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A Simple Predictive Model for El Niño

By James Case

The heating and cooling of tropical oceans are major determinants of global climate. Because the Sun's rays strike more vertically near the equator, tropical waters absorb more solar heat than others, and they transfer a substantial fraction of it to the air immediately above them. The warming air expands, rises, and drifts away from the equator, making room for cooler surface air to migrate toward the equator from north and south. The resulting cycle combines with the Earth's counterclockwise rotation to form a giant heat engine that powers the high jet streams, the (usually) westward-blowing equatorial trade winds, and important ocean currents, such as the (Atlantic) Gulf Stream and (Pacific) Humboldt.

The westering trade winds gather moisture at the eastern edges of the tropical oceans and deposit it far to the west, leaving (for instance) the west coast of South America relatively dry while sustaining the rain forests of New Guinea and Indonesia. Those same trade winds push warm surface water ahead of them, sometimes causing the sea level in the western Pacific to exceed that in the eastern Pacific by as much as a meter. The resulting pressure differential causes cool, deep, nutrient-rich water to rise toward the surface of the eastern Pacific, where it has historically sustained rich commercial fisheries.

Severe disruptions in this chain of causality are politically, economically, and scientifically significant. As they cannot be prevented, early warning systems are extremely desirable.

A Phenomenon in Need of Quantitative Understanding

Toward the end of the 19th century, Peruvian fishermen began to notice that the cold north-flowing ocean current in which they usually fished was occasionally displaced by a southerly flow of warmer water; as a result, their accustomed prey was migrating southward, beyond the reach of the fleet. Because the onset of such conditions tended to occur around Christmas, they dubbed the development "El Niño" (boy child). The subsequent (and significantly less consequential) arrival of unusually cold offshore waters has come to be known more recently as "La Niña" (girl child).

El Niño and La Niña have now been officially defined as "sustained sea surface temperature anomalies in excess of half a degree centigrade across the tropical Pacific Ocean." Persistence of either condition for more than five months is classified as an El Niño or La Niña "episode." Occurrences of shorter duration are mere "conditions." Such events have occurred at irregular intervals of two to seven years, each lasting a year or two. The earliest recorded El Niño began in 1790 and lasted until 1793, but evidence of the phenomenon has been found in the fossil record as far back as the Holocene period. Recent El Niños occurred in 1986–87, 1991–92, 1993, 1994, 1997–98, 2002–03, 2004–05, and 2006–07. The El Niño of 1997–98 was unusually powerful, raising Pacific air temperatures more than 1.5º C above normal. It has been speculated that the intensity and/or frequency of El Niño events will increase with global warming, although no consensus has been reached on the matter.

In 1928, after a careful study of British Empire records, Gilbert Walker concluded that El Niño episodes are associated with the Southern Oscillation, an alternating pattern of tropical atmospheric pressures on the eastern and western sides of the Pacific. Today, the strength of this oscillation is given by the Southern Oscillation Index, a daily record of the difference in surface air pressure between Tahiti and Darwin, Australia. El Niño episodes correspond to negative values of the index, La Niña episodes to positive values. The former are ordinarily accompanied by sustained warming in the central and eastern parts of the tropical Pacific, a decrease in the strength of the cross-Pacific trade winds, and a reduction in rainfall over eastern and northern Australia.

A powerful El Niño happened to coincide with the International Geophysical Year of 1957, during which conditions at sea were monitored as never before; thanks to weather-recording stations put in place for the year, the effects of the El Niño event were documented not only on the coasts of Peru and Ecuador, but all the way across the tropical Pacific. The new data revived interest in the phenomena, and led eventually to the publication—by Jacob Bjerknes in 1969—of a seminal paper [1] describing a mechanism whereby the Southern Oscillation appeared to influence El Niño.

Computer models of the oceans began to appear in the 1970s. Early models treated the ocean as an upper layer of uniform temperature, separated by a thermocline (surface of discontinuity) from the uniformly cold depths. The models were sufficient to show that westerly winds over the equatorial Pacific could in principle cause sea surface temperatures to increase in the east. Subsequent models, allowing ocean temperatures to vary both horizontally and vertically, reproduced the main oceanographic aspects of the El Niño Southern Oscillation when provided with historical wind data for the period of interest.

