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2006 Abel Symposium: Mathematics and Computation

By Nick Trefethen

Younger and older numerical mathematicians mingled at the 2006 Abel Symposium in Ålesund, Norway. From left to right: Anna-Karin Tornberg, David Bindel, Paul Tupper, Peter Lax, Ingrid Daubechies, and Brynjulf Owren. Photograph by H. Munthe-Kaas.

The Abel Prizes have become famous: Since 2003 we have celebrated Serre, Atiyah and Singer, Lax, and Carleson. But there is also another side to the largesse of Norway's Niels Henrik Abel Memorial Fund: the annual Abel Symposium, administered by the Norwegian Mathematical Society. Each summer, this fund supports the gathering of a small group of mathematicians in a particular area of specialization. This year, at the end of May in Ålesund, Norway, the subject was "Mathematics and Computation, A Contemporary View."

Ålesund is a beautiful seaside town at the head of a system of fjords. The flight from Oslo showed participants the grandeur of Norway's mountains, covered with snow at this season and looking as magnificent as the Alps or the Rockies. One of us had missed a connection and had contemplated renting a car and attempting the trip by road. A glance out the window showed what a mistake that would have been!

Brynjulf Owren of the Norwegian University of Science and Technology in Trondheim and Hans Munthe-Kaas of the University of Bergen organized the symposium, with assistance from Ron DeVore (South Carolina), Arieh Iserles (Cambridge), Peter Olver (Minnesota), and Nick Trefethen (Oxford). There were about 30 participants from outside Norway and a slightly smaller number of Norwegians. If you have a chance to bring together a few dozen scholars to discuss a subject as large as mathematics and computation, what do you do? The selection of people will inevitably be arbitrary. In this case the solution was a determined blend of young and old, ranging in age from about 28 to 80:

Thursday: Doug Arnold, David Bindel, Folkmar Bornemann, Franco Brezzi, Albert Cohen, Wolfgang Dahmen, Ioana Dumitriu, Alan Edelman, Bjørn Engquist

Friday: Anna Gilbert, Leslie Greengard, Tom Hou, Peter Lax, Christian Lubich, Nilima Nigam, Guillermo Sapiro, Steve Smale

Saturday: Rob Stevenson, Eitan Tadmor, Mike Todd, Anna-Karin Tornberg, Paul Tupper, Divakar Viswanath, Shing-Tung Yau.

You will see many familiar names on this list; many of those you don't recognize are destined to become familiar in the future. How to coordinate such a team? In the end it was decided that the best coordination was none at all, and the participants spoke exactly as listed above, in alphabetical order, from Arnold to Yau. Their sole assignment: to present exciting research and, if so inclined, to speculate about the future. A few attendees, including Ingrid Daubechies and Ernst Hairer, modestly declined to speak, and the Norwegians too stayed in their seats, despite the presence of such eminent figures as Petter Bjørstad, Syvert Nørsett, and Trond Steihaug.

Of course, such a small group could never fully cover a large field. In this particular collection, PDE, data analysis, and dynamics, for example, were better represented than linear algebra, optimization, or non-numerical mathematical computation.

It would not do to try to summarize 24 exciting and varied talks. Instead, let us take a look at the talks of just two of the youngest participants—former winners, as it happens, of the Putnam exam (Dumitriu) and the Fox Prize in Numerical Analysis and the SIAM 100-Dollar, 100-Digit Challenge (Tupper).

Ioana Dumitriu is a postdoc at UC Berkeley, soon to take up a position at the University of Washington. In her half-hour slot, Dumitriu decided to give two talks: her own and that of her Berkeley colleague Olga Holtz, who had planned to attend the meeting but had had to pull out for visa reasons. Dumitriu achieved this feat memorably, describing first her work with Demmel and Holtz on the question of when a family of polynomials can be evaluated to high relative accuracy. A developing theory answers this question in many cases by making use of the theory of varieties. The second half of Dumitriu's talk concerned the hot subject of the new group-theoretic algorithms for $n \times n$ matrix multiplication introduced by Cohn, R. Kleinberg, Szegedy, and Umans (see "Toward an Optimal Algorithm for Matrix Multiplication," by Sara Robinson,

SIAM News, November 2005; http://www.siam.org/news/news.php?id=174). Will the new algorithms, as conjectured, eventually bring Coppersmith and Winograd's $O(n^{2.376})$ complexity down to $O(n^{2+\varepsilon})$? Nobody knows, but here are two new results from Dumitriu's joint work with Demmel, Holtz, and Kleinberg: The group-theoretic algorithms are numerically stable; and if there exists any algorithm at all with exponent ω, then there exists a group-theoretic algorithm with exponent $\omega + \varepsilon$ for any $\varepsilon > 0$.

Dumitriu took up the challenge to look ahead, ending her talk with a controversial opinion. "In the future scientific computing will be done exclusively by people with a strong theoretical background . . . and moreover, the future is already here!"

Toward the other end of the alphabet was Paul Tupper, now a junior faculty member at McGill University. The theme of his talk, "A Difficult Open Conjecture in the Analysis of Molecular Dynamics," was a longstanding problem that has been the focus of a good deal of recent attention: What is the validity of computations of chaotic systems, when the chaos inevitably causes the individual trajectories to be incorrect? Tupper considered the particular problem of a system of particles interacting with a Lennard–Jones potential modeled by a second-order Störmer–Verlet discretization in time. In such a computation, the individual trajectories are certainly not right. Yet averaged quantities appear to converge, and in fact they seem to do so exponentially, with errors shrinking at a rate of $\exp(-\text{const}/\Delta t)$ as $\Delta t \to 0$. Can this be true? If so, how does it happen? Tupper had formulated a precise conjecture. There were people in the room who began by thinking he must be wrong and ended by thinking he might be right.

And Tupper too took up the challenge. Reminding us that his training was partly in physics, he proposed the following "physicist's view" of numerical computation:

> "Algorithms should be used, not analyzed."
>
> "If you can prove something interesting about a system, that is because the system is trivial."

It is a token of the warmth of this memorable symposium that at the end many of us were happy to agree that yes, maybe Dumitriu and Tupper were both right.

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