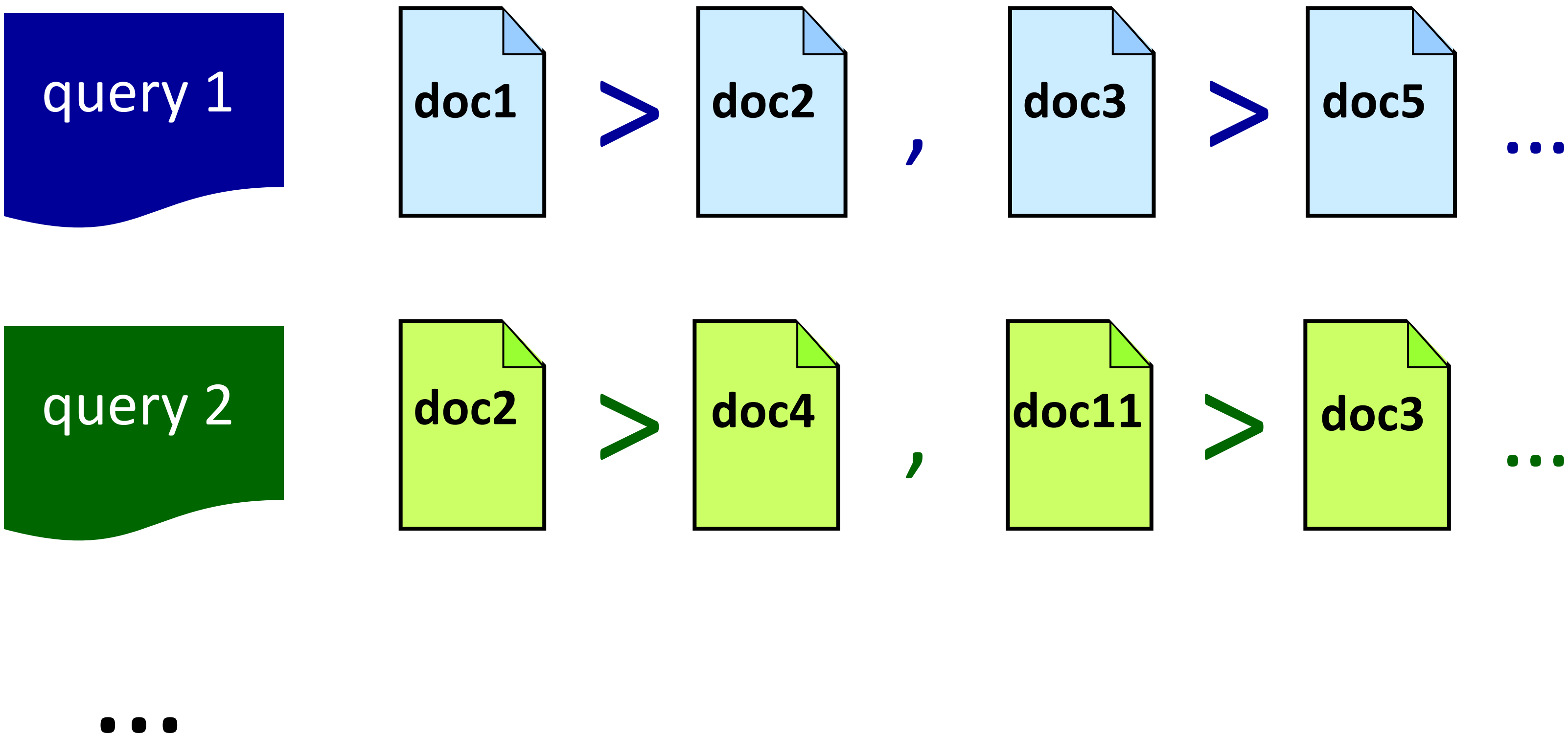
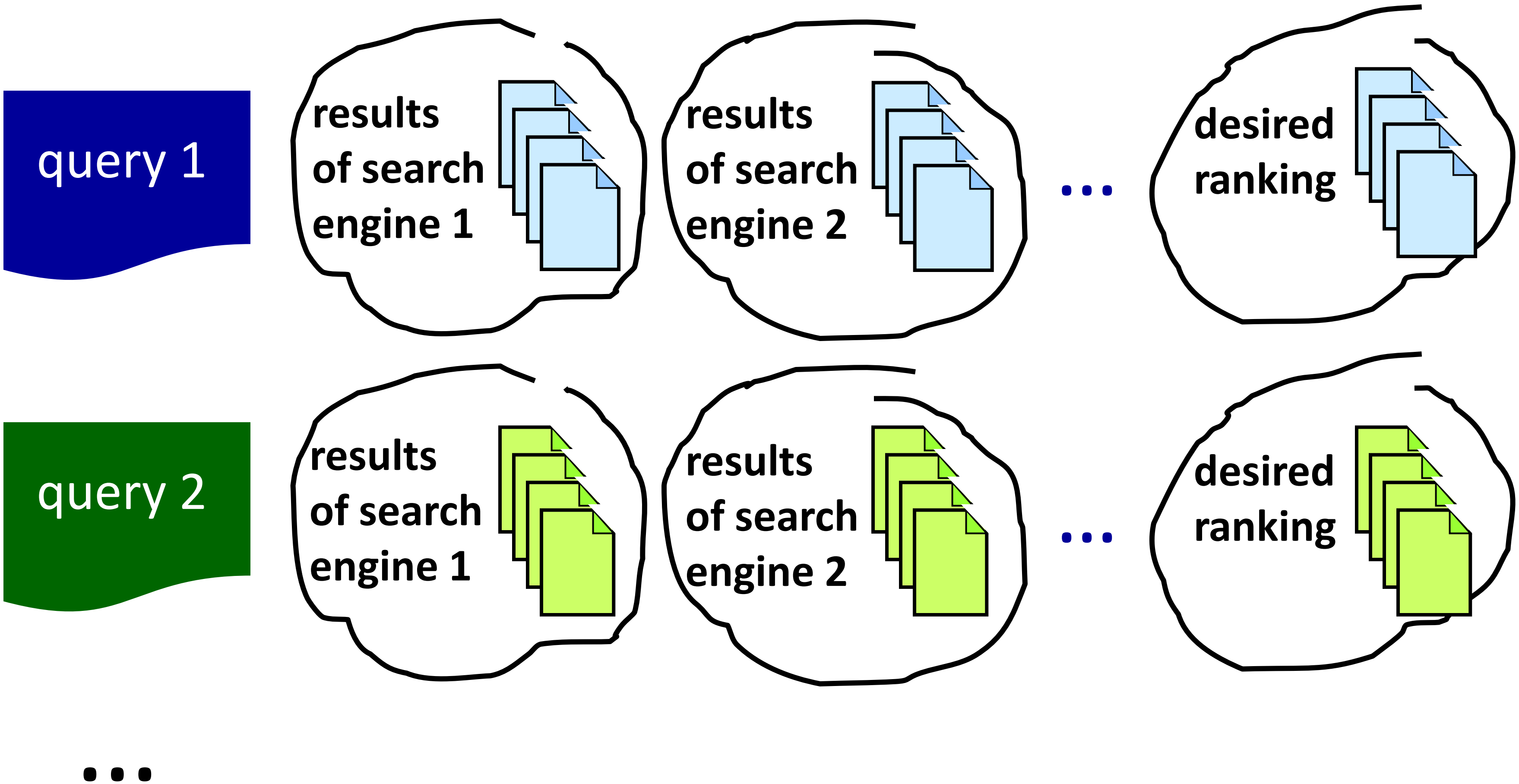
...

Subset Ranking



Rank Aggregation



Types of Ranking Problems

Instance Ranking

Label Ranking

Subset Ranking

Rank Aggregation

?

This tutorial

Tutorial Road Map

Part I: Theory & Algorithms

Bipartite Ranking
k-partite Ranking
Ranking with Real-Valued Labels
General Instance Ranking

RankSVM
RankBoost
RankNet

Part II: Applications

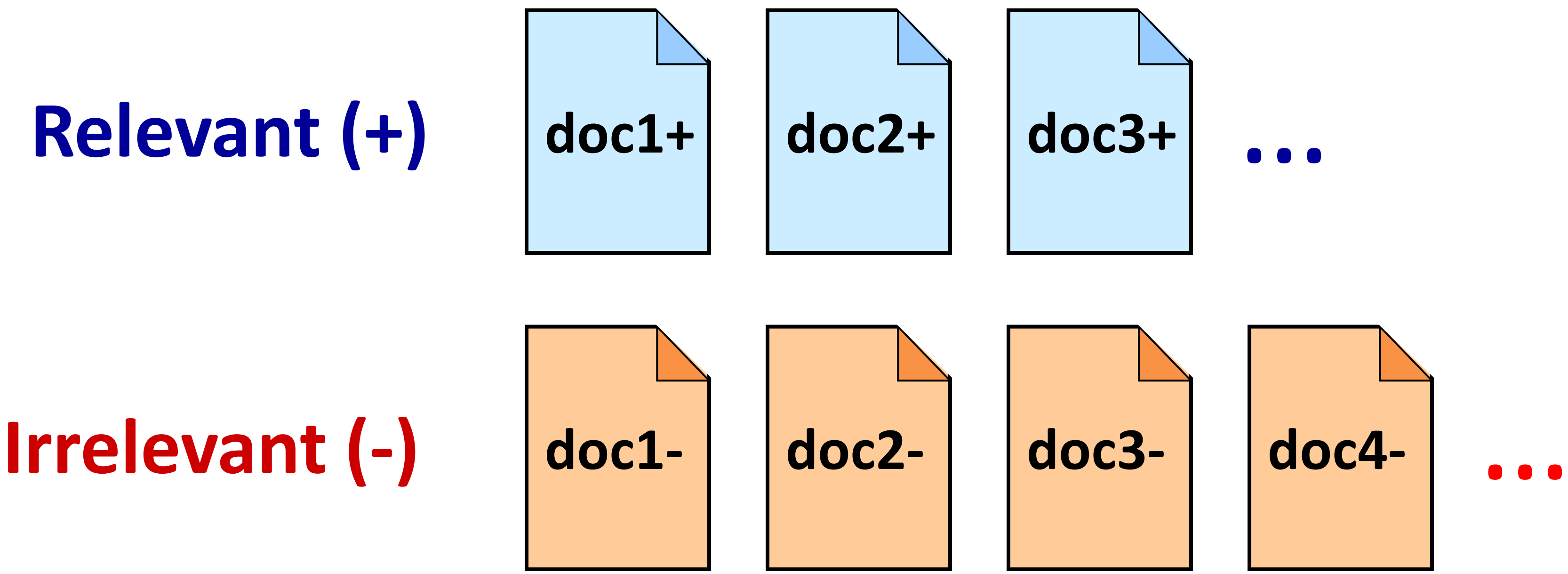
Applications to Bioinformatics
Applications to Drug Discovery
Subset Ranking and Applications to Information Retrieval

Further Reading & Resources

Part I
Theory & Algorithms

[for Instance Ranking]

Bipartite Ranking

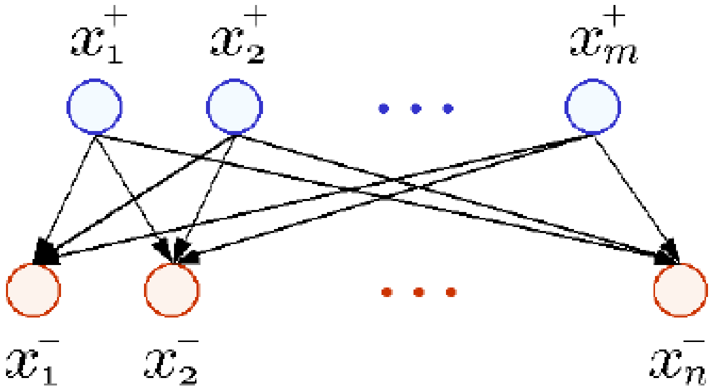


Bipartite Ranking

- Instance space X
- Input: Training sample $S = (S_+, S_-)$:

$$S_+ = (x_1^+, \ldots, x_m^+) \in X^m \quad \text{(positive examples)}$$
$$S_- = (x_1^-, \ldots, x_n^-) \in X^n \quad \text{(negative examples)}$$

- Output: Ranking function $f : X \rightarrow \mathbb{R}$



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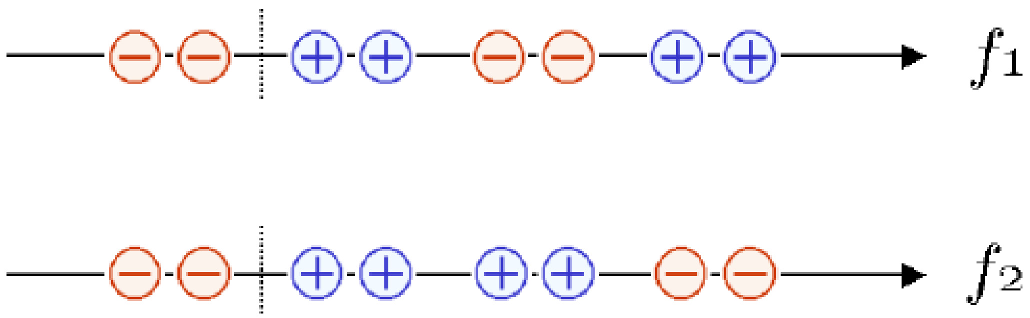
- Output: Ranking function $f : X \rightarrow \mathbb{R}$

- Expected error: $\mathbf{er}(f) = \mathbf{P}_{(x,x')\sim\mathcal{D}_+\times\mathcal{D}_-}\left[f(x) < f(x')\right]$
- Empirical error: $\widehat{\mathbf{er}}_S(f) = \dfrac{1}{mn}\displaystyle\sum_{i=1}^{m}\sum_{j=1}^{n} \mathbf{1}(f(x_i^+) < f(x_j^-))$

Is Bipartite Ranking Different from Binary Classification?

