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Numbers: Their Genealogies, Personalities, and Functions

Numerical Notation. *By Stephen Chrisomalis, Cambridge University Press, Cambridge, UK, 2010, 486 pages, $95.00.*

What's your number?
Cucumber.
—Children's Song

All is number.
—Pythagoras

BOOK REVIEW
By Philip J. Davis

One supposes that thousands of years ago mathematics took off when humans moved from vague quantitative expressions, such as "a pile of," "a lot of," "many," "innumerable," "a myriad of," "a host of golden daffodils," to exact designations, such as "sixty-eight coins in the fountain." But counting, which led to arithmetic, may have shared the primitive stage with spacing and distancing, which led to geometry. The combination of the two characterizes a significant part of today's mathematics.

Numbers, numbers, numbers. You can enunciate numbers in natural languages: "sixty-two" or "zwei und sechzig." You can write them down in some system: 62 or LXII. You can combine them arithmetically: LXII + X = LXXII. You can invent new names and representations for them: 10^{100} = a googol, 10^{135} = a quinquagintillion. You can compare numbers: $3 < 4$. You can estimate them: The population of the world is around six billion.

What are numbers for? The cardinals answer the question "how much?" and are associated with the arithmetic operations; ordinals answer the question "how far along?" and are associated with the notions of less than and greater than. Numbers can also serve as identification tags, answering "which of many?" and in this capacity are associated with the notions of equality and non-equality.

But quantity, order, and tags hardly exhaust the uses—often simultaneous—to which humans have put numbers. Numbers can embody good or bad luck. In craps, 7 and 11 are good; 2, 3, and 12 are bad. Numbers can be associated with alphabets: a = 1, b = 2, . . . ; $\alpha = 1$, $\beta = 2$ . . . . One can then proceed from specific words or people to numbers carrying non-quantitative interpretations. "The number of the beast" = 666, and it is an easy exercise to derive 666 out of "Nero Caesar" or out of the names of other people you love to hate.

According to an ancient Chinese tradition, even numbers embody the female principle in nature, odd numbers the male. Over the years and over the continents, numerology has remained alive, spawning many systems of mystic beliefs and hosts of believers and relevant practices. Numbers can be the carriers of religious imagery that goes beyond either of the beliefs just mentioned, and has engendered long-lasting and acrimonious disputes, such as that concerning the nature of the Trinity. Today, in the USA, 9/11 and 911 evoke different actions and reactions. Humans have thus exhibited unlimited imagination in having their numbers serve a wide variety of purposes and elicit a wide variety of responses. It's hard to imagine civilized life without numbers.

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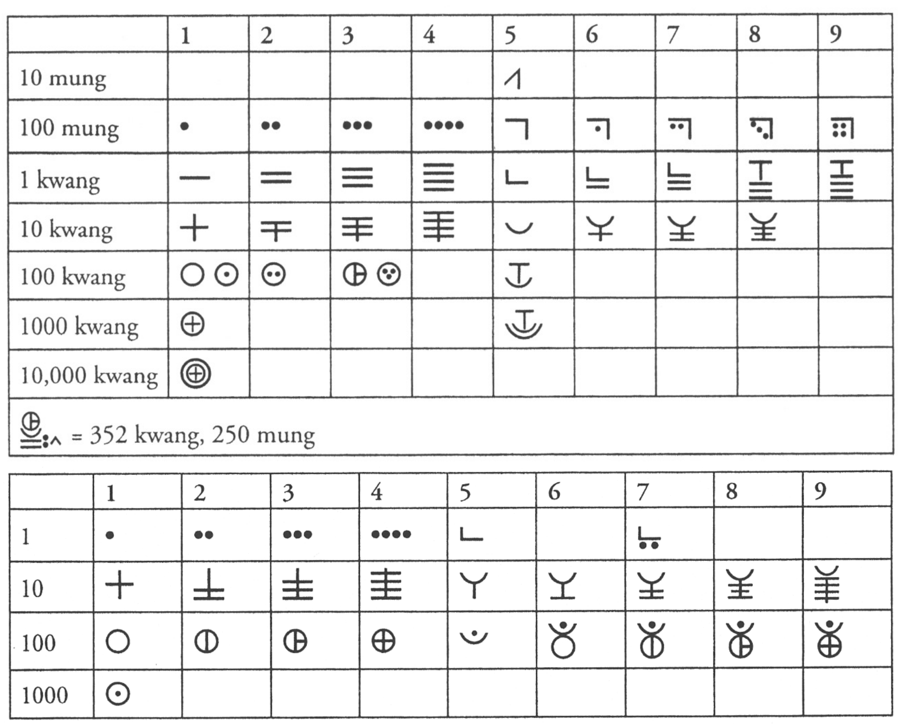
There is hardly a civilization, however primitive or advanced, that has not had some sort of number system, and this splendid book, written by Stephen Chrisomalis, a linguistic anthropologist at Wayne State University, is exceedingly rich in examples and discussions. The book details easily a hundred such systems existing over space and time, with discussions of their origin, development, transmission, social utility, and inner meaning.

