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From Coding Theory to Robotics to Topology, Applications Abound for $SI(AG)^2$

By Michelle Sipics

With the creation this year of the SIAM Activity Group on Algebraic Geometry, or $SI(AG)^2$, SIAM has expanded the number of its activity groups to 17.

The group has elected four officers, each to a two-year term: Frank Sottile (Texas A&M University), chair; Elizabeth Allman (University of Alaska, Fairbanks), vice chair; Daniel Bates (Colorado State University), program director; and Seth Sullivant (North Carolina State University), secretary. $SI(AG)^2$ will organize minisymposia for the SIAM Annual Meeting and hold its own biennial meetings, the first of which is scheduled for 2011.

SIAM's guidelines for the creation of a new SIAG require that the subject be "of current importance, broad enough to generate significant participation, yet focused enough to assure that the SIAG's goals are achieved." For $SI(AG)^2$, areas of interest range from coding theory to robotics to topology and algebraic topology, and beyond. The new group, according to SIAM president Doug Arnold, far exceeds the requirements.

"Algebraic geometry is often viewed as inaccessible and abstruse, but its fundamental object of study is an algebraic variety—the solution set for a system of polynomial equations—which is certainly an elementary and basic notion, and one which arises in many applications," Arnold says. "There is also a great deal of interest among its practitioners, even some who are not closely connected to applications, in computing, and so computational algebraic geometry, both algorithm development and implementation, is very active," he says, noting that algebraic geometry can also play important roles in statistics, optimization, control, bioinformatics, even evolutionary biology.

Arnold, who was director of the Institute for Mathematics and its Applications at the University of Minnesota from 2001 to 2008, recalls that a widespread and growing interest in computational algebraic geometry had become apparent to the IMA by 2002; the eventual result was an annual program, from September 2006 through June 2007, on applications of algebraic geometry. Events held during the year included workshops on emerging applications in biology and statistics (a report on the evolutionary biology sessions ran in the July/August 2007 issue of *SIAM News*) and on software for algebraic geometry, along with tutorials on methods of algebraic geometry in engineering and optimization.

It was the success of the IMA annual program that led Arnold to suggest the new activity group. The idea, presented to a small group of participants in the program, met with unanimous enthusiasm, and unanimity ruled again last July, when the SIAM Board and Council approved the creation of the group.

That level of interest has continued through the launch of $SI(AG)^2$, says Frank Sottile, who points out that a number of people have already joined or rejoined SIAM because of the new SIAG.

At an AMS meeting this fall, he met Ruriko Yoshida, a statistician at the University of Kentucky. On learning of the new group, Sottile reports, Yoshida "thought it was a great idea; . . . she'd already made plans for running a symposium at [next year's] annual meeting and she'd joined SIAM—all this from her BlackBerry on the way home."

As a student of J.A. De Loera (one of the plenary speakers scheduled for the 2010 SIAM Annual Meeting), Yoshida worked on LattE, a software package for computing integer points in polyhedra; she now works on applications of algebraic methods in statistics. In a talk during the IMA program, she presented an analysis of the evolution of cool-season grasses together with symbiotic fungi; using phylogenetic trees and most-recent-common-ancestor (MRCA) pairs, Yoshida and her colleagues estimated the probability of co-divergence between the trees. She hopes to organize a double session on algebraic statistics for the 2010 Annual Meeting.

Two of the $SI(AG)^2$ officers—Elizabeth Allman and Seth Sullivant—also work in phylogenetics. Sullivant, who is one of 16 recipients of this year's Packard Fellowship for Science and Engineering, studies the geometry of statistical models, including those in evolutionary biology. Allman, like Yoshida, is interested in probabilistic modeling of sequence evolution along a tree.

Such models, Allman explains, are used in statistical analyses of aligned sequence data—like that arising from DNA from a number of different organisms. After such a model has been specified, she says, "phylogenetic trees depicting the inferred evolutionary relationships between the organisms can be constructed. The algebra comes in because the probabilistic models actually define algebraic varieties, and so tools from algebraic geometry can be brought into play." Allman presented a talk on the subject during the IMA program. Titled "Phylogenetic Models: Algebra and Evolution," the talk was especially memorable for the speaker's offer of a personally caught and smoked Alaskan salmon to the solver of an open problem involving molecular phylogenetics. (The offer still stands, Allman notes.)

Recently, she has also become interested in coalescent theory and in modeling the coalescent process in population genetics.

"Gene trees for a set of organisms are often in conflict with one another," Allman explains, "making it therefore hard to infer the species-level evolutionary relationships, since gene trees are constructed from only a small portion of an entire genome." In coalescent theory, it is possible to construct an evolutionary tree for a number of species from conflicting gene tree input.

Coalescent theory relies on mathematics to trace separate lineages back to a common ancestor. The theory allows researchers to estimate many of the parameters of population genetics, including the expected time to coalescence (the joining of lineages). Algebraic geometry is instrumental, Allman points out, because many coalescent models are parameterized by polynomials, and therefore also give rise to algebraic varieties. Interestingly, semi-algebraic considerations come up too, and to prove theorems it is usually necessary to look at both polynomial equations and polynomial inequalities.

Additional plans for next summer's SIAM Annual Meeting include a $SI(AG)^2$ minisymposium on numerical computation in algebraic geometry, organized by Dan Bates. The session, Bates says, is inspired by another plenary talk, by kinematician Charles Wampler of General Motors.

"Certain problems in the mechanical engineering subfield called kinematics can be turned into polynomial systems," Bates explains. "Over the past 20 years or so, a new branch of computational algebraic geometry, called numerical algebraic geometry, has been developed." Bates credits these developments largely to the joint efforts of Jan Verschelde, a computational mathematician, Wampler, and Andrew Sommese, an algebraic geometer.

"Though much of the original motivation of numerical algebraic geometry came from problems in kinematics, the field has now grown to include quite a few numerical and symbolic–numeric methods for studying the solution sets of polynomial systems," Bates says. "There are applications both within math and to science and engineering. . . . [Wampler] will speak on this connection, and the proposed minisymposium will likely include speakers with problems and techniques within kinematics and some from numerical algebraic geometry."

Sottile looks forward to $SI(AG)^2$ activities that can address both the connections between algebraic geometry and other fields, as Wampler's talk will highlight, as well as gaps in the current professional "coverage" of the topic. While other professional societies have groups that touch on algebraic geometry, Sottile says, those groups often leave room for broader coverage—or, as he hopes, for collaboration with the new SIAG to further their collective cause.

"There are activities [for] coding in the Institute of Electrical and Electronics Engineers, graphics in SIAM's geometric design group, and symbolic computation in the Association for Computing Machinery," Sottile says. Yet "the parts of these disciplines that involve algebraic geometry or related techniques are either niches in their larger disciplines, or do not encompass the full range of applications of algebraic geometry that we see happening right now.

"This SIAG will fill that gap, and it has the potential to make meaningful connections with these other societies."

He also emphasizes the broad focus intended for the SIAG: "We're very clear when we talk to people that [this is] really an umbrella notion. There are a lot of people using ideas from traditional parts of pure mathematics in applications, and we'd like to be a home for them now."

Similarly, Sottile hopes that $SI(AG)^2$ will help to displace the stereotypical view of algebraic geometry as only tenuously connected with applied math.

In most academic departments, what most algebraic geometers do is not considered "applications of math," he says. "This group helps legitimize what we've been doing—working on applications just like what the people officially called applied mathematicians do in our departments. Many, many people think this will help serve as a focus for their work."

For more information about $SI(AG)^2$, visit http://www.siam.org/activity/ag/.

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