Example 1



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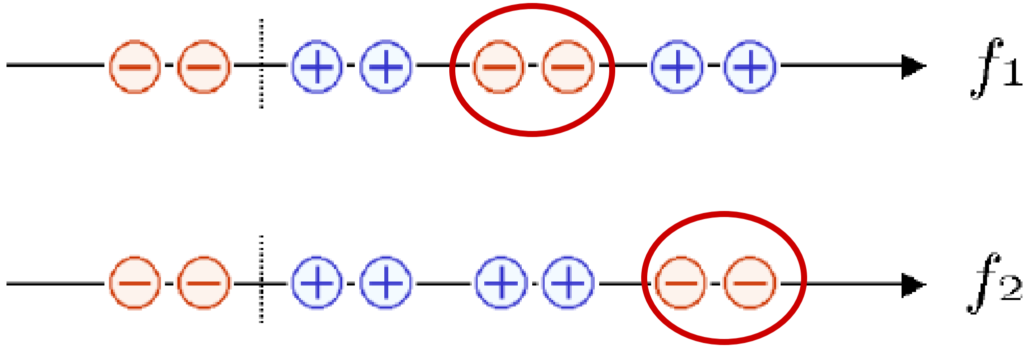

Classification error = $\frac{1}{4}$

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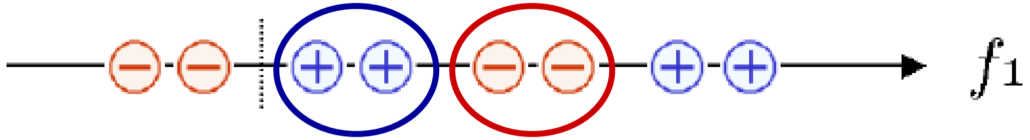
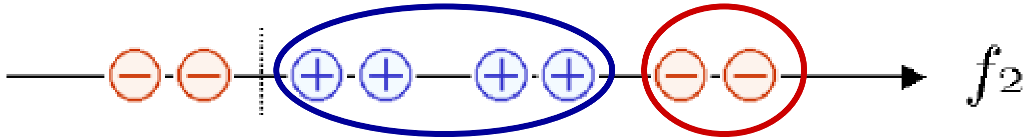


Classification error = $\frac{1}{4}$

Ranking error = $\frac{1}{4}$

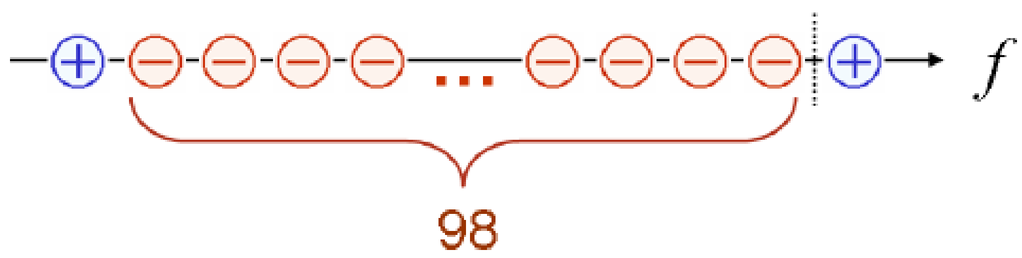


Classification error = $\frac{1}{4}$

Ranking error = $\frac{1}{2}$

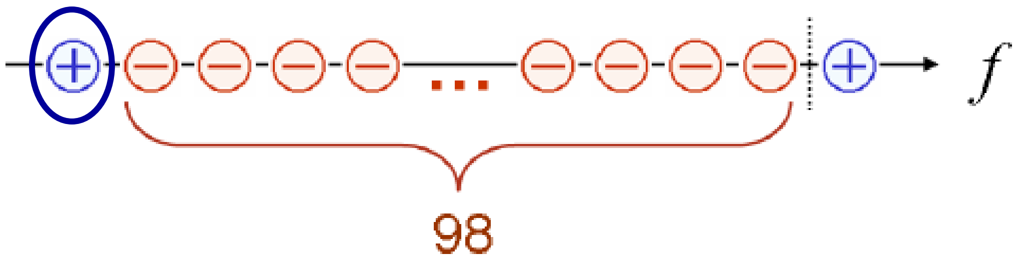
Is Bipartite Ranking Different from Binary Classification?

Example 2



Is Bipartite Ranking Different from Binary Classification?

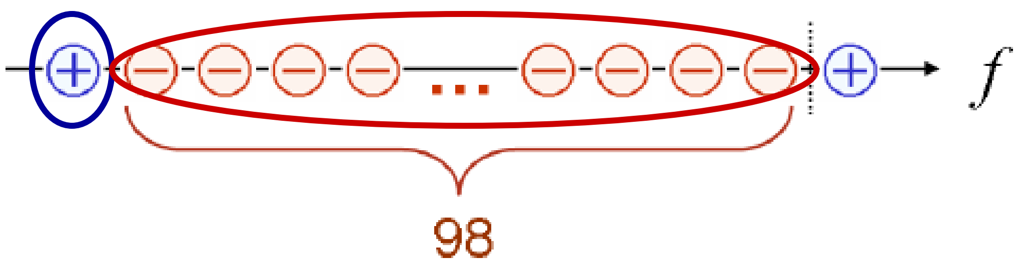
Example 2



Classification error = $\frac{1}{100}$

Is Bipartite Ranking Different from Binary Classification?

Example 2



Classification error = $\frac{1}{100}$

Ranking error = $\frac{1}{2}$

Bipartite Ranking: Basic Algorithmic Framework

Minimize a convex upper bound on the empirical ranking error, possibly with some regularization, over some class of ranking functions:

$$\min_{f \in \mathcal{F}} \left[\frac{1}{mn} \sum_{i=1}^{m} \sum_{j=1}^{n} \ell(f, x_i^+, x_j^-) + \lambda N(f) \right]$$

where

$\ell(f, x_i^+, x_j^-)$	:	convex upper bound on $\mathbf{1}(f(x_i^+) < f(x_j^-))$
$N(f)$	:	regularizer
$\lambda > 0$	:	regularization parameter
$\mathcal{F}$	:	class of ranking functions

Bipartite RankSVM Algorithm

$$\min_{f \in \mathcal{F}_K} \left[\frac{1}{mn} \sum_{i=1}^{m} \sum_{j=1}^{n} \ell_{\text{hinge}}(f, x_i^+, x_j^-) + \frac{\lambda}{2} \|f\|_K^2 \right]$$

$$\ell_{\text{hinge}}(f, x_i^+, x_j^-) = \left(1 - \left(f(x_i^+) - f(x_j^-)\right)\right)_+ \quad \left[u_+ = \max(u, 0)\right]$$

$\mathcal{F}_K$ = reproducing kernel Hilbert space (RKHS) with kernel function K

$$N(f) = \frac{\|f\|_K^2}{2}$$

[Herbrich et al, 2000; Joachims, 2002; Rakotomamonjy, 2004]

Bipartite RankSVM Algorithm

Introducing slack variables and taking the Lagrangian dual results in the following convex quadratic program (QP) over mn variables $\{\alpha_{ij} : 1 \leq i \leq m, 1 \leq j \leq n\}$:

$$\min_{\alpha} \left[\frac{1}{2} \sum_{i=1}^{m} \sum_{j=1}^{n} \sum_{k=1}^{m} \sum_{l=1}^{n} \alpha_{ij}\alpha_{kl}\, \phi(x_i^+, x_j^-, x_k^+, x_l^-) - \sum_{i=1}^{m} \sum_{j=1}^{n} \alpha_{ij} \right]$$

subject to $0 \leq \alpha_{ij} \leq C \quad \forall\, i, j$

where

$$\phi(x_i^+, x_j^-, x_k^+, x_l^-) = \left(K(x_i^+, x_k^+) - K(x_i^+, x_l^-) - K(x_j^-, x_k^+) + K(x_j^-, x_l^-) \right)$$

$$C = \frac{1}{\lambda m n}$$

Can be solved using a standard QP solver, or more efficient methods (e.g. Chapelle & Keerthi, 2010).

Bipartite RankBoost Algorithm

$$\min_{f \in \mathcal{L}(\mathcal{F}_{\text{base}})} \left[\frac{1}{mn} \sum_{i=1}^{m} \sum_{j=1}^{n} \ell_{\text{exp}}(f, x_i^+, x_j^-) \right]$$

$$\ell_{\text{exp}}(f, x_i^+, x_j^-) \;=\; \exp\Big(-\Big(f(x_i^+) - f(x_j^-)\Big)\Big)$$

$\mathcal{L}(\mathcal{F}_{\text{base}})$ = linear combinations of functions in some base class $\mathcal{F}_{\text{base}}$

[Freund et al, 2003]

Bipartite RankBoost Algorithm

Input: $(S_+, S_-) \in X^m \times X^n$.

Initialize: $D_1(x_i^+, x_j^-) = \dfrac{1}{mn}$ for all $i \in \{1, \ldots, m\},\ j \in \{1, \ldots, n\}$.

For $t = 1, \ldots, T$:

- Train weak learner using distribution D_t; get weak ranker $f_t \in \mathcal{F}_{\text{base}}$.
- Choose $\alpha_t \in \mathbb{R}$.
- Update: $D_{t+1}(x_i^+, x_j^-) = \dfrac{1}{Z_t} D_t(x_i^+, x_j^-) \exp\left(-\alpha_t \left(f_t(x_i^+) - f_t(x_j^-)\right)\right)$

  where $Z_t = \displaystyle\sum_{i=1}^{m} \sum_{j=1}^{n} D_t(x_i^+, x_j^-) \exp\left(-\alpha_t \left(f_t(x_i^+) - f_t(x_j^-)\right)\right)$.

Output final ranking: $f(x) = \displaystyle\sum_{t=1}^{T} \alpha_t f_t(x)$.

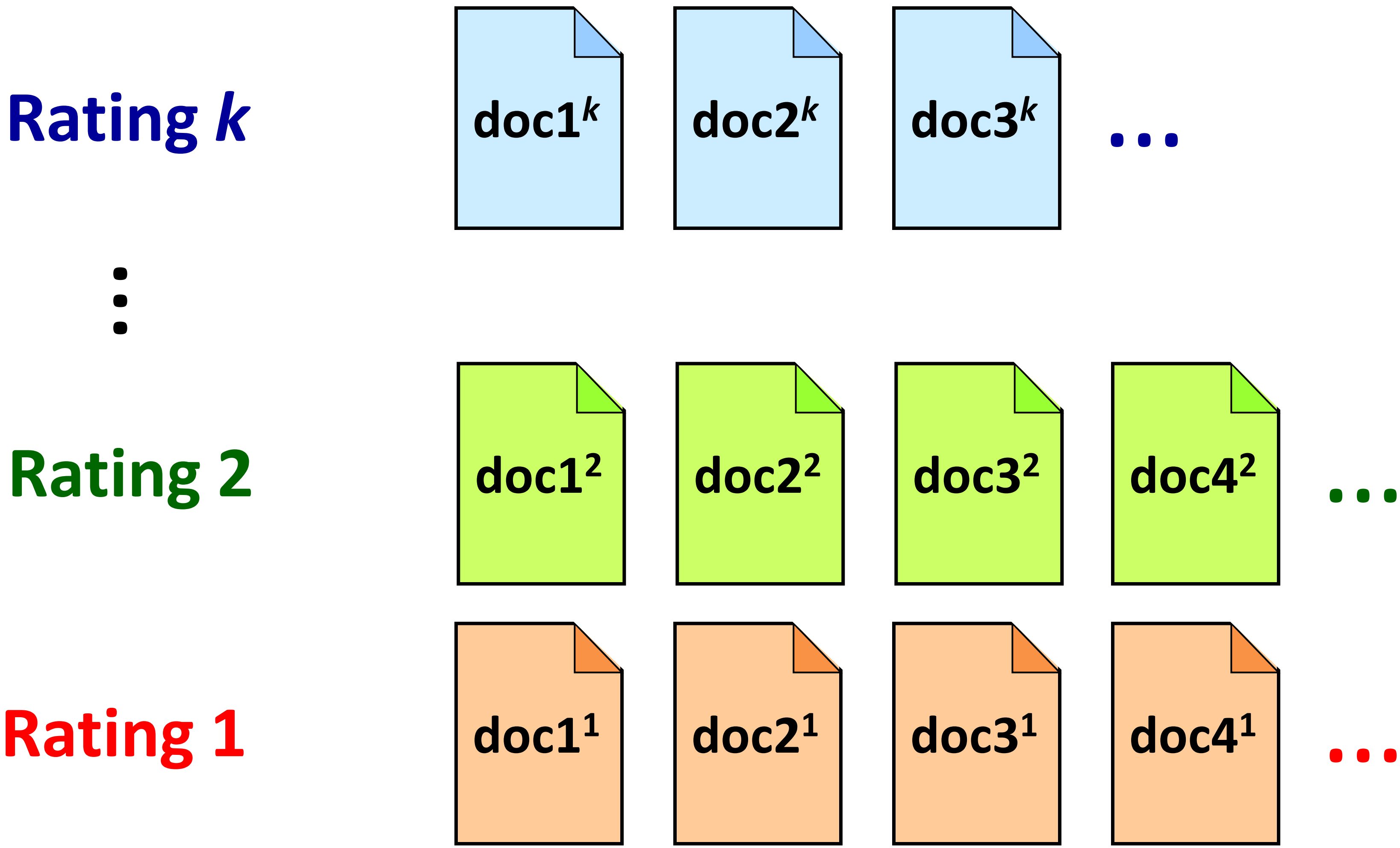
Bipartite RankNet Algorithm

$$\min_{f \in \mathcal{F}_{\text{neural}}} \left[\frac{1}{mn} \sum_{i=1}^{m} \sum_{j=1}^{n} \ell_{\text{logistic}}(f, x_i^+, x_j^-) \right]$$

$$\begin{aligned} \ell_{\text{logistic}}(f, x_i^+, x_j^-) &= \log\left(1 + \exp\left(-\left(f(x_i^+) - f(x_j^-)\right)\right)\right) \\ \mathcal{F}_{\text{neural}} &= \text{functions represented by some class of} \\ & \quad \text{neural networks} \end{aligned}$$

[Burges et al, 2005]

k-partite Ranking



k-partite Ranking

- Instance space X
- Input: Training sample $S = (S_1, S_2, \ldots, S_k)$:

$$S_k = (x_1^k, \ldots, x_{n_k}^k) \in X^{n_k} \quad \text{(examples of rating } k\text{)}$$
$$\vdots$$
$$S_2 = (x_1^2, \ldots, x_{n_2}^2) \in X^{n_2} \quad \text{(examples of rating } 2\text{)}$$
$$S_1 = (x_1^1, \ldots, x_{n_1}^1) \in X^{n_1} \quad \text{(examples of rating } 1\text{)}$$

- Output: Ranking function $f : X \rightarrow \mathbb{R}$

- Empirical error:

$$\widehat{\mathbf{er}}_S(f) = \left(\frac{1}{\sum\limits_{1 \le a < b \le k} n_a n_b} \right) \sum_{1 \le a < b \le k} \sum_{i=1}^{n_b} \sum_{j=1}^{n_a} (b-a)\, \mathbf{1}\left(f(x_i^b) < f(x_j^a)\right)$$

k-partite Ranking: Basic Algorithmic Framework

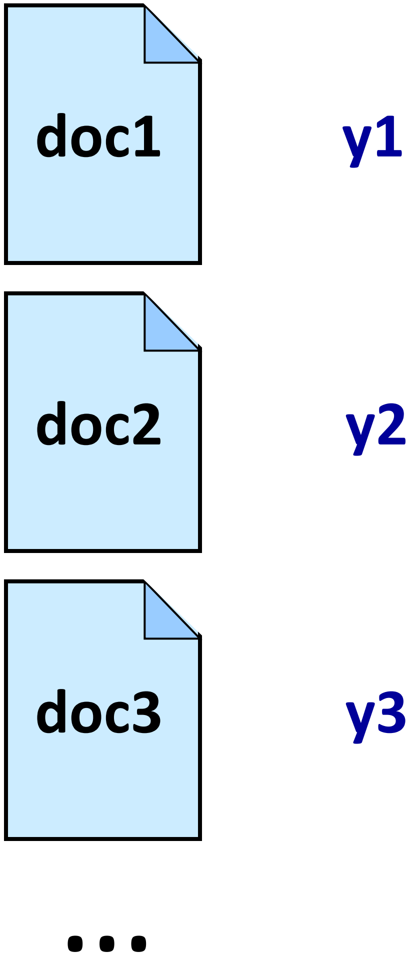
Minimize a convex upper bound on the empirical ranking error, possibly with some regularization, over some class of ranking functions:

$$\min_{f \in \mathcal{F}} \left[\left(\frac{1}{\sum_{1 \leq a < b \leq k} n_a n_b} \right) \sum_{1 \leq a < b \leq k} \sum_{i=1}^{n_b} \sum_{j=1}^{n_a} \ell(f, x_i^b, x_j^a, (b-a)) + \lambda N(f) \right]$$

where

$$\begin{aligned}
\ell(f, x_i^b, x_j^a, (b-a)) \quad &: \quad \text{convex upper bound on } (b-a)\,\mathbf{1}(f(x_i^b) < f(x_j^a)) \\
N(f) \quad &: \quad \text{regularizer} \\
\lambda > 0 \quad &: \quad \text{regularization parameter} \\
\mathcal{F} \quad &: \quad \text{class of ranking functions}
\end{aligned}$$

