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SIAG/LA Prize Recognizes More Accurate, Faster SVD Algorithm

By James Demmel

Zlatko Drmač of the University of Zagreb, Croatia, and Krešimir Veselić of Fern Universität in Hagen, Germany, are the recipients of the 2009 SIAM Activity Group on Linear Algebra Prize. The researchers were recognized for their two-part paper "New Fast and Accurate Jacobi SVD Algorithm" [1,2]. The prize was awarded during the SIAM Conference on Applied Linear Algebra, held in Monterey Bay, California, October 26–29.

"The paper by Drmač and Veselić represents the culmination of a long thread of research on accurate linear algebra algorithms," the prize committee wrote in the citation. "The goal of all these algorithms is to compute some quantity of interest, like singular values, with high relative accuracy. This is in contrast to the usual accuracy guaranteed by normwise backward stability: high absolute accuracy, which may guarantee no correct digits at all in tiny singular values. The algorithm designed by Drmač and Veselić is the first such algorithm that is not just more accurate but faster than the conventional backward stable algorithm, in this case reduction to bidiagonal form followed by bidiagonal QR." The members of the prize committee were Michele Benzi, chair (Emory University), Inderjit Dhillon (University of Texas, Austin), Peter Lancaster (University of Calgary, Canada), Julio Moro (University Carlos III, Madrid, Spain), and Henk van der Vorst (University of Utrecht, the Netherlands).

Zlatko Drmač (left), co-recipient (with Krešimir Veselić, not shown) of the 2009 SIAG/LA Prize, accepted the congratulations of prize committee chair Michele Benzi in October at the SIAM Conference on Applied Linear Algebra. Photo by Nicholas Higham.

The singular value decomposition is one of the most widely used matrix factorizations in science and engineering. The first effective algorithm, published by Golub and Kahan in 1965, has been extensively researched and optimized ever since, and so the ability to make significant improvements today is remarkable. As for many other matrix algorithms, there are two metrics of success: accuracy and speed. The traditional accuracy metric for matrix computations is backward stability: getting exactly the right answer for a problem differing from the input slightly in norm. In particular, this means that the larger singular values are computed quite accurately, whereas the tinier ones may have no leading digits correct.

But several authors began to identify special situations that warranted much more accurate computation of the SVD, because in these situations small relative uncertainties in the matrix entries caused only small *relative* uncertainties in the singular values, no matter how tiny. Several proposed algorithms achieved this higher accuracy; one of them actually went faster than the conventional SVD (work for which this author and W. Kahan received the second SIAG/LA Prize, in 1991). But the more widely applicable high-accuracy algorithms, including variants of the very old Jacobi method, were woefully slower than the algorithms that were backward stable only, and thus little used. In the words of Kahan, "the fast drives out the slow, even if the fast is wrong."

The remarkable achievement of Drmač and Veselić is to attain the high accuracy described above while going *two times faster* than the longtime standard workhorse SVD algorithm: bidiagonalization followed by QR iteration to compute all the singular values and vectors. Their success resulted from a number of innovations and optimizations, some applied to the Jacobi method itself and others as preconditioning steps designed to make Jacobi converge more quickly. For good measure, they found, and fixed, a subtle bug in the LAPACK implementation of the Businger–Golub QR decomposition with column-pivoting that dates back to its LINPACK implementation. Their code was released as part of LAPACK version 3.2.1.

Zlatko Drmač presented the work at the conference. Organized by a committee chaired by Esmond Ng of Lawrence Berkeley National Laboratory, the conference was the tenth in a successful series that began in Raleigh in 1982. The next SIAM Conference on Applied Linear Algebra will take place in 2012 in Valencia, Spain.

References

[1] Z. Drmač and K. Veselić, *New fast and accurate Jacobi SVD algorithm: Part I*, SIAM J. Matrix Anal. Appl., 29 (2008), 1322–42.
[2] Z. Drmač and K. Veselić, *New fast and accurate Jacobi SVD algorithm: Part II*, SIAM J. Matrix Anal. Appl., 29 (2008), 1343–62.

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