From *SIAM News*, Volume 38, Number 8, October 2005

Can Science Outperform the Shamans in Global Financial Markets?

The (Mis)Behavior of Markets: A Fractal View of Risk, Ruin, and Reward. *By Benoît Mandelbrot and Richard L. Hudson, Basic Books, New York, 2004, 328 pages, $27.50.*

Benoît Mandelbrot's career path has come full circle. What began with a study of cotton prices during the late 1950s, and branched out into fractal and multifractal geometry during the 1960s through the 1990s, has lately returned to the analysis of economic time series. Along the way, Mandelbrot (by his own admission) left his mark on statistical physics, cosmology, meteorology, hydrology, geomorphology, anatomy, taxonomy, neurology, linguistics, information technology, computer graphics, and (of course) mathematics. In his latest book, with journalist Richard L. Hudson as co-author, he combines the story of his remarkable career with a message for the public at large.

Mandelbrot and Hudson first met in 1997, when the latter was managing editor of the European edition of *The Wall Street Journal*. Mandelbrot showed up one day at Hudson's Brussels office to convince him that the *Journal* was systematically misrepresenting the way that markets work by giving undue credit to conventional wisdom. Taking the path of least journalistic resistance, the publisher decided to print what the apparent "mad scientist" had to say. A year later, having endured no embarrassing consequences, Hudson invited Mandelbrot to speak about risk, ruin, and reward at a business conference the *Journal* was organizing for A-list European financiers, en-trepreneurs, and CEOs. Mandelbrot stole the show, making more sense to many in the audience than their own CFOs. According to the organizers' post-conference audience-feedback survey, Mandelbrot finished in a tie with Microsoft's Steve Ballmer—a legendary crowd-pleaser in such situations—for best speaker of the day.

BOOK REVIEW

By James Case

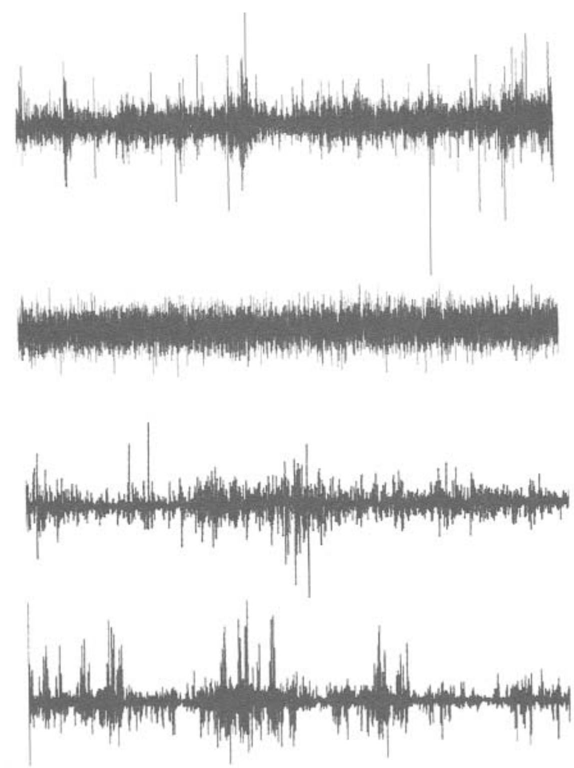

Figure 1. *Four depictions of price changes over time. The first and third are authentic; the second and fourth are fakes. Figures from* The (Mis)Behavior of Markets.

Mandelbrot and Hudson have produced an eminently readable—if somewhat quirky—book about risk, ruin, reward, and modern theories of finance. It is unusual, in a book by multiple authors, to find statements like "I was spellbound. By the end of that long Metro ride, I had the topic for half my thesis." But Hudson makes it clear in a "prelude" signed by himself alone that the ideas in the book under review are Mandelbrot's, and that the first-person voice that occurs from time to time is his as well.