Ranking with Real-Valued Labels



Ranking with Real-Valued Labels

- Instance space X
- Real-valued labels $Y = \mathbb{R}$
- Input: Training sample $S = ((x_1, y_1), \ldots, (x_m, y_m)) \in (X \times \mathbb{R})^m$
- Output: Ranking function $f : X \to \mathbb{R}$
- Empirical error:

$$\widehat{\mathbf{er}}_S(f) = \frac{1}{\binom{m}{2}} \sum_{1 \leq i < j \leq m} |y_i - y_j| \, \mathbf{1}\left((y_i - y_j)(f(x_i) - f(x_j)) < 0\right)$$

Ranking with Real-Valued Labels: Basic Algorithmic Framework

Minimize a convex upper bound on the empirical ranking error, possibly with some regularization, over some class of ranking functions:

$$\min_{f \in \mathcal{F}} \left[\frac{1}{\binom{m}{2}} \sum_{1 \le i < j \le m} \ell(f, (x_i, y_i), (x_j, y_j)) \; + \; \lambda N(f) \right]$$

where

$$\begin{array}{rcl}
\ell(f, (x_i, y_i), (x_j, y_j)) & : & \text{convex upper bound on} \\
 & & |y_i - y_j| \, \mathbf{1}\left((y_i - y_j)(f(x_i) - f(x_j)) < 0\right) \\
N(f) & : & \text{regularizer} \\
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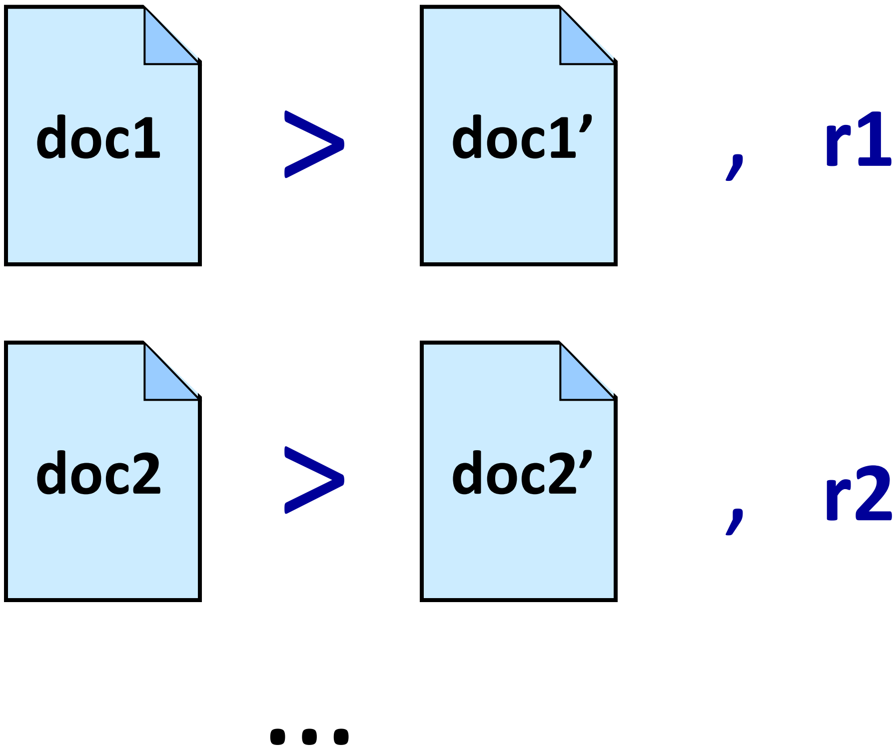
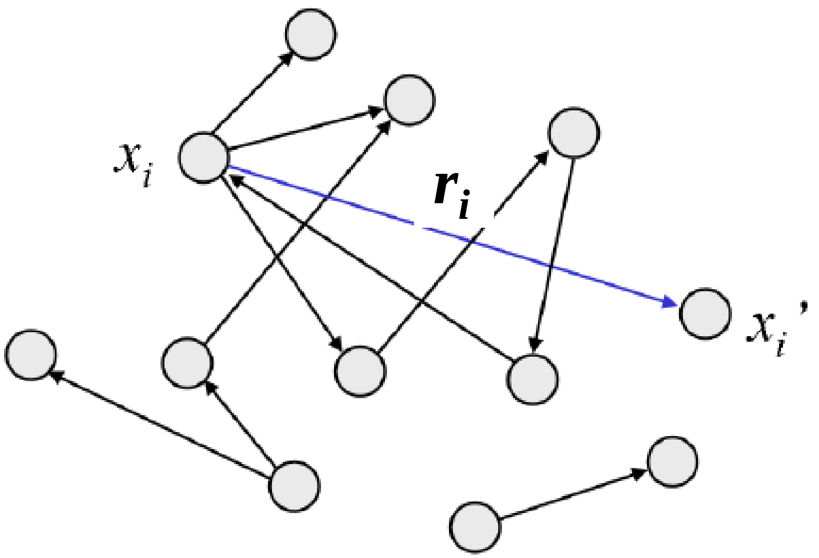
General Instance Ranking



General Instance Ranking

- Instance space X
- Input: Training sample $S = ((x_1, x'_1, r_1), \ldots, (x_m, x'_m, r_m)) \in (X^2 \times \mathbb{R}_+)^m$
- Output: Ranking function $f : X \rightarrow \mathbb{R}$



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- Output: Ranking function $f : X \to \mathbb{R}$

- Empirical error: $\widehat{\mathbf{er}}_S(f) = \frac{1}{m} \sum_{i=1}^{m} r_i \, \mathbf{1}(f(x_i) < f(x'_i))$

General Instance Ranking: Basic Algorithmic Framework

Minimize a convex upper bound on the empirical ranking error, possibly with some regularization, over some class of ranking functions:

$$\min_{f \in \mathcal{F}} \left[\frac{1}{m} \sum_{i=1}^{m} \ell(f, x_i, x_i', r_i) + \lambda N(f) \right]$$

where

$\ell(f, x_i, x_i', r_i)$	:	convex upper bound on $r_i \mathbf{1}(f(x_i) < f(x_i'))$
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General RankSVM Algorithm

$$\min_{f \in \mathcal{F}_K} \left[\frac{1}{m} \sum_{i=1}^{m} \ell_{\text{hinge}}(f, x_i, x_i', r_i) + \frac{\lambda}{2} \|f\|_K^2 \right]$$

$$\ell_{\text{hinge}}(f, x_i, x_i', r_i) = \Big(r_i - \big(f(x_i) - f(x_i') \big) \Big)_+ \quad \big[u_+ = \max(u, 0) \big]$$

$\mathcal{F}_K$ = reproducing kernel Hilbert space (RKHS) with kernel function K

$$N(f) = \frac{\|f\|_K^2}{2}$$

[Herbrich et al, 2000; Joachims, 2002]

General RankBoost Algorithm

$$\min_{f \in \mathcal{L}(\mathcal{F}_{\text{base}})} \left[\frac{1}{m} \sum_{i=1}^{m} \ell_{\text{exp}}(f, x_i, x_i', r_i) \right]$$

$$\begin{aligned} \ell_{\text{exp}}(f, x_i, x_i', r_i) &= r_i \exp\left(-\left(f(x_i) - f(x_i')\right)\right) \\ \mathcal{L}(\mathcal{F}_{\text{base}}) &= \text{linear combinations of functions in some} \\ & \quad \text{base class } \mathcal{F}_{\text{base}} \end{aligned}$$

[Freund et al, 2003]

General RankNet Algorithm

$$\min_{f \in \mathcal{F}_{\text{neural}}} \left[\frac{1}{m} \sum_{i=1}^{m} \ell_{\text{logistic}}(f, x_i, x_i', r_i) \right]$$

$$\begin{aligned} \ell_{\text{logistic}}(f, x_i, x_i', r_i) &= r_i \log\left(1 + \exp\left(-\left(f(x_i) - f(x_i')\right)\right)\right) \\ \mathcal{F}_{\text{neural}} &= \text{functions represented by some class of} \\ &\quad \text{neural networks} \end{aligned}$$

[Burges et al, 2005]

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Bipartite Ranking
k-partite Ranking
Ranking with Real-Valued Labels
General Instance Ranking

RankSVM
RankBoost
RankNet

Part II: Applications

Applications to Bioinformatics
Applications to Drug Discovery
Subset Ranking and Applications to Information Retrieval

Further Reading & Resources

Part II
Applications

[and Subset Ranking]

Application to Bioinformatics

nature

Searching for genetic determinants in the new millennium

N.J. Risch

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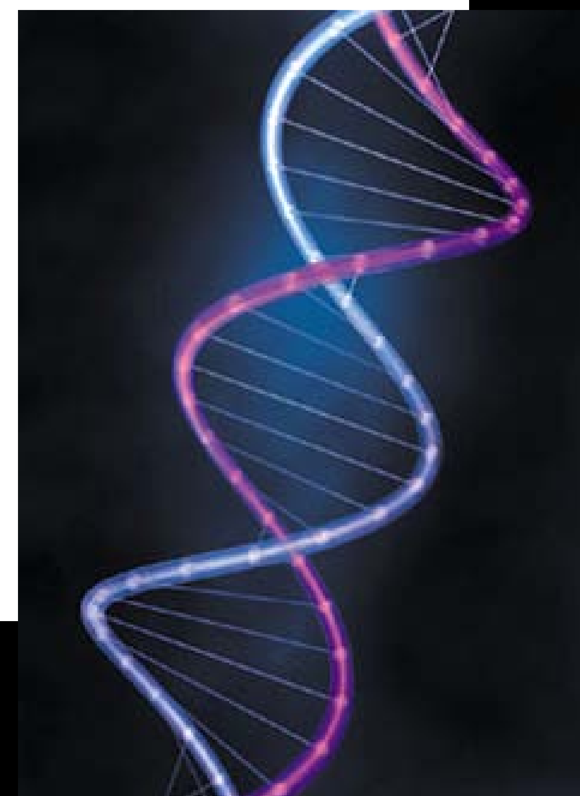

Nature **405**:847–856, 2000

Application to Bioinformatics

Searching for genetic determinants in the new millennium

N.J. Risch

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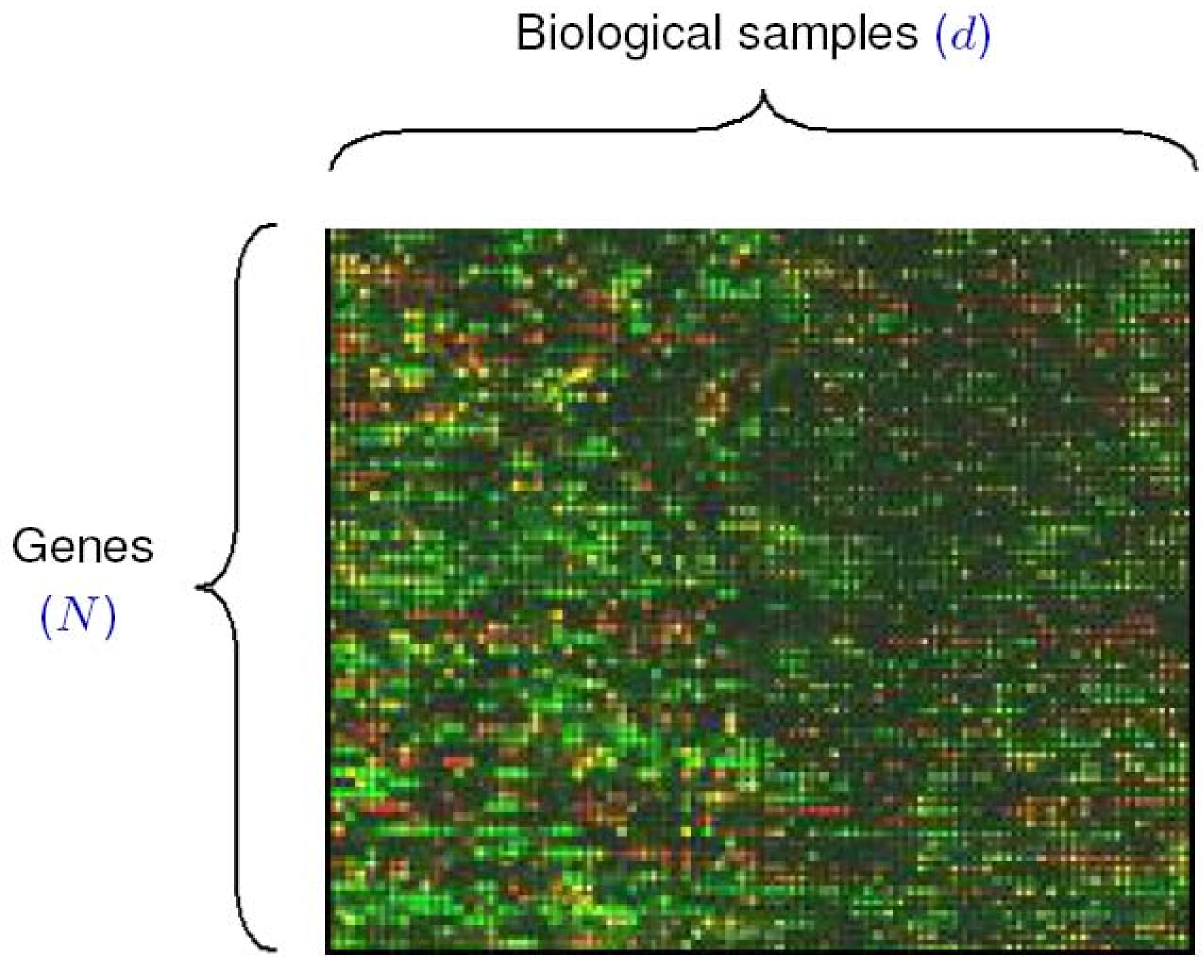
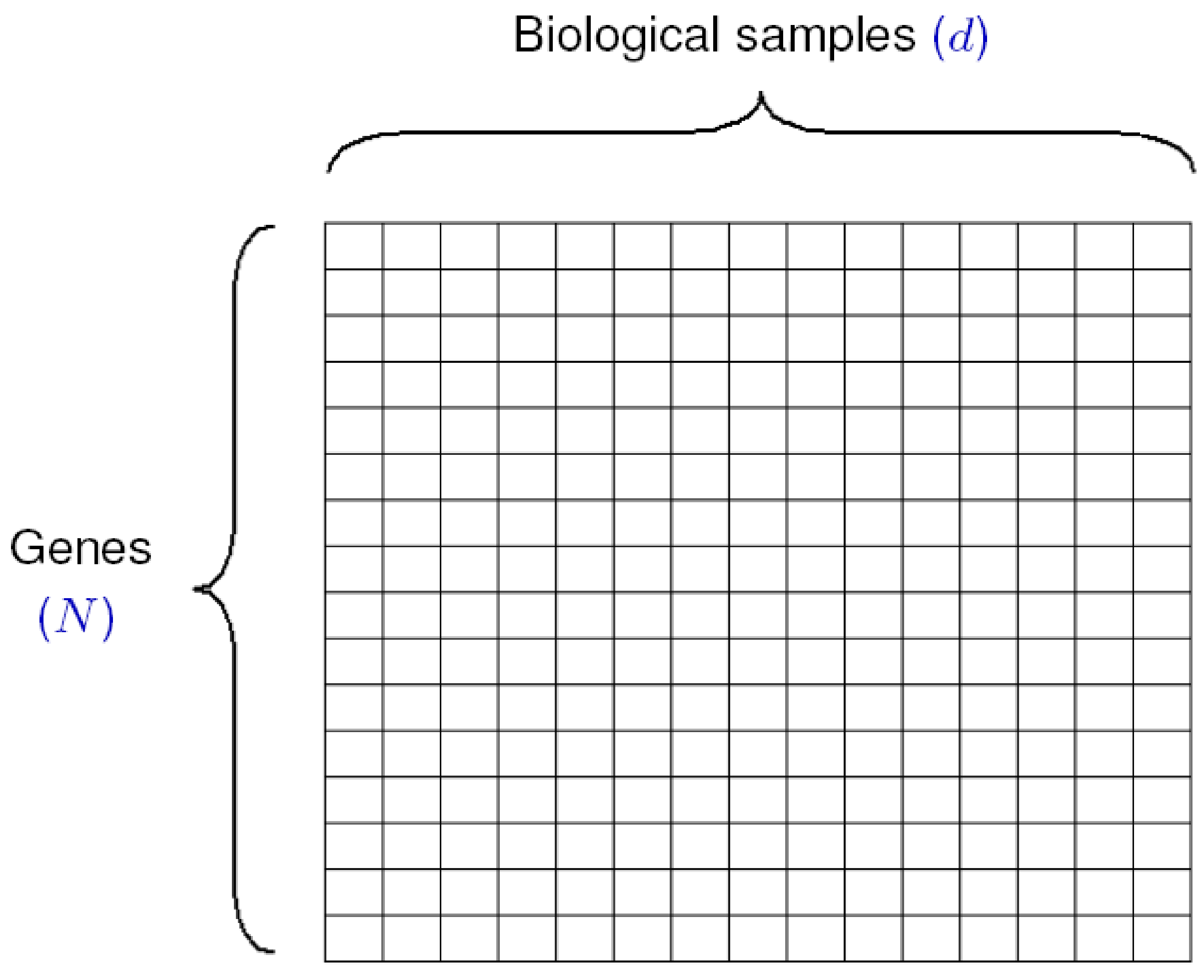
Identifying Genes Relevant to a Disease Using Microarrray Gene Expression Data



