From *SIAM News*, Volume 43, Number 4, May 2010

The Classic Beauty of Symmetry With a Modern Twist of Chaos

Symmetry in Chaos: A Search for Pattern in Mathematics, Art, and Nature, Second Edition. *By Michael Field and Martin Golubitsky, SIAM, Philadelphia, 2009, xiv+199 pages, $59.00.*

There's so much to like about this book. It's packed with stunning images, but beyond that, it's written with real heart and personality. The examples are apt, the explanations are clear and friendly, there are little jokes sprinkled throughout, and the mathematical ideas—symmetry, group theory, bifurcation theory, chaos—are inherently appealing, especially in these authors' hands.

BOOK REVIEW
By Steven Strogatz

The subject is dynamics in the presence of symmetry. Part of the motivation comes from scientific applications, ranging from chemical pattern formation to the behavior of turbulent fluids. But the essence of the book is more playful than that. It's an exploration of the links between art, nature, mathematics,and computation.

In mathematical terms, the book focuses on the simplest setting in which both symmetry and chaos arise: the dynamics of iterated mappings of the complex plane, where the mappings are chosen to possess various combinations of rotational, reflectional, and translational symmetries. For example, the most basic family of maps the authors consider has the form

$$F(z) = (\lambda + \alpha z\bar{z} + \beta \operatorname{Re}(z^n) + i\omega)z + \gamma \bar{z}^{n-1},$$

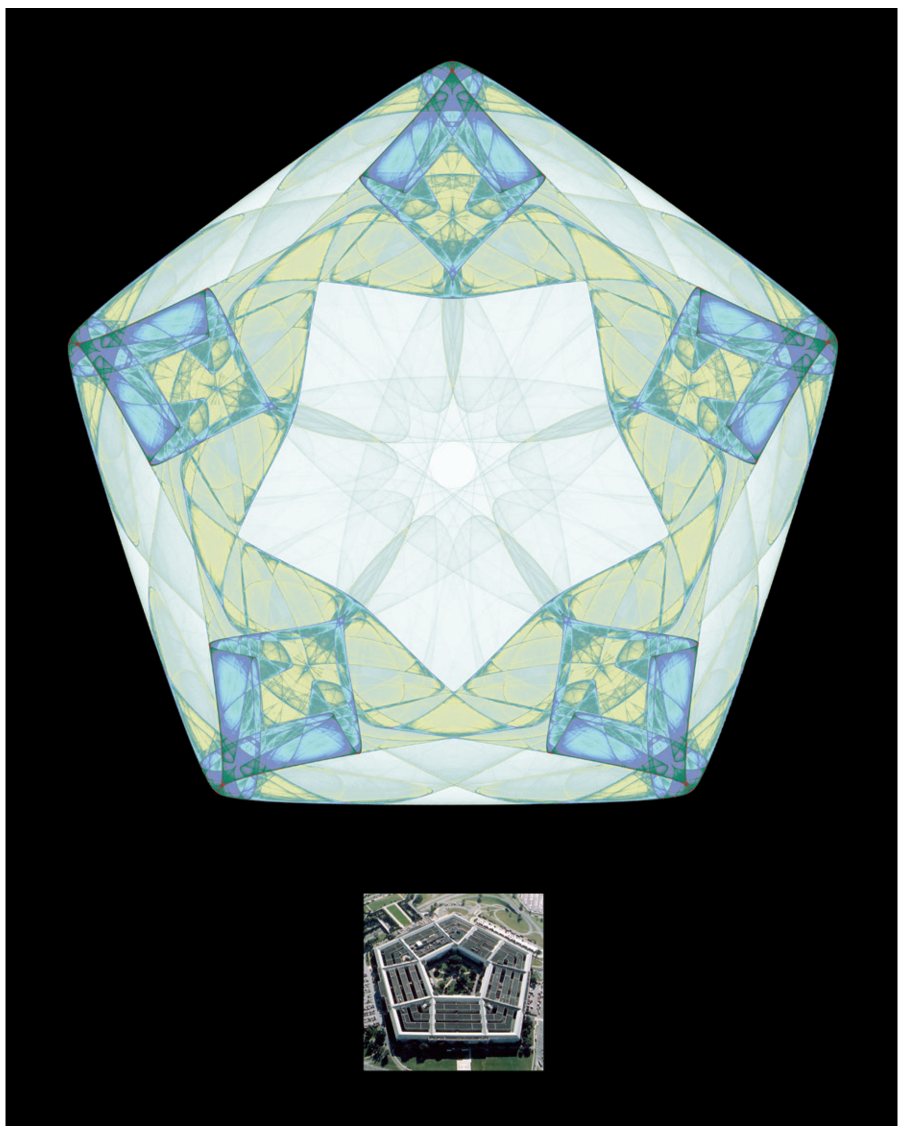
where α, β, γ, λ, and ω are real parameters and the overbar denotes complex conjugate. This formula has n-fold dihedral symmetry (the full symmetry of the regular n-gon) if $\omega = 0$, or n-fold rotational symmetry if $\omega \neq 0$.