Mandelbrot was born in Warsaw in 1924, and was largely home-schooled by an uncle. In 1936, the family moved to Paris, where another uncle was a professor of mathematics. During the Nazi occupation, the teenager was sent first to a small town in the countryside—where he kept busy tending horses and mending tools—and later (after acquiring a fake ID and some touched-up ration coupons) to a school in Lyons. It was there that he discovered his gift for geometry, and arrived at his conviction that pictures are an indispensable tool for mathematical research and analysis. This was not a popular conviction in the fall of 1945 at École Polytechnique, where he was able at last to begin his university education, just as the small world of French mathematics was falling under the spell of N. Bourbaki. But nothing Mandelbrot has learned then or since compels him to recant the opinion he formed in Lyons.

To illustrate the sort of graphic analysis Mandelbrot favors, the book offers four depictions of price changes over time and asks the reader to distinguish between the two real ones and the two fakes (see Figure 1). The second, which shows prices varying more or less uniformly over time, stands out from the rest. It was generated by a continuous random walk model, in which the lion's share of the price changes are drawn from the central portion of a Gaussian normal distribution. What few outliers there are barely stand out from the rest of the data. In contrast are the first and third price series, which are authentic. The first depicts changes in the price of IBM stock between 1959 and 1996; the third charts relative changes in the dollar-to-Deutschemark exchange rate over the same period. In these and all other empirical price series, price swings are highly erratic, the large ones being numerous and clustered.

The fourth series is fictitious, generated by Mandelbrot's latest model of "how financial markets work." It seems to contain about as many large changes (both positive and negative) as the two real ones, as well as a similar degree of clustering. Mandelbrot describes the model by which it was generated as "fractional Brownian motion in multi-fractal time." While the new model cannot yet be used to pick stocks, trade derivative securities, or evaluate options, he seems hopeful that further research by his successors will in time make such applications possible.

Mandelbrot observes that the modern theory of finance is largely the work of five men: Louis Bachelier, Harry Markowitz, William Sharpe, and the team of Fisher Black and Myron Scholes. He devotes an entire chapter to Bachelier, another to the other four, and a third to his case against the theory he attributes mainly to the five. Of them all, he clearly finds Bachelier the most sympathetic.

The other four, the authors write, "were secure in their professions and honored by their peers; their importance was to have made the boldest strokes that completed the canvass begun by Bachelier." They concede that many others contributed to the theory, and that they may have overlooked some deserving individuals.

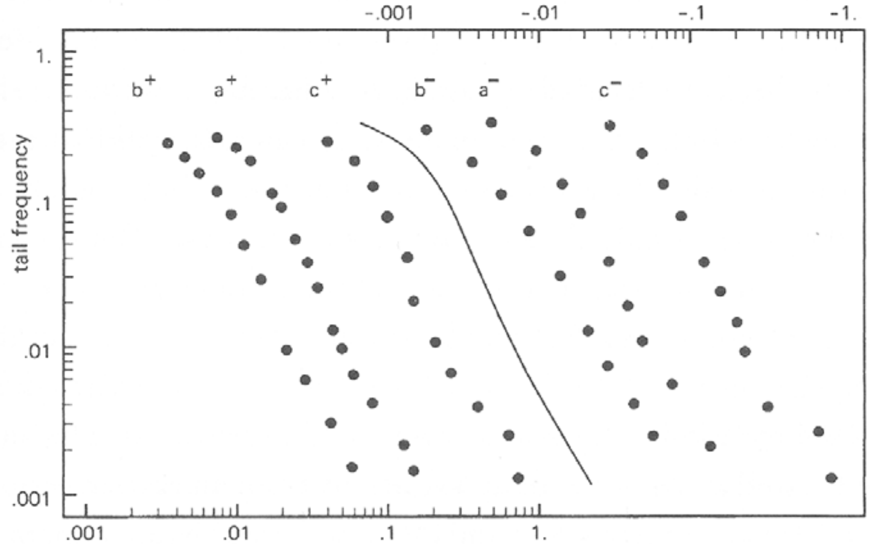

Figure 2. *A plot of three sets of data on cotton prices, with separate analyses of the positive and negative changes of logarithm of price (from work Mandelbrot did in 1963), provided evidence of power-law behavior and an L-stable distribution.*

Born in Le Havre in 1870 to well-to-do parents, Bachelier was forced to interrupt his education and enter the family (wine) business when, shortly before his 20th birthday, both of his parents died. Soon thereafter, he was drafted into the military, where, the authors tell us, his records show him to have been "a comely young man, five foot nine, with blond hair, blue eyes, and an aquiline nose." Only at the age of 22 was he able to enter the University of Paris—then open to any high school graduate and a far cry from the prestigious grandes écoles he might have attended had his parents lived—as a student of mathematics. Although his academic performance there was mediocre, he was taken on as a doctoral student by the great Poincaré, under whom he produced a thesis titled *Théorie de la Spéculation*.