$N \gg d$

Identifying Genes Relevant to a Disease Using Microarrray Gene Expression Data



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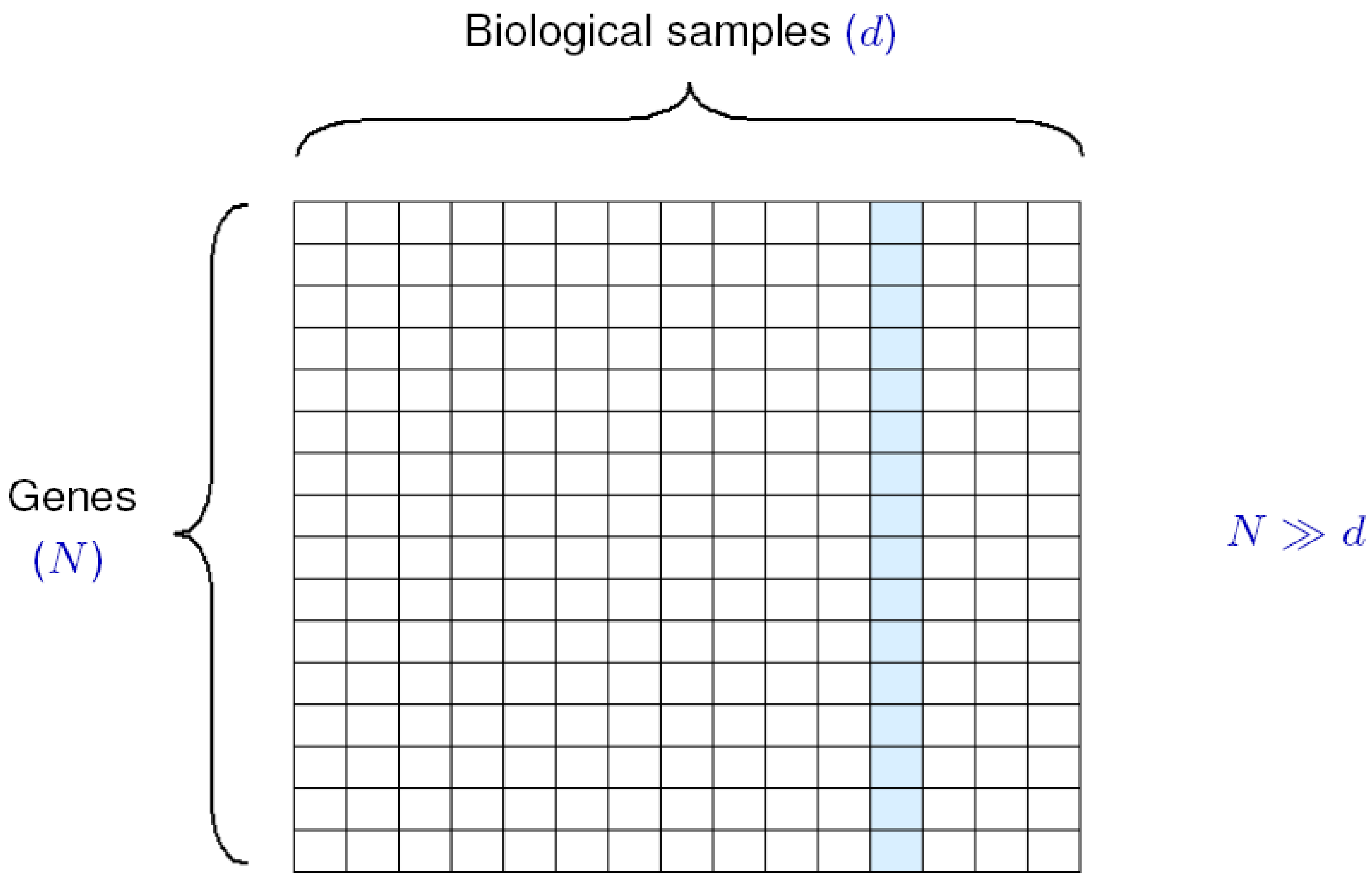

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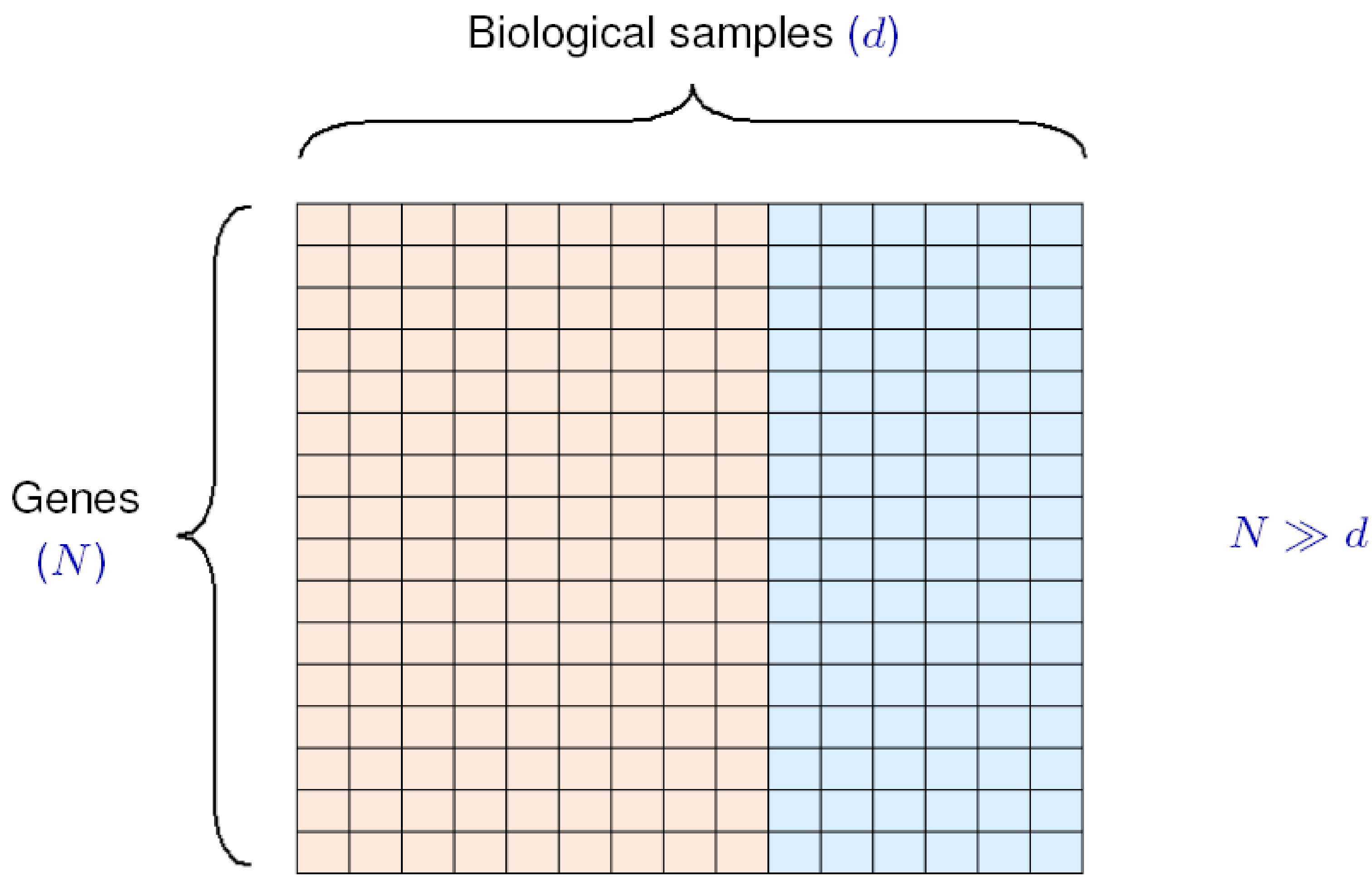

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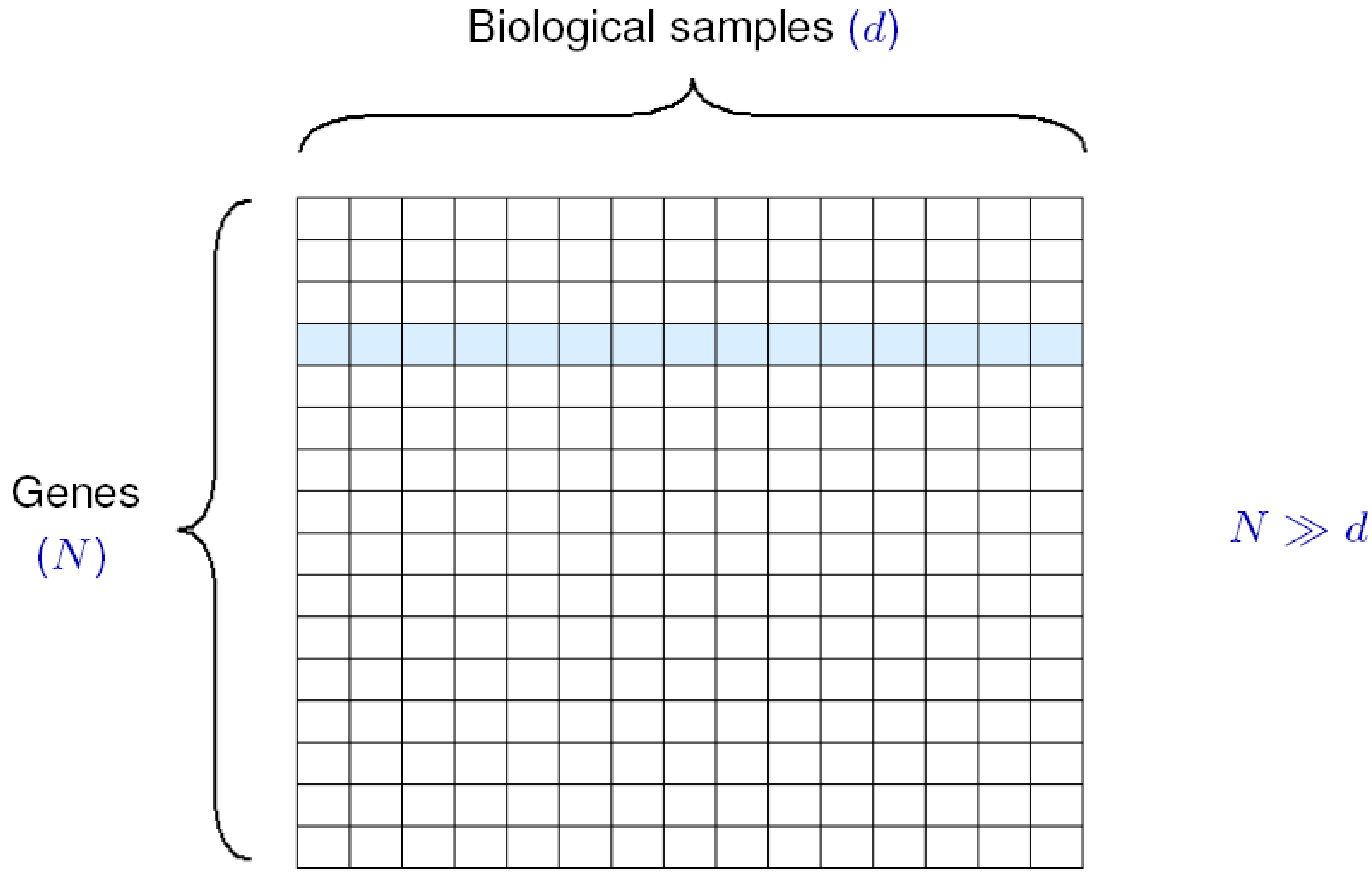