Chrisomalis sketches a typology for number systems. Within it we have, among others, the "additive–cumulative Roman," as in 1492 = MCCCCLXXXXII or—more compactly, if we allow subtraction—MCDXCII. Or we have the "additive–multiplicative traditional Chinese," as in $1492 = 1 \times 1000 + 4 \times 100 + 9 \times 10 + 2$ (with the Chinese symbols for the digits).

The book details the evolutionary history of eight major number phylogenies—Hieroglyphic, Levantine, Italic, Alphabetic, South Asian, Mesopotamian, East Asian, Mesoamerican—with a full chapter devoted to each. The chapter



Examples of Ryukyu numerals, a Japanese numerical notation system used in the late 19th century: money (top) and firewood bundles (bottom). From Numerical Notation.

on East Asian numbers, which can be taken as typical in its coverage, gives the graphical symbols employed in eight different notational systems. To this is added a great deal about the history of the systems and the uses to which they have been put.

With this book, Chrisomalis has made a significant contribution to the social and historical analysis of number representations. He identifies 17 factors that have influenced changes in numerical notational systems. Among them are: New functions sometimes make old systems obsolete. A system might be adopted because a large number of nearby social groups have been using it productively. A system might be imposed under conditions of political, economic, or cultural domination and, when challenged, might be defended, again for cultural or political reasons. A system might be retained for reasons of literary prestige (such as Roman numerals on buildings).

Numerical notation systems have lifetimes, Chrisomalis points out; in an interesting table he lists the lifetimes of 28 systems. As examples: Use of the Egyptian hieroglyphic system extended from 3250 to 400 BCE, that of the Armenian from 400 CE to 1650. Thirteen of them, including the Hebrew alphabetic, the classical Roman, the Cyrillic, the traditional Chinese, and, of course, the Arabic positional that we employ, are alive and well today.

Expounding what he calls the macrohistory of numerals, Chrisomalis writes that the period from 3000 to 800 BCE "saw the invention of the earliest systems in Egypt, Mesopotamia, the Indus Valley and China." He then fast forwards to the period from 1500 to the present: "This was the only period in which there was a prolonged decline in the number of systems in use. . . . The simplest but overly simplistic explanation for [the decline views it] as a consequence of European imperial expansion." To this he adds: "The rise of the capitalist world-system was the most significant event in the history of numerical notation." Combine this with Max Weber's* thesis on the rise of capitalism and the "Protestant ethic" and you have an interesting theory of mathematical history to chew over.

The book mentions such ancient aids to computation as pebble boards, abacuses, and tables, but hardly discusses the manner in which electronic computers and other recent technologies have influenced the representation of numbers. In this connection one might cite the new prominence given to binary and hexadecimal, as well as floating-point, representations, though they can be traced back to some mechanical computing machines of the 1930s. The bar codes now printed on practically all products for identification purposes are representations of numbers adapted for scanners.

Much more esoterically, we also have "exact" representations of algebraic numbers *r*, given as triples [P,L,U], where P is a polynomial that has *r* as a root and L and U are rational lower and upper bounds that suffice to distinguish *r* from any other root of P. Accompanying this, there has emerged a large literature of "computational algebra systems" discussing the manipulation of such expressions.

The conclusion, easily reached, is that as technology progresses and as the social and physical worlds become increasingly mathematized, numerical notation systems will grow apace, both in quantity and in variety. Their exposition should engage and challenge future authors.

*Max Weber (1864–1920), famous German sociologist and political economist.

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