Then as now, the authors write, French academia was a cliquish, elitist institution in which outsiders like Bachelier were poorly tolerated. So, original though it was, Bachelier's thesis on the evolution of the prices of government bonds at the Paris exchange (Bourse) did not impress the panel of judges before whom he was obliged to defend it. Finance was then a distasteful trade, not a fit subject for scientific inquiry. The panel awarded him a respectable "mention honorable," rather than the "très honorable" that would have been the launching pad for an august mathematical career. As a result, he spent twenty some years (with time out for military service during World War I) as a high school teacher, an adjunct lecturer at the Universities of Paris, Besançon (near the Swiss border), Dijon, and Rennes, before securing a professorial chair at Besançon in 1926. Fortunately, his thesis appeared in a major journal, and was not lost to posterity.

Only after the crash of 1929 did the study of financial markets become a respectable topic of scientific investigation. The early work—like that of Alfred Cowles during the 1930s—was mainly empirical. Not until the aftermath of World War II did theoretical work begin. Of those who participated, the four named earlier did the most to create the modern theory of finance—the one described in the book under review as a "coin tossing view" of the subject. The whole of chapter five is devoted to a case against that view.

Mandelbrot made three key discoveries, separated in each case by the passage of several years. From his original study of cotton prices—chosen because they constituted one of the few century-long price series then available to researchers—he realized that price changes are not normally distributed. Indeed, when he graphed the positive (and separately the negative) price changes against their frequencies on log–log paper, he obtained plots of the sort shown in Figure 2, suggesting that the two were related by an inverse power law. This led him to the study of Vilfredo Pareto's work on income distribution—he had never before studied any economics—and to re-examine the probability distributions whose characteristic functions are given by

$$\log f(t) = i\delta t - \gamma|t|^{\alpha}\,[1 + i\beta(t/|t|)\tan(\alpha\pi/2)].$$

Mandelbrot now calls them "L-stable distributions" in honor of Paul Levy, who isolated them during the 1930s, and whose lectures he himself had attended while a student in Paris. The most important parameter is α, which determines the fatness of the tails: When $\alpha = 2$ and $\beta = 0$, we have the standard normal distribution; when $\alpha = 1$ and $\beta = 0$, we have the Cauchy distribution with its famously fat tails.

The fatness of the tails is particularly important in calculations of the "value at risk" (VAR) when a moderately risky asset is held, such as a position in the dollar market for euro-futures. Under the assumption that price changes are normally distributed, the chance that a portfolio could fall by as much as 12% might be only 5%. Yet if a fat-tailed distribution is assumed, the expected value of losses greater than (say) 12% can suddenly become infinite, meaning that there is no longer any limit on the amount of harm such an investment can cause. A single bank can suddenly lose more than its entire worth, forcing the bank to default on its obligations to other banks, which in turn must do likewise, precipitating an international crisis. It is part of Mandelbrot's message to the general public that modern financial theory—which posits normal distributions at every turn—vastly underestimates the likelihood of such catastrophe.

Just as his work on cotton prices was beginning to cause a stir, in about 1963, Mandelbrot stumbled on his second key discovery, in the work of British hydrologist H.E. Hurst. Dispatched to Egypt in the early years of the 20th century, Hurst had promptly been assigned to a project intended to realize the vast economic potential of the Nile by generating electricity and taming its floods. His team traveled the length and breadth of the drainage basin, mapping the meanderings of its tributaries on a scale of 1:50,000, measuring the fall in altitude between specific points, and sinking screw-piles deep into the permanent subsoil to anchor flood gauges. They soon learned that, while the month-to-month variations within a given year are predictable, the year-to-year variations are not. The Nile discharges ranged from 151 billion cubic meters in a wet year like 1878–79 to a mere 42 billion in the drought year of 1913–14. Moreover, the wet years, like the dry ones, tended to occur in clusters.