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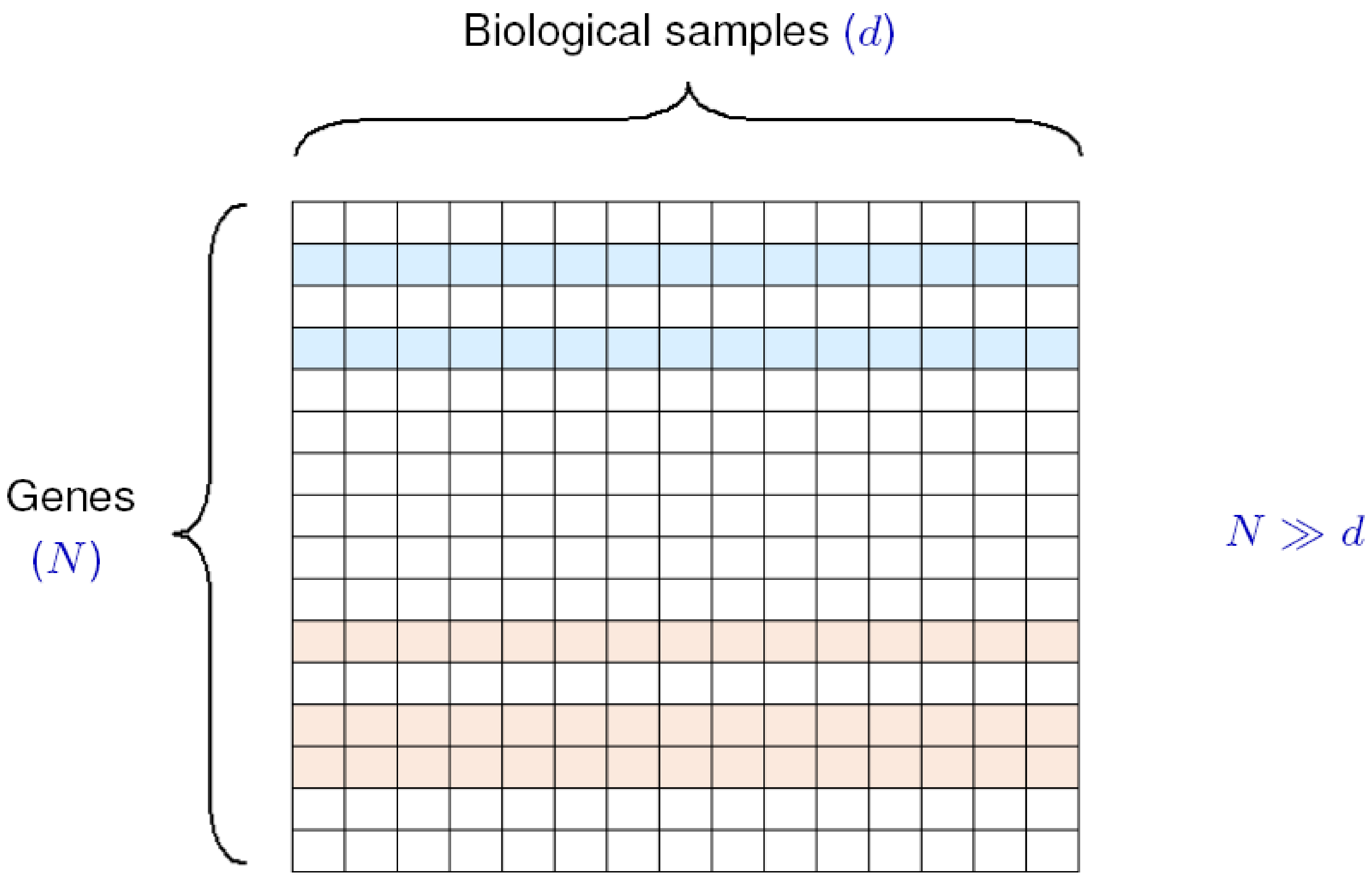
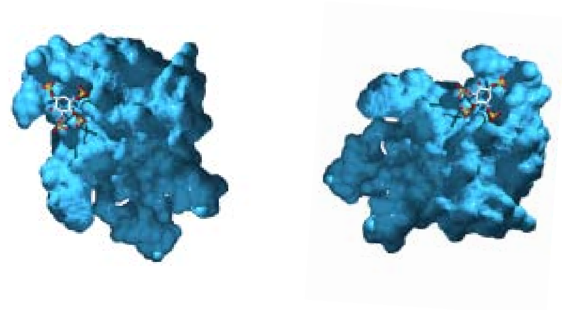


Formulation as a Bipartite Ranking Problem

Relevant



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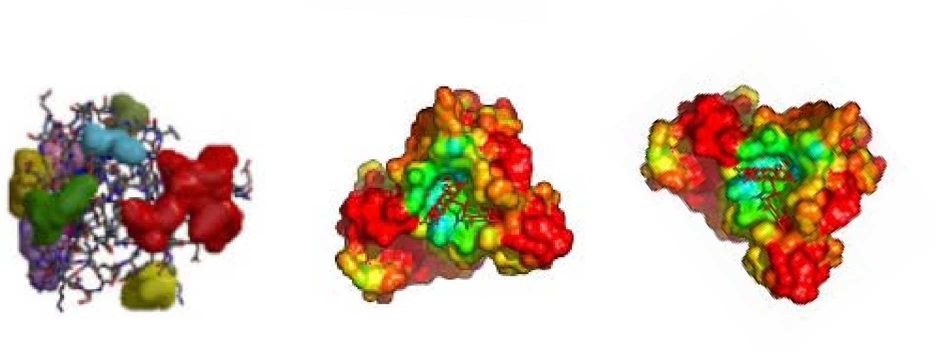
Not relevant



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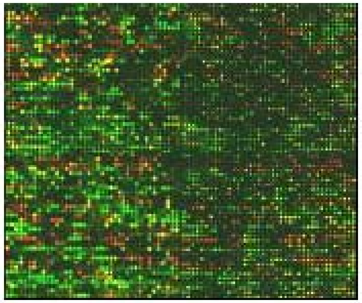
Microarray Gene Expression Data Sets

[Golub et al, 1999; Alon et al, 1999]

Data Set	No. of Genes	No. of Tissue Samples	Notes
Leukemia	7129	72	25 AML / 47 ALL
Colon cancer	2000	62	40 tumor / 22 normal



Selection of Training Genes

Leukemia	Colon cancer
Positive genes: Markers for AML/ALL	**Positive genes: Markers for colon cancer**
Myeloperoxidase CD13 CD33 HOXA9 Homeo box A9 V-myb CD19 CD10 (CALLA) TCL1 (T cell leukemia) C-myb Deoxyhypusine synthase	Phospholipase A2 Keratin 6 isoform PTP-H1 TF-IIIA V-raf oncogene MAPK kinase 1 CEA Oncoprotein 18 PEP carboxykinase ERK kinase 1
Negative genes	**Negative genes**
157 genes involved in physiological cellular functions	56 genes involved in physiological cellular functions

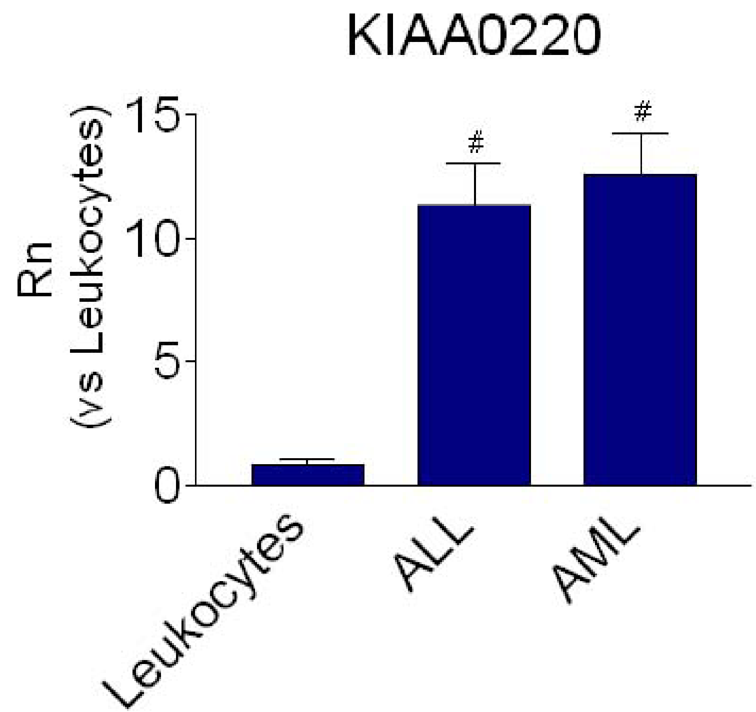
Top-Ranking Genes for Leukemia Returned by RankBoost

♦ Known marker; 🔹 Potential marker;
🟥 Known therapeutic target; 🟦 Potential therapeutic target;
x No link found.

	Gene	Relevance Summary	t-Statistic Rank	Pearson Rank
1.	KIAA0220	🟦	6628	2461
2.	G-gamma globin	♦	3578	3567
3.	Delta-globin	♦	3663	3532
4.	Brain-expressed HHCPA78 homolog	🟦	6734	2390
5.	Myeloperoxidase	♦	139	6573
6.	Disulfide isomerase precursor	🟦	6650	575
7.	Nucleophosmin	♦	405	1115
8.	CD34	♦	6732	643
9.	Elongation factor-1β	x	4460	3413
10.	CD24	♦	81	1
11.	60S ribosomal protein L23	🟦	1950	73
12.	5-aminolevulinic acid synthase	🟦	4750	3351

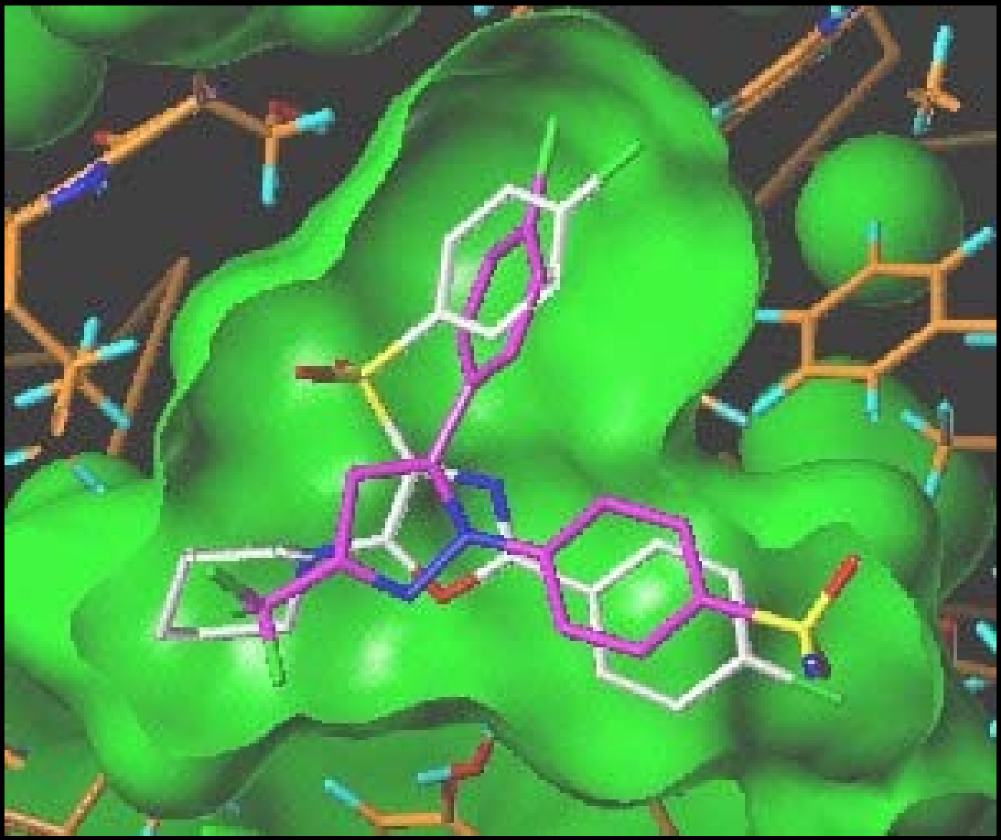
[Agarwal & Sengupta, 2009]

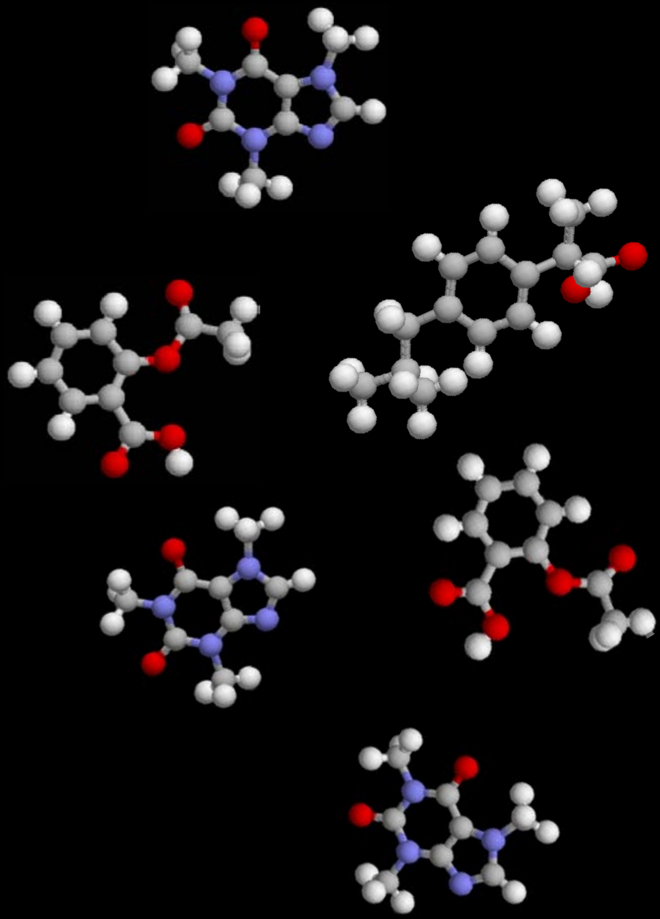
Biological Validation



[Agarwal et al, 2010]

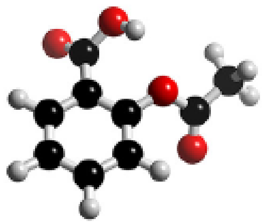
Application to Drug Discovery



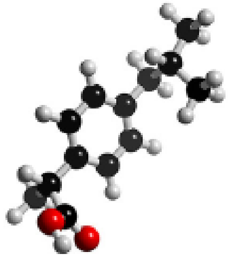


Problem: Millions of structures in a chemical library. How do we identify the most promising ones?

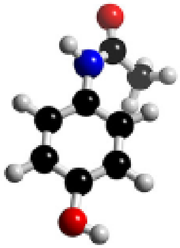
Formulation as a Ranking Problem with Real-Valued Labels



$\mathrm{pIC}_{50} = 5.6718$



$\mathrm{pIC}_{50} = 8.2991$



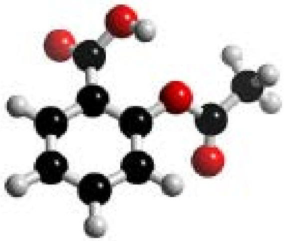
$\mathrm{pIC}_{50} = 4.1317$

...