Then, for an artfully chosen set of parameters, Field and Golubitsky choose an initial point z_0 and generate its orbit via $z_{k+1} = F(z_k)$. After a few thousand iterates, the system settles onto a strange attractor and hops chaotically on it. Of course, a slightly different initial condition would lead to a wildly different orbit—this is chaos, after all—but that doesn't matter in their examples; the attractor stays the same, and so do its long-term statistical properties.

The idea, then, is to probe the statistics of the attractor. As the orbit hops around on it, are some parts of the attractor visited more often than others, and if so, how much more often? To visualize this, Field and Golubitsky color the computer screen as follows: After discarding an initial transient, they iterate the map anywhere from 20 to 100 million times, and then count how many times a given pixel on the screen is visited. By coloring the pixels according to how often they're hit, the authors obtain gorgeous images of the attractor's invariant measure.

But this description doesn't do justice to the book. You have to look at it, and read it. The images have a classical beauty, thanks to their symmetry, but with a modern twist of chaos and its attendant intricacy.

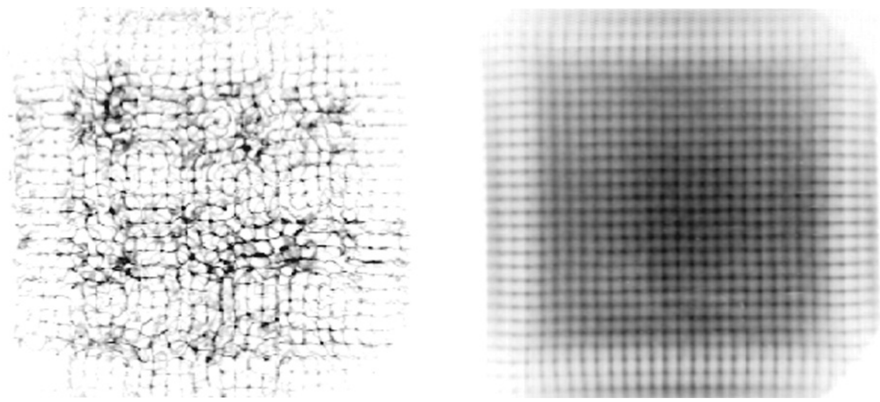
In case you're wondering how this edition compares to the first, I'd say a great book has gotten even better. The changes are slight but significant. There's a new discussion of work by Jerry Gollub and his co-workers about observations in Faraday fluid experiments of the phenomenon of "patterns on average," a nifty theoretical idea that originated in these abstract studies of iterated maps and has now been seen in the lab. There are also many little meticulous improvements over the first edition—for instance, the images in a figure comparing the symmetry of a three-pointed star with its symmetric chaotic counterpart have been reoriented so that their lines of symmetry now coincide. Dozens of similar touches appear throughout the book—they



The Pentagon and the Pentagon Attractor. The latter exemplifies the visual puns sprinkled throughout Symmetry in Chaos. *What you see may depend on your political persuasion.*

may be small individually, but collectively they make the reader's life easier. I wish all authors did that.

Steven Strogatz is a professor of applied mathematics and director of the Center for Applied Mathematics at Cornell University.



New to the second edition: shadow graphs from a time-instantaneous high-amplitude picture (left) and time-average picture (right) from the Faraday experiment of Jerry Gollub and colleagues. Illustrations from Symmetry in Chaos.