A devastating El Niño in 1982–83 was blamed for some 2100 deaths worldwide and more than $13 billion in damage. In its wake, scientists concluded that a more quantitative understanding of El Niño was needed. In 1985, the Tropical Ocean–Global Atmosphere program of the World Climate Research Programme began to look at the interactions of the oceans and atmosphere worldwide. Among other things, the program deployed an array of moored and satellite-tracked buoys capable of relaying data (ocean currents, sea level, and water temperatures from the surface to as far as 500 meters below the surface, along with air temperatures, humidity, and wind direction and speed) to orbiting satellites in real time. So valuable were the data thus obtained that a system of 70 buoys known as the Tropical Atmosphere–Ocean array continues to monitor conditions in the equatorial Pacific Ocean and atmosphere.

Development of a Model

It was suggested [3] in 1976 that satisfactory models of the equatorial oceans might be constructed by regarding rapidly and randomly changing atmospheric conditions as perturbations of the slow-moving ocean currents below. What now seems to be the simplest method for doing so involves the stochastic ordinary differential equation

$$d\boldsymbol{T}/dt = \boldsymbol{BT} + \xi, \qquad (1)$$

where $\boldsymbol{T} = (T_1, T_2, \ldots, T_n)$ is a state vector in which T_i represents the sea surface temperature at location i, $\xi = (\xi_1, \xi_2, \ldots, \xi_n)$ is a vector of random perturbations, and $\boldsymbol{B}$ is an $n \times n$ matrix of real constants [4]. Such equations are simplified versions of the partial differential equation

$$\partial \boldsymbol{T}/\partial t = (\boldsymbol{u} \bullet \nabla)\boldsymbol{T} + \boldsymbol{F}, \qquad (2)$$

in which the $\boldsymbol{u} \bullet \nabla$ operator corresponds to advection by ocean currents and the random vector $\boldsymbol{F}$ aggregates the effects of wind stress forcing, evaporation, etc.

Cecile Penland, a physical scientist at the National Oceanic and Atmospheric Administration and a contributor to the Forecasters' Forum section of the *Climate Diagnostics Bulletin* of the National Weather Service, has been involved in this work for many years. In an invited talk at the 2008 SIAM Annual Meeting, she described a particularly efficient method for predicting the onset of an El Niño some seven to nine months in advance. Her model accomplishes the reduction of (2) to (1) by assuming that the temperature distributions $\boldsymbol{T}(\boldsymbol{x},t)$ of interest lie in a subspace of C^1 spanned by 20 empirical orthogonal functions [4]. The most interesting of them is depicted in Figure 1b, representing a full-blown El Niño. Figure 1a shows a "red-alert" warning condition, close approximations of which tend to precede an El Niño event by roughly eight months. The patterns 1a and 1b are in fact the right and left eigenvectors of the operator $\exp(\boldsymbol{B}\tau)$, with τ representing a lead time of about eight months.

Much computational effort has been devoted to equations like (1). Because their solutions rarely possess time derivatives, they are better written in the form

$$d\boldsymbol{x} = \boldsymbol{F}(\boldsymbol{x},t)dt + \boldsymbol{G}(\boldsymbol{x},t) \circ d\boldsymbol{W}, \qquad (3)$$

where the "$\circ$" indicates that the integration is to be performed in the sense of Stratonovitch, rather than that of Itô, and $\boldsymbol{W}$ represents a vector of independent Wiener processes. The distinction between Stratonovitch and Itô integration rules has been shown to be important; the choice depends on the nature of the problem to be solved. The choice of numerical integration techniques is also important: In hydrodynamic cases, Heun's Stratonovitch technique is more appropriate than Euler's Itô scheme. Penland's colleagues at NOAA's Climate Diagnostics Center in Boulder, Colorado, favor a pair of Runge–Kutta-type methods developed by Brian Ewald and Roger Temam in about 2003 [2]; the methods were designed to fit with the existing Galerkin architecture of currently operational weather prediction models.