Cheminformatics Data Sets

[Sutherland et al, 2004]

Data Set	No. of Compounds	No. of Chemical (2.5D) Descriptors	pIC_{50} Values
DHFR inhibitors	361	70	3.3 – 9.8
COX2 inhibitors	292	74	4.0 – 9.0



DHFR Results Using RankSVM

2.5D chemical descriptors
Gaussian kernel

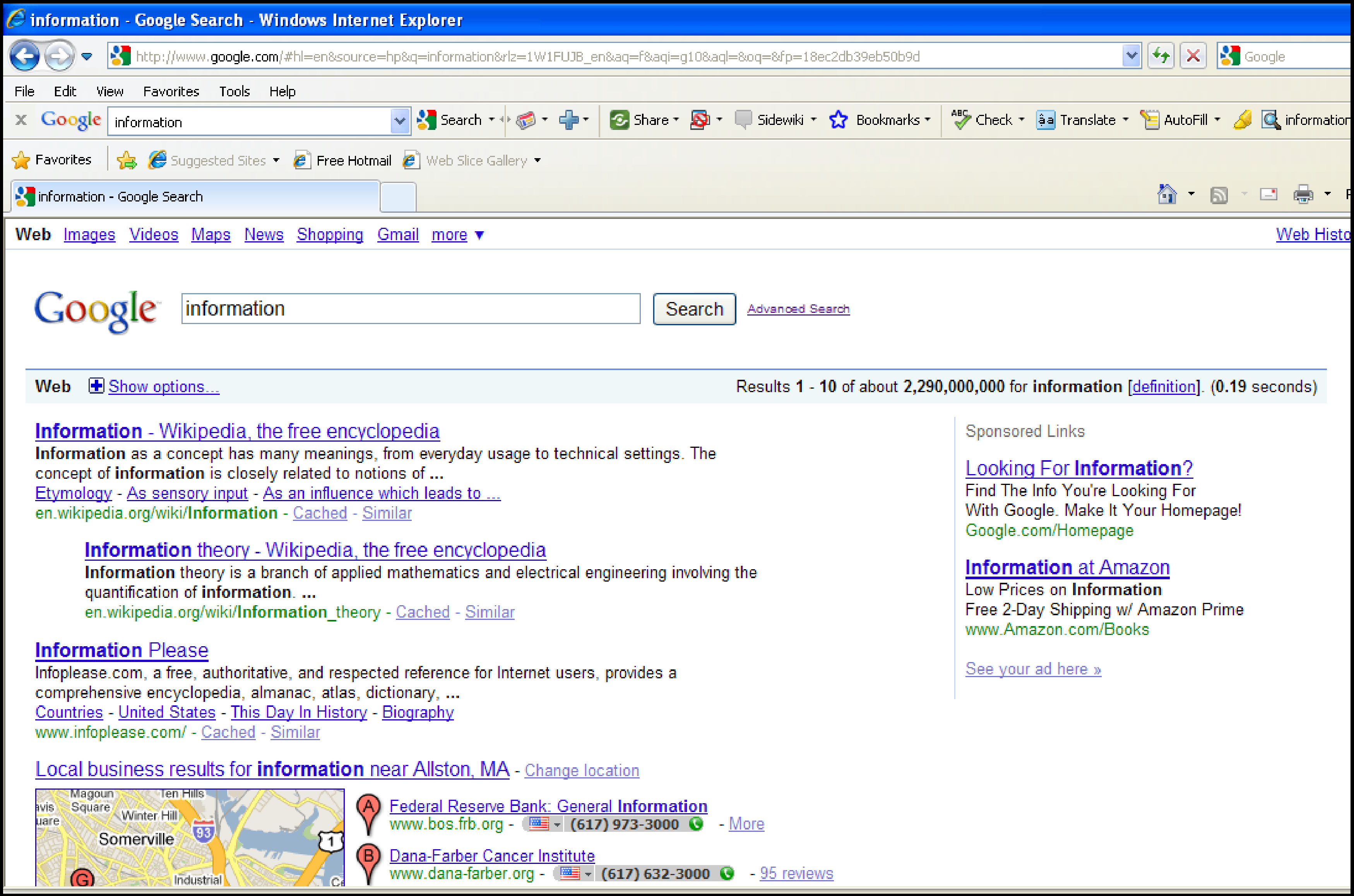
Training size	Ranking error	
	SVR	RankSVM
24	0.4755	**0.4601**
48	**0.3430**	0.3509
72	0.2840	**0.2726**
96	0.2483	**0.2351**
120	0.2171	**0.2121**
144	**0.2023**	0.2032
168	0.2019	**0.1817**
192	0.1808	**0.1749**
216	0.1816	**0.1722**
237	0.1714	**0.1681**

FP2 molecular fingerprints
Tanimoto kernel

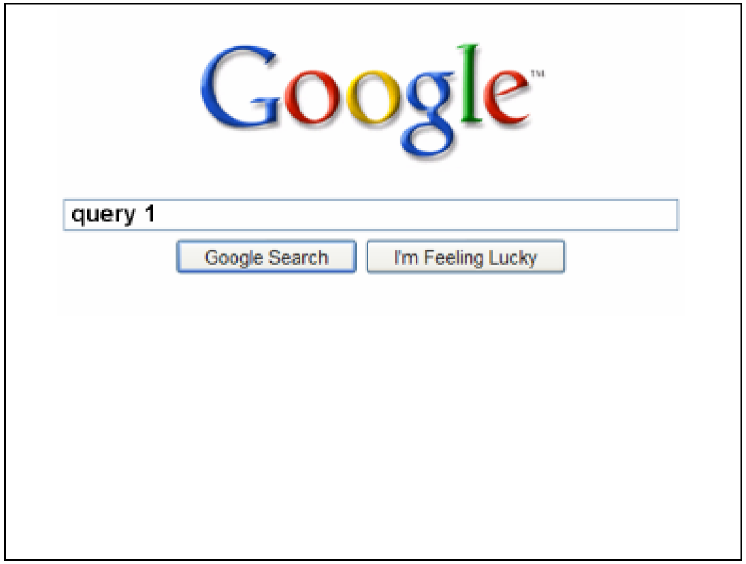
Training size	Ranking error	
	SVR	RankSVM
24	0.3793	**0.3546**
48	0.2905	**0.2896**
72	0.2517	**0.2421**
96	0.2343	**0.2201**
120	0.2147	**0.2052**
144	0.2166	**0.1988**
168	0.2096	**0.1966**
192	0.2056	**0.1962**
216	0.1907	**0.1787**
237	0.1924	**0.1798**

[Agarwal et al, 2010]

Application to Information Retrieval (IR)