Widening his perspective, Hurst gathered flood records from every conceivable source, beginning with the Nile and continuing through discharges from Lake Huron and the Truckee River in California, water levels in Sweden's Dalalven Lake, rainfall measurements for locations from Adelaide, Australia, to Washington, DC, and a host of other weather-related phenomena. In time, he assembled a total of 5915 yearly measurements of 51 natural phenomena. In almost all he discovered that the difference between the all-time-high and all-time-low annual measurements grew in proportion to an unexpectedly large power of the number of years in the series. If the measurements were normally distributed about their means, that power would be about 1/2. Yet he kept finding (as for the Nile) powers in excess of 2/3.

Hurst devised a statistic for measuring the degree of clustering in a time series and published his results between 1951 and 1955, when well into his seventies. When Mandelbrot learned of the work, he concocted a theory with which Hurst's findings were consistent and incorporated it into his theory of price variations. What matters to anyone concerned with long-term damage control, Mandelbrot realized, is not only the size of the largest variations, but also the sequence in which they occur.

Mandelbrot's third key insight concerns the difference between clock time and (local) market time. Some days on the trading floor are hectic: Phones ring off the hook, prices fluctuate wildly, gains and losses mount. Time seems to fly by. It is on such days that fortunes are won and lost. On other days, customers seem catatonic. Trading volumes are thin, prices hardly change, long lunches appeal. Nobody sees a chance to make any real money, and time hangs heavy on the hands of one and all. The 11th chapter of the book under review, "The Multifractal Nature of Trading Time," presents Mandelbrot's analysis of this phenomenon, and his unconventional wisdom concerning financial markets.

Dismissing the "standard" analyses of financial time series in terms of Brownian motion and Gaussian random variables as "not merely wrong" but "dangerously wrong," he issued a challenge to Federal Reserve chairman Alan Greenspan, New York State attorney general Eliot Spitzer, and then SEC chairman William Donaldson to provide funds for fundamental research on financial markets. A mere 5% of the $432.5 million pledged in 2003 by Wall Street firms for "independent" research as part of their settlement of fraud charges brought by Spitzer's office would, in his opinion, be a good place to start.

Though convinced that most of the settlement money will go to the flock of media and ratings firms prepared to spin off "independent subsidiaries" as soon as the eligibility requirements for sharing the loot are announced, Mandelbrot still dares to hope that some will trickle down to industry and academic researchers inclined to investigate the fat-tailed distributions, long-term time dependence, bursts of volatility, abrupt (even discontinuous) price movements, and other non-Brownian phenomena that he and others have documented over the years. A failure to fund more such research will, in his opinion, expose the global financial system to additional crisis risk of the sort that became almost commonplace during the 1990s.

Concerning such risk, Mandelbrot quotes the notable/quotable Lawrence Summers—former secretary of the treasury and current president of Harvard University—to the effect that the global financial system endured no less than six genuine crises during the generally prosperous 1990s: Mexico in 1995; Thailand, Indonesia, and South Korea in 1997–98; Russia in 1998; and Brazil in 1998–99. He might also have mentioned the narrowly averted bankruptcy of Long-Term Capital Management, the hedge fund that threatened during the summer of 1998 to bring down several of the larger banks in Germany, Switzerland, and the U.S. Mandelbrot sees no reason to suppose that the risk of such disasters will abate as long as the world entrusts its financial well-being to shamans—his characterization of central bankers—instead of scientists.

Mandelbrot is by no means sure that science is ready to outperform the shamans. Indeed, he is quick to acknowledge an opinion expressed by Wassily Leontief, a Harvard economist and 1973 Nobel Prize winner: "In no field of empirical enquiry has so massive and sophisticated a statistical machinery been used with such indifferent results." Mandelbrot maintains only that researchers like himself have made important strides toward understanding the monumental risks inherent in global financial markets, and that more such research is urgently needed.

James Case writes from Baltimore, Maryland.