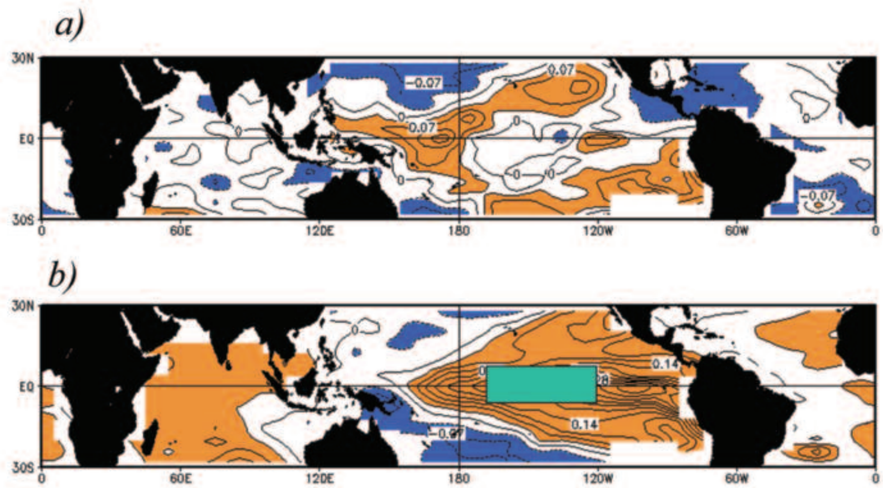

Figure 1. *The transient growth possible in a multidimensional linear system occurs when an El Niño develops. Linear inverse modeling predicts that an optimal pattern (a) precedes a mature El Niño pattern (b) by about eight months. From Cecile Penland's talk at the 2008 SIAM Annual Meeting.*

Any of the relevant temperature distributions $\boldsymbol{T}(\boldsymbol{x},t)$ can be decomposed as

$$\boldsymbol{T}(\boldsymbol{x},t) = \boldsymbol{T}_{\text{avg}}(\boldsymbol{x}) + \boldsymbol{T}_{\text{annual}}(\boldsymbol{x},t) + \boldsymbol{T}'(\boldsymbol{x},t), \quad (4)$$

in which the long-term average temperature $\boldsymbol{T}_{\text{avg}}(\boldsymbol{x})$ exceeds in magnitude both the (periodic) average annual variation $\boldsymbol{T}_{\text{annual}}(\boldsymbol{x},t)$ and the instantaneous temperature anomaly $\boldsymbol{T}'(\boldsymbol{x},t)$. It makes sense, then, to regard $\boldsymbol{T}_{\text{avg}}$ and $\boldsymbol{T}_{\text{annual}}$ as empirically known, to linearize (2) about $\boldsymbol{T}_{\text{avg}}$, and to concentrate on predicting the sea surface temperature anomalies $\boldsymbol{T}'(\boldsymbol{x},t)$.

The quantity ξ that appears in (1) is an example of "additive noise," which is at best an approximation of the real situation. Penland was quick to point out in her talk that a more accurate equation for sea surface temperature anomalies is of the form

$$d\boldsymbol{T}/dt = \boldsymbol{BT} + (\boldsymbol{AT} + \boldsymbol{C})\xi_1 + \boldsymbol{D}\xi_2, \qquad (5)$$

in which the primes have been suppressed and the components of the "noise" vector ξ_1 are multiplied by components of the state vector $\boldsymbol{T}$, introducing a complication known as "multiplicative noise" into the system. Equation (5) can nevertheless be transformed into an equation that yields to the same solution method as does (1). In particular, the best estimate of $\boldsymbol{T}(t+\tau)$ is $\boldsymbol{G}(\tau)\boldsymbol{T}_0(t)$, where $\boldsymbol{G}(\tau) = \exp\{(\boldsymbol{B}+\boldsymbol{A}^2/2)\tau\}$ and the $n \times n$ covariance matrix of predictions is of the form

$$\Sigma(t,\tau) = \boldsymbol{M}(t+\tau) - \boldsymbol{G}(\tau)\boldsymbol{M}(t)\boldsymbol{G}^T(\tau), \qquad (6)$$

in which $\boldsymbol{M}(t) = \boldsymbol{COV}(\boldsymbol{T},\boldsymbol{T})$ could be found by solving the (deterministic) matrix ODE

$$dM(t)/dt = BM(t) + M(t)B^T + Q(t), \quad (7)$$

if it were not already known empirically. Here, $\boldsymbol{B}$ is a given matrix of constants and $\boldsymbol{Q}(t)$ a given (periodic) matrix function of time. Because all the requisite computations are relatively standard, the result is a remarkably simple way to predict El Niño events seven to nine months in advance. What seems truly remarkable, given the demonstrated importance of the data collected by buoys and the like from far above and far below the ocean surface, is that a model involving but a single PDE with but a single unknown function $\boldsymbol{T}(\boldsymbol{x},t)$ of just two space variables can be so predictive.

References

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