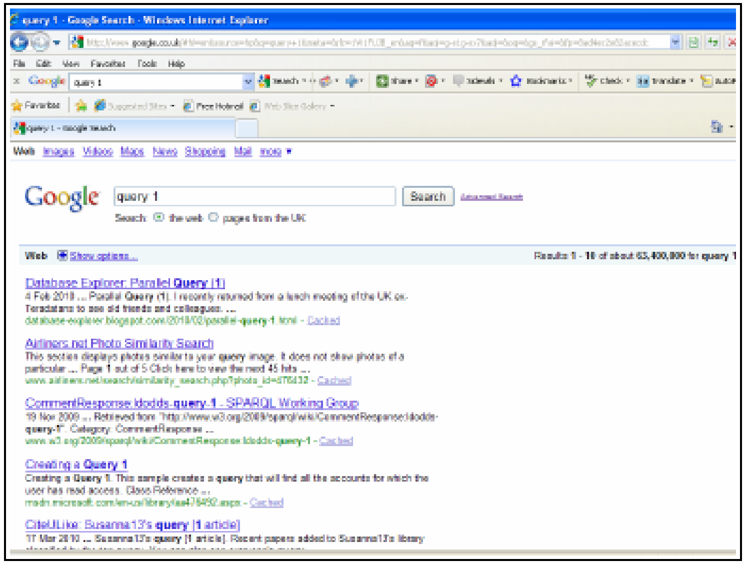


Learning to Rank in IR



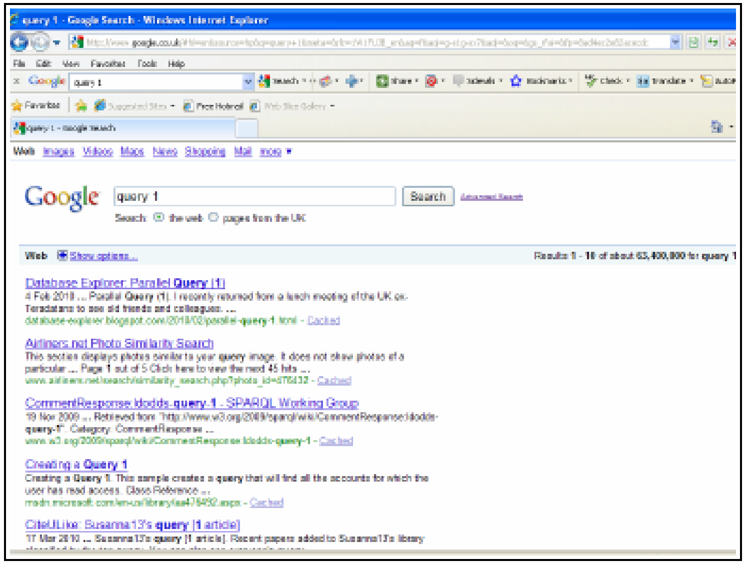
q1

Learning to Rank in IR



q1

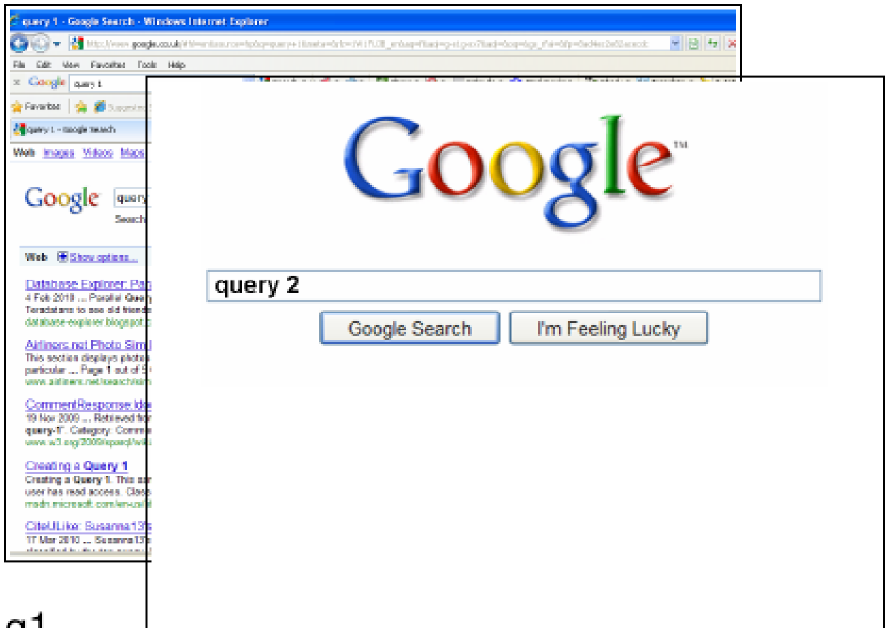
Learning to Rank in IR



q1

rel1

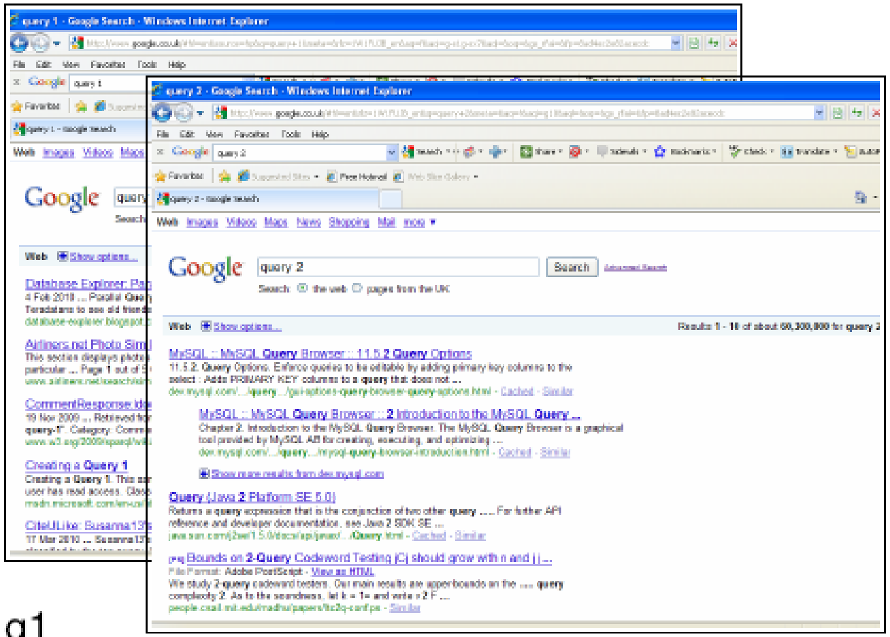
Learning to Rank in IR



q1

rel1 q2

Learning to Rank in IR

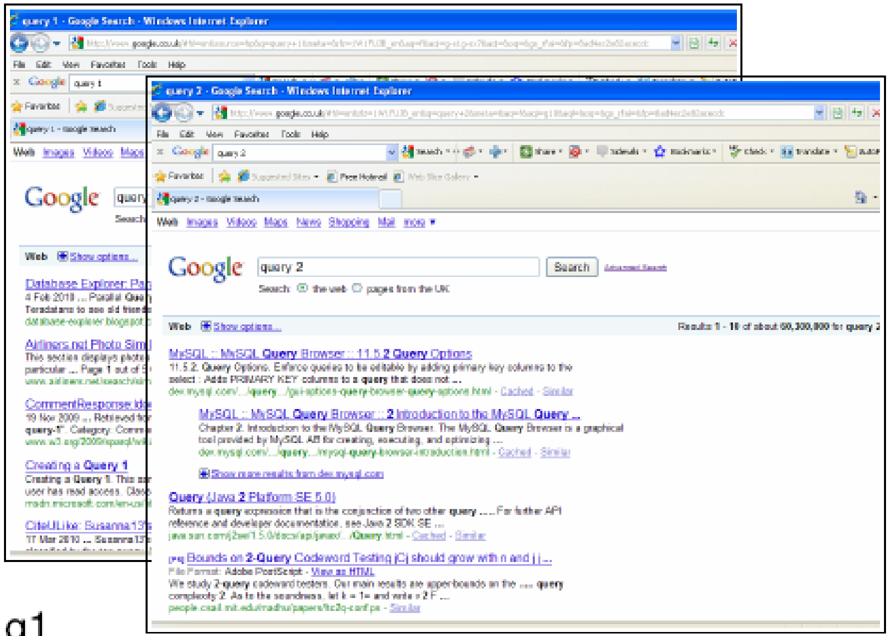


q1

rel1

q2

Learning to Rank in IR



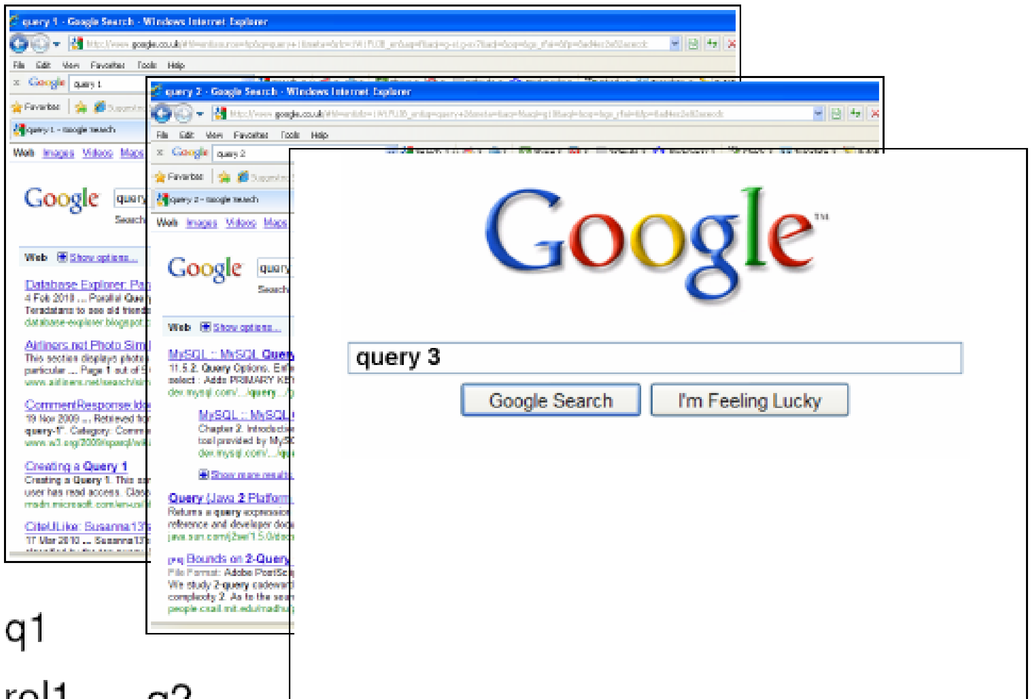
q1

rel1

q2

rel2

Learning to Rank in IR

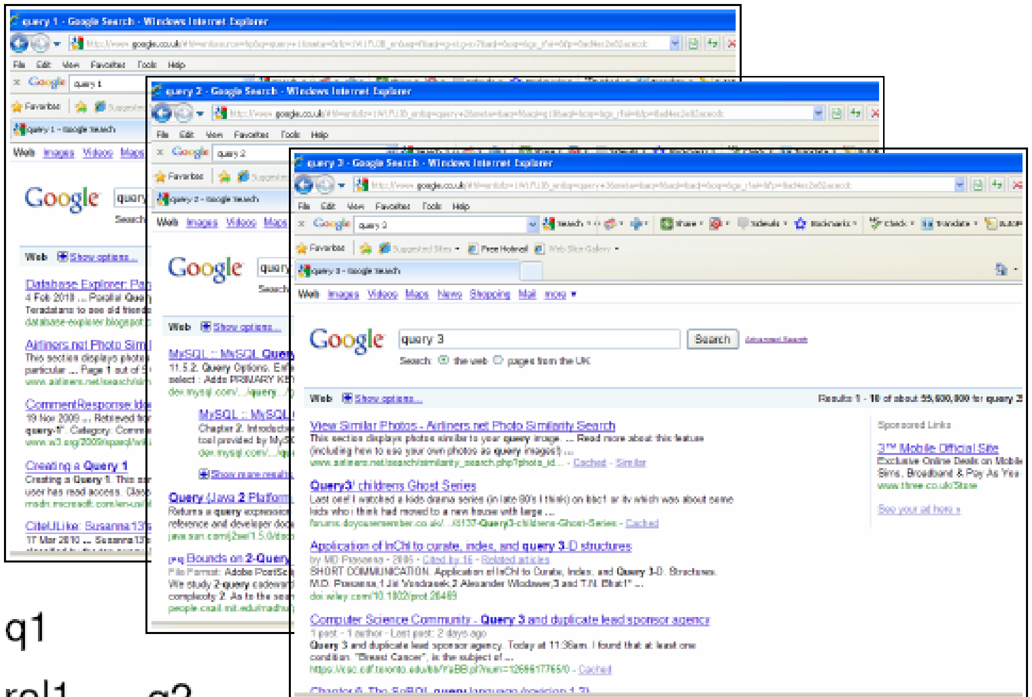


q1

rel1 q2

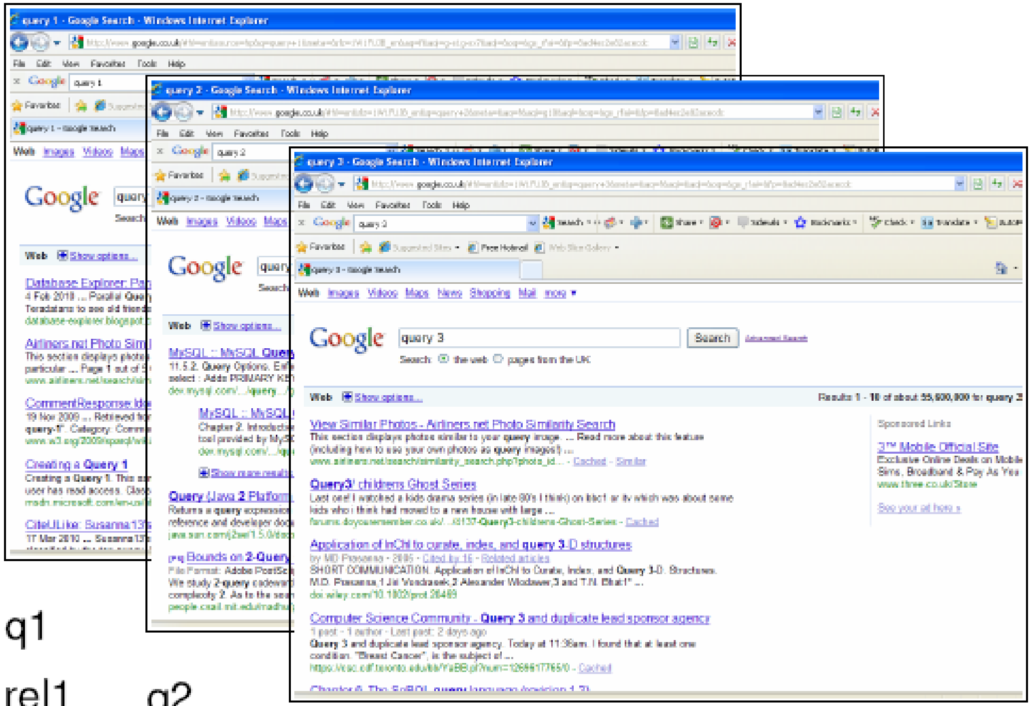
rel2 q3

Learning to Rank in IR



q1 rel1 q2 rel2 q3

Learning to Rank in IR



q1

rel1

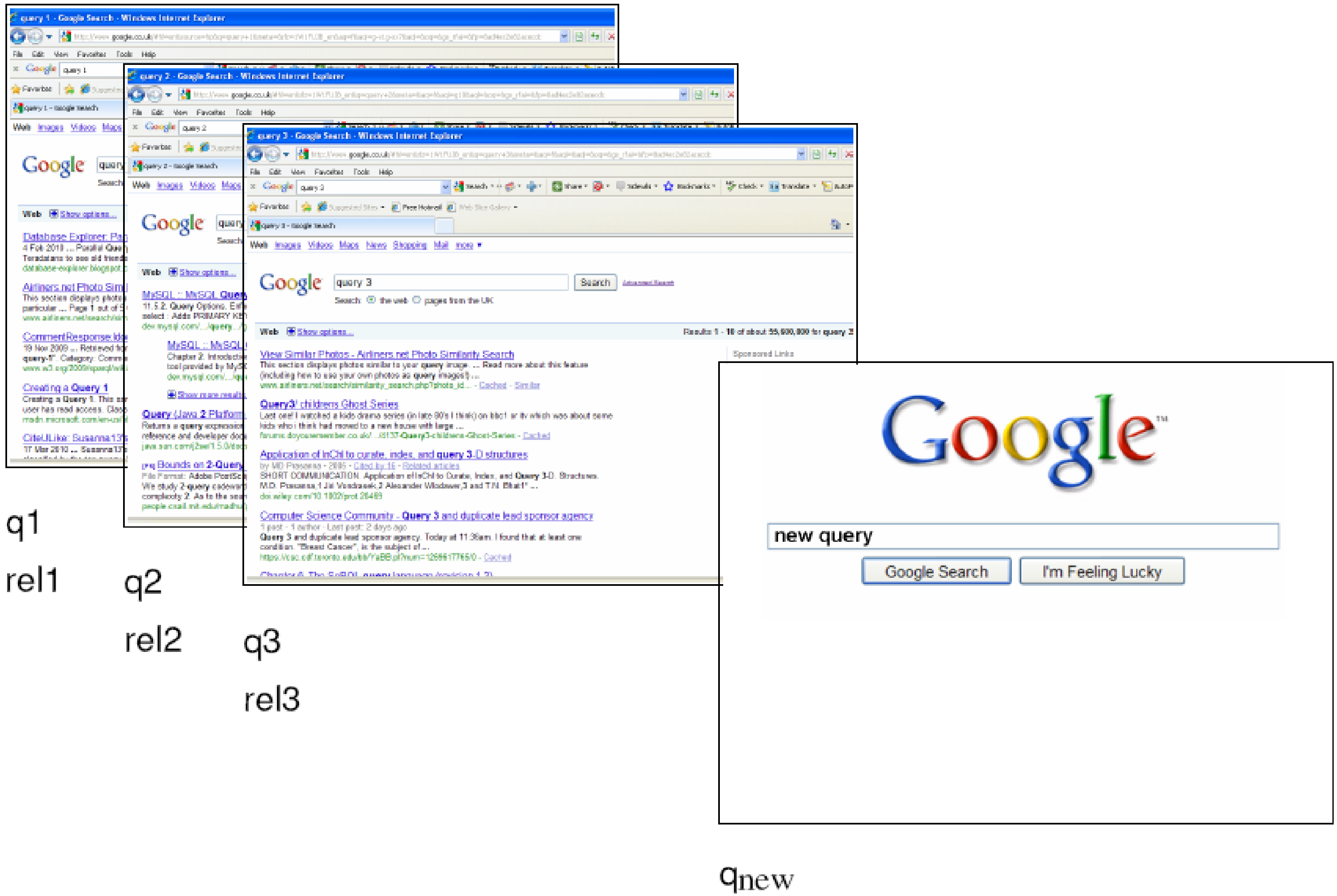
q2

rel2

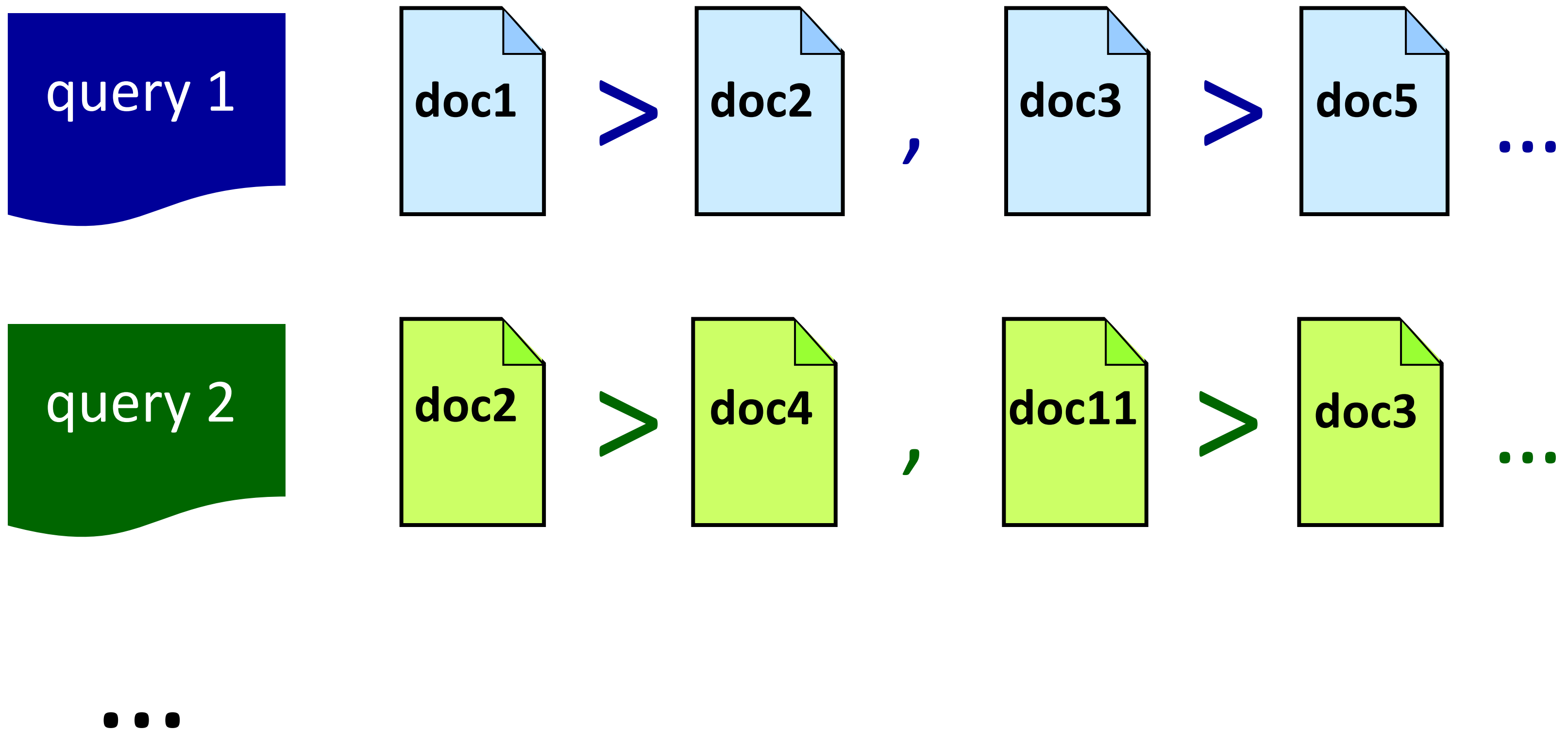
q3

rel3

Learning to Rank in IR



General Subset Ranking



General Subset Ranking

- Query space Q
- Document space D
- Query-document feature mapping $\phi : Q \times D \to \mathbb{R}^d$
- Input: Training sample $S = (S^1, \ldots, S^m)$:

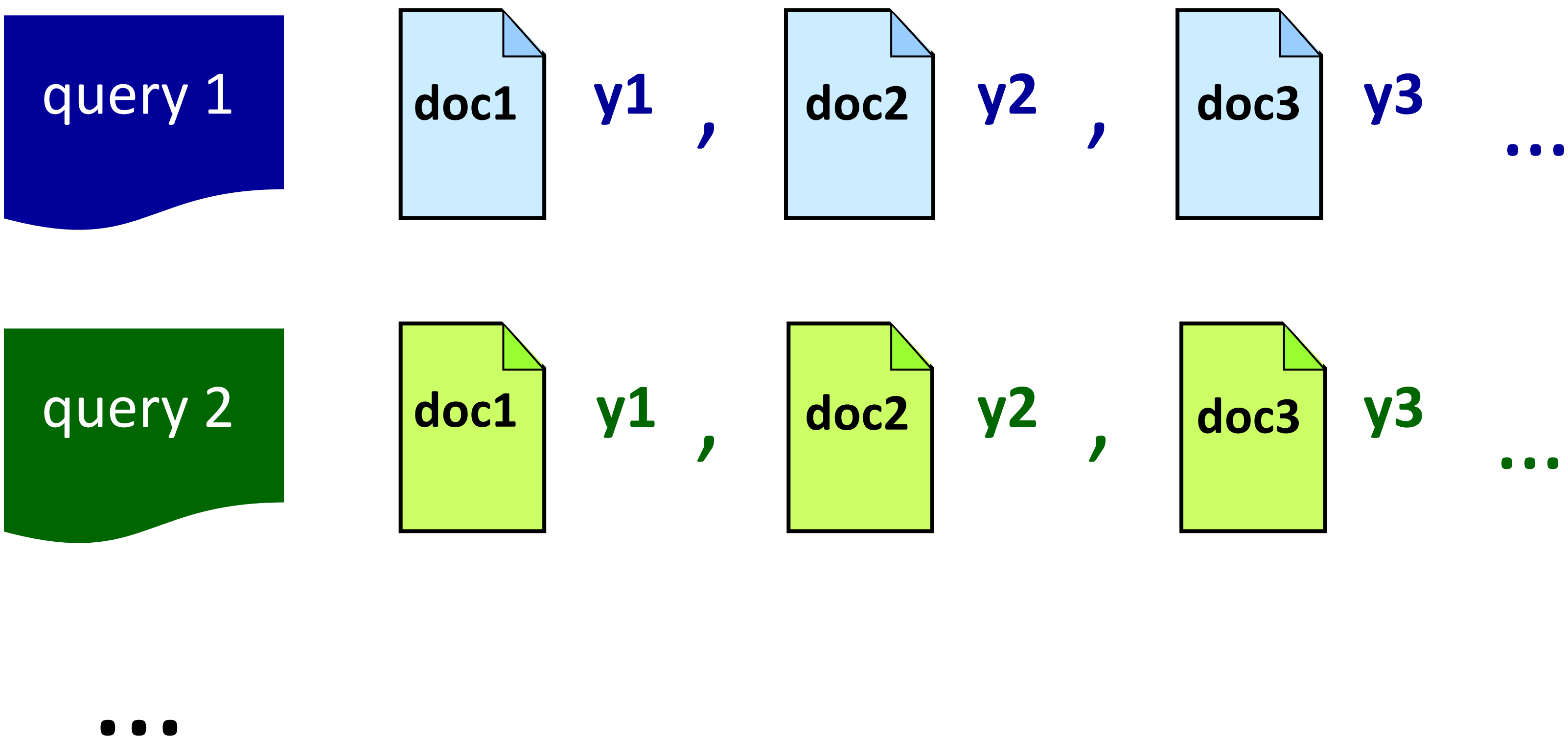
  $$S^i = \Big((\phi_1^i, {\phi_1^i}'), \ldots, (\phi_{n_i}^i, {\phi_{n_i}^i}') \Big) \in (\mathbb{R}^d \times \mathbb{R}^d)^{n_i}$$

  where

  $$\phi_j^i = \phi(q^i, d_j^i), \quad {\phi_j^i}' = \phi(q^i, {d_j^i}')$$

- Output: Ranking function $f : \mathbb{R}^d \to \mathbb{R}$

Subset Ranking with Real-Valued Relevance Labels



Subset Ranking with Real-Valued Relevance Labels

- Query space Q
- Document space D
- Query-document feature mapping $\phi : Q \times D \to \mathbb{R}^d$
- Input: Training sample $S = (S^1, \ldots, S^m)$:

  $$S^i = \Big((\phi_1^i, y_1^i), \ldots, (\phi_{n_i}^i, y_{n_i}^i) \Big) \in (\mathbb{R}^d \times \mathbb{R})^{n_i}$$

  where

  $$\phi_j^i = \phi(q^i, d_j^i), \quad y_j^i = \text{relevance of } d_j^i \text{ to } q^i$$

- Output: Ranking function $f : \mathbb{R}^d \to \mathbb{R}$

RankSVM Applied to IR/Subset Ranking

Standard RankSVM

$$\min_{f \in \mathcal{F}_K} \left[\left(\frac{1}{\sum_{i=1}^{m} \binom{n_i}{2}} \right) \sum_{i=1}^{m} \sum_{1 \le j < k \le n_i} \ell_{\text{hinge}} \left(f, (\phi_j^i, y_j^i), (\phi_k^i, y_k^i) \right) + \frac{\lambda}{2} \|f\|_K^2 \right]$$

$$\ell_{\text{hinge}} \left(f, (\phi_j^i, y_j^i), (\phi_k^i, y_k^i) \right) = \left(1 - \left(\text{sign}(y_j^i - y_k^i) \cdot (f(\phi_j^i) - f(\phi_k^i)) \right) \right)_+ ,$$

convex upper bound on

$$\mathbf{1} \left((y_j^i - y_k^i)(f(\phi_j^i) - f(\phi_k^i)) < 0 \right)$$

[Joachims, 2002]

RankSVM Applied to IR/Subset Ranking

RankSVM with Query Normalization & Relevance Weighting

$$\min_{f \in \mathcal{F}_K} \left[\frac{1}{m} \sum_{i=1}^{m} \left[\frac{1}{\binom{n_i}{2}} \sum_{1 \leq j < k \leq n_i} \ell^{\text{rel}}_{\text{hinge}} \left(f, (\phi^i_j, y^i_j), (\phi^i_k, y^i_k) \right) + \frac{\lambda}{2} \|f\|^2_K \right] \right]$$

$$\ell^{\text{rel}}_{\text{hinge}} \left(f, (\phi^i_j, y^i_j), (\phi^i_k, y^i_k) \right) = \left(|y^i_j - y^i_k| - \left(\text{sign}(y^i_j - y^i_k) \cdot (f(\phi^i_j) - f(\phi^i_k)) \right) \right)_+ ,$$

convex upper bound on

$$|y^i_j - y^i_k| \, \mathbf{1} \left((y^i_j - y^i_k)(f(\phi^i_j) - f(\phi^i_k)) < 0 \right)$$

[Agarwal & Collins, 2010; also Cao et al, 2006]

Ranking Performance Measures in IR

Mean Average Precision (MAP)

Binary Labels: $y_j \in \{0, 1\}$

$$\text{MAP}_S(f) = \frac{1}{m}\sum_{i=1}^{m}\left[\frac{1}{\left|\{j : y_j^i = 1\}\right|}\sum_{j:y_j^i=1}\text{prec}^i_{r_j^i}(f)\right]$$

$$r_j^i = \text{rank of document } d_j^i \text{ for query } q^i$$

$$\text{prec}^i_r(f) = \text{fraction of positives in top } r \text{ documents for query } q^i$$

Ranking Performance Measures in IR

Normalized Discounted Cumulative Gain (NDCG)

General Real-Valued Labels: $y_j \in \mathbb{R}$

$$\mathrm{NDCG}_S(f) = \frac{1}{m}\sum_{i=1}^{m}\left[\frac{1}{Z_i}\sum_{r=1}^{n_i}\frac{2^{y^i_{\pi^i_r}}-1}{\log_2(r+1)}\right]$$

$$\pi^i_r = \text{index of document ranked at position } r \text{ for query } q^i$$

$$Z_i = \text{normalization constant}$$

$$\mathrm{NDCG@}k_S(f) = \frac{1}{m}\sum_{i=1}^{m}\left[\frac{1}{Z_i}\sum_{r=1}^{k}\frac{2^{y^i_{\pi^i_r}}-1}{\log_2(r+1)}\right]$$

Ranking Algorithms for Optimizing MAP/NDCG

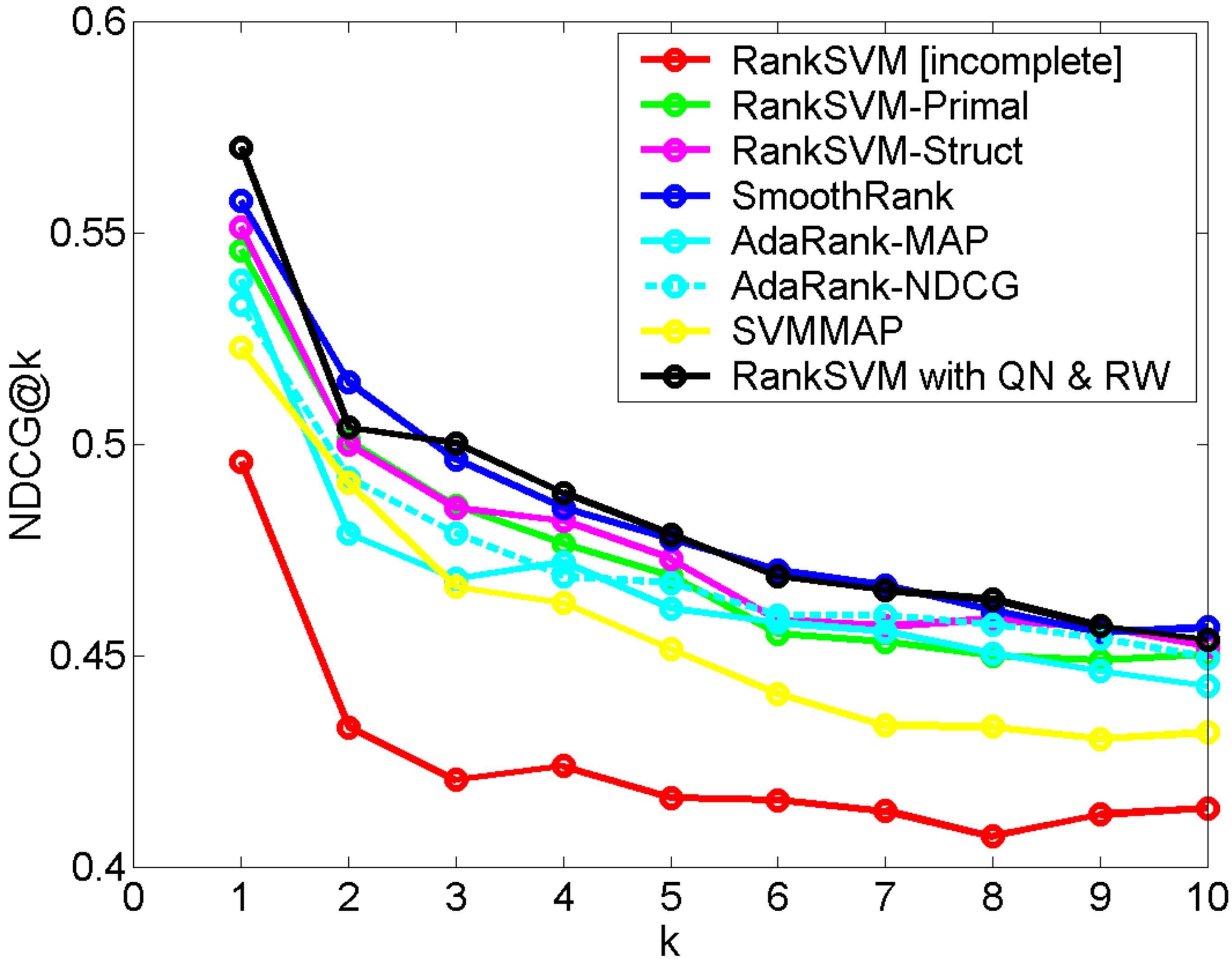
- SVMMAP [Yue et al. 2007]
- SVMNDCG [Chapelle et al. 2007]
- LambdaRank [Burges et al. 2007]
- AdaRank [Xu & Li 2007]
- Regression-based algorithm [Cossock & Zhang 2008]
- SoftRank [Taylor et al. 2008]
- SmoothRank [Chapelle & Wu 2010]

LETOR 3.0/OHSUMED Data Set

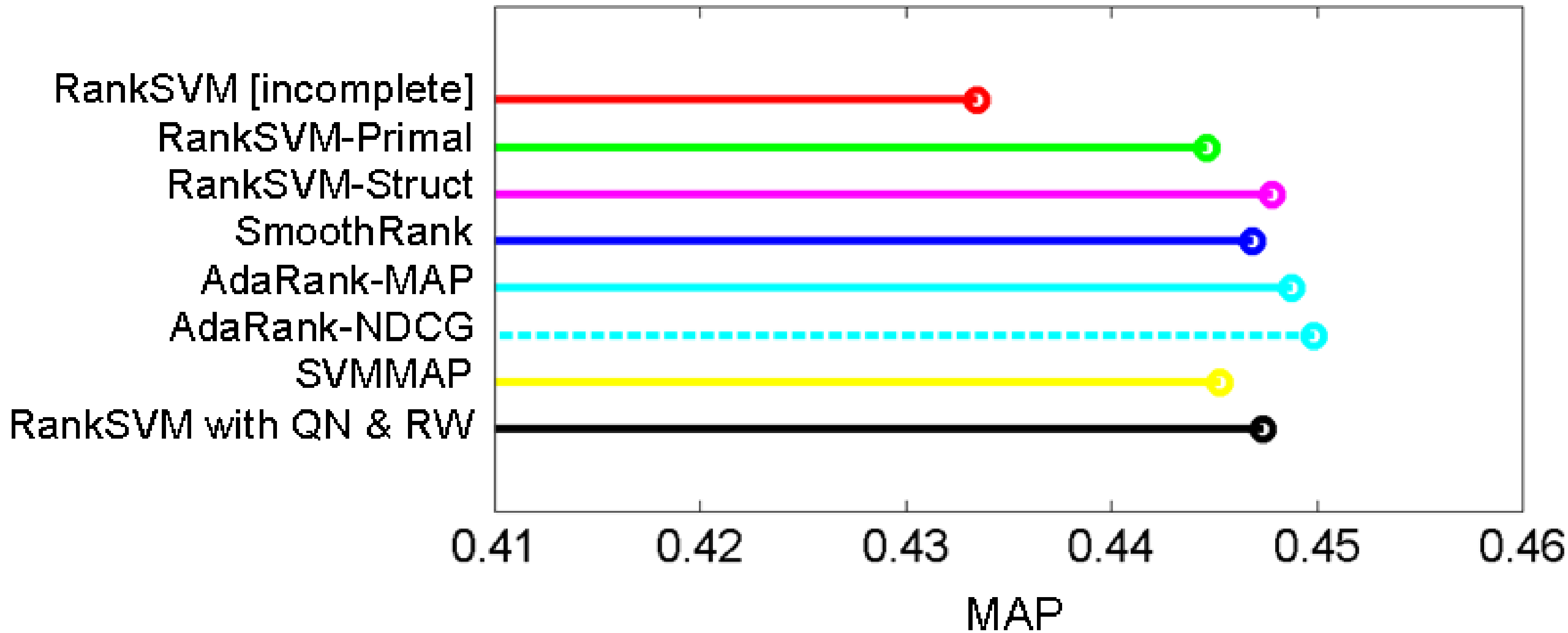
[Liu et al, 2007]

No. of Queries	Relevance Labels	Total no. of Query-Doc Pairs	Avg. no. of Docs/Query	No. of Features
106	2 : definitely relevant 1 : partially relevant 0 : not relevant	16, 140	152	45

OHSUMED Results – NDCG



OHSUMED Results – MAP



Further Reading & Resources

[Incomplete!]

Early Papers on Ranking

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S. Agarwal, T. Graepel, R. Herbrich, S. Har-Peled, D. Roth, Generalization bounds for the area under the ROC curve, *Journal of Machine Learning Research*, 6:393—425, 2005.

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S. Agarwal, D. Dugar, and S. Sengupta, Ranking chemical structures for drug discovery: A new machine learning approach. *Journal of Chemical Information and Modeling*, DOI 10.1021/ci9003865, 2010.

Other Applications

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Collaborative Filtering

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Learning to Rank

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Learning to Rank for Information Retrieval

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The Mathematics of Ranking

Tutorial Articles & Books

Tie-Yan Liu, Learning to Rank for Information Retrieval, Foundations & Trends in Information Retrieval, 